MULTIPLE-MEASUREMENT VECTOR MODEL AND ITS APPLICATION TO THROUGH-THE-WALL RADAR IMAGING

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ABSTRACT
This paper addresses the problem of Through-the-Wall Radar Imaging (TWRI) using the Multiple-Measurement Vector (MMV) compressive sensing model. TWR image formation is reformulated as a compressed sensing (CS) problem, seeking a sparse representation in the spatial domain. In traditional CS-based through-the-wall radar imaging (TWRI) methods, the measurement matrix is vectorized so that a single measurement vector (SMV) model is applied to generate a sparse solution, which represents a scene comprising point-like targets. For multiple measurement TWRI problems, the SMV model may produce a sub-optimum sparse solution. On the other hand, the proposed MMV model for TWRI generates a more sparse scene by processing all the measurements simultaneously. To evaluate the effectiveness of the proposed method, it is applied to fuse multiple polarization data to form the radar image. Based on simulated data with different number of measurements and noise levels, the proposed MMV-based TWRI method produces better TWR images in terms of image quality and detection accuracy.

Index Terms— Through-the-Wall Radar Imaging, Multiple-Measurement Vectors, Compressed Sensing, Multiple Polarizations.

1. INTRODUCTION
Through-the-Wall Radar Imaging (TWRI) is emerging as a viable sensing and imaging technology, which makes use of electromagnetic waves below the S-band to penetrate through building wall materials to illuminate an indoor scene. Much attention has been paid to TWRI in recent years due to its numerous civilian and military applications (see [1, 2, 3] and references therein for more details). In TWRI, the delay-and-sum (DS) beamforming method is commonly employed for image formation [4]. To achieve high resolution in both down-range and cross-range in stepped-frequency radar systems, large bandwidth signal and long antenna arrays are required. However, this increases the time of data acquisition and processing. Therefore, to have a practical TWRI system, there is a need to develop fast and efficient image formation techniques.

Recently, Compressed sensing (CS) has been considered for TWR image formation due to its ability to perform data acquisition and compression simultaneously [5, 6, 7]. Until now, most of the CS methods for TWR image formation are based on Single-Measurement Vector (SMV) model, in which the observation/measurement is processed as a vector. This is appropriate for one imaging system at one location using single polarization and assuming frequency-independent target RCS. Deviations from these assumptions require the extension of the TWRI problem by considering multiple measurements.

The contribution of this paper is the application of the multiple-measurement vector algorithm [8] for reconstructing TWR images. The proposed method is an enhancement of the traditional SMV-based TWRI image formation method when multiple measurements are available. The validity of the proposed MMV model is evaluated in fusing multiple-polarization data for TWRI. It is the first attempt to solve the TWR image formation using the MMV model.

The remainder of the paper is organized as follows. Delay-and-sum beamforming method for TWRI is described in Section 2. Section 3 presents the new MMV model for image formation. In Section 4, experimental results are given to show the effectiveness of the MMV method compared to SMV. Finally, Section 5 presents concluding remarks.

2. TWR IMAGE FORMATION
In traditional TWRI, the scene is scanned at several locations, then delay-and-sum (DS) beamforming is applied to the collected measurements to form the image [4]. In the stepped-frequency approach, the scene is scanned with an M-array of co-located transmit/receive antennas, each operating with a discrete set of N frequencies, \( \{ f_n : n = 0, 1, \ldots, N - 1 \} \). In two-dimensional imaging, the antenna array is placed parallel to the wall and the scene is scanned in a horizontal (or
vertical) direction. Here we assume a monostatic operation and the wall parameters are known or can be estimated. Given $P$ targets in the scene, the received signal at the $m$th antenna location for a transmitted frequency $f_n$ can be expressed as

$$z_{mn} = \sum_{p=1}^{P} \sigma_p \exp(-j2\pi f_n \tau_{m,p})$$

(1)

where $\sigma_p$ is the reflection coefficient of the $p$th target, and $\tau_{m,p}$ denotes the round-trip propagation delay between the $p$th target and the $m$th transmit/receive antenna.

To form the image, the target space is partitioned into $N_x \times N_y$ discrete positions (or pixels), where $N_x$ and $N_y$ denote the spatial resolutions along the $x$- and $y$-axis, respectively. In conventional DS beamforming, the complex amplitude of the $kl$-th image pixel $I_{kl}$ is obtained by combining all the signals collected at the $M$ receivers,

$$I_{kl} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} z_{mn} \exp(j2\pi f_n \tau_{m,kl})$$

(2)

where $\tau_{m,kl}$ is the focusing delay between the $kl$-th pixel and the $m$th antenna location.

More recently, compressed sensing (CS) theory has been applied to TWRI due to the increasing demand for higher image resolution and faster data acquisition. The CS-based method can be applied in the spatial or frequency domain to generate a sparse scene representation from a small number of selected measurements [5, 7]. Assuming the scene contains a set of $P$ point-like targets whose number is much smaller than the total number of pixels ($P \ll N_x \times N_y$), then TWRI image formation can be reformulated as a CS problem. Let $s_{kl}$ denote an indicator function, defined as

$$s_{kl} = \begin{cases} \sigma_p, & \text{if a target } p \text{ exists at the } kl \text{-th pixel;} \\ 0, & \text{otherwise.} \end{cases}$$

(3)

If the elements $s_{kl}$ are lexicographical ordered into a column vector $s$, the measurement $z_{mn}$ given in Eq. (1) can be written as $z_{mn} = \Psi_{mn}s$, where $\Psi_{mn}$ is a row vector. Similarly, the vector $z$, obtained by a lexicographical ordering of all elements $z_{mn}$, can be expressed as

$$z = \Psi s,$$

(4)

where $\Psi$ is an $NM \times N_xN_y$ matrix whose rows are given by $\Psi_{mn} = \exp(-j2\pi f_n \tau_{m,kl})$.

Equation (4) above contains the full measurement set, consisting of the collected measurements at all antenna locations and all frequencies. However, compressive sensing theory allows the reconstruction of a sparse signal $s$ from a reduced set of non-adaptive linear measurements $y = \Phi s$, where $\Phi$ is a known dictionary. In TWRI, the reduced set of measurements may be obtained with a randomly generated selection matrix $\Phi$ of size $K \times MN$ (with $K \ll MN$) that contains only one non-zero element in each row and each column:

$$y = \Phi z = \Phi \Psi s = As,$$

(5)

where the dictionary $A = \Phi \Psi$. According to CS theory, the sparse signal $s$ can be recovered by solving the following convex optimization problem:

$$\min \| s \|_1 \quad \text{subject to} \quad y = As.$$  

(6)

This is known as the single measurement vector CS model, where all measurements are order into a single vector. The signal $s$ can be recovered almost perfectly provided the number of measurements $K$ is $O(P(\mu^2(\Phi, \Psi)) \log(N_xN_y))$ [9], where $P$ is the number of sparse targets, $\mu(\Phi, \Psi)$ is the coherence between $\Phi$ and $\Psi$ and $N_xN_y$ is the length of $s$, which is the total number of pixels in the scene.

3. TWR IMAGE FORMATION USING MMV

In the above CS-based method for TWRI, the received signals from the radar system are lexicographically ordered to create a single measurement vector for the SMV model. However, one may cast the problem differently and consider another strategy of rearranging the received signals; for example, each column of the measurement matrix can represent the received signal from each antenna. In this case, the TWR image formation becomes a Multiple-Measurement Vector (MMV) problem. In this section, we tackle the problem of multiple polarization TWRI using the MMV model.

3.1. MMV Model

Consider a $K \times L$ measurement matrix $Y$, comprising $L$ measurement vectors, and a known dictionary $\mathcal{A}$ containing $Q$ atoms. The MMV model aims to find a sparse $Q \times L$ matrix $S$ by solving the following problem [8]:

$$\min \mathcal{R}(S) \quad \text{subject to} \quad Y = AS$$

(7)

where $\mathcal{R}(S)$ denotes the sparsity rank of the matrix $S$, which is the number of non-zero rows in $S$. In other words, the aim is to find a sparse matrix $S$, whose columns possess the same sparsity profile. When the noise is taken into account, the problem becomes:

$$\min \mathcal{R}(S) \quad \text{subject to} \quad \| Y - AS \|_2 \leq \epsilon$$

(8)

where $\epsilon$ bounds the amount of noise in the data.

3.2. Multiple-Polarization Data

Multiple polarization results from either the transmitted signals or the received radiation. In [10], the authors describe an adaptive polarization contrast technique, in which the principal components (PCs) of the background and targets are extracted using the statistics of the original polarization data.
However, the PC calculation is time-consuming, especially when cross- and dual-polarizations and more antennas are involved. Here, we propose to combine the measurements from several multi-polarization channels using the MMV model.

Without loss of generality, consider $L$ polarization channels, where the collected data for the $i$th channel is lexicographically ordered in a vector $x_i$; the corresponding scene is represented as a column vector $s_i$. Then using a reduced set of measurements, see Eq. (5), we have

$$y_i = \Phi_i x_i = \Phi_i \Psi s_i = A_i s_i, \quad i \in [1, L].$$

Clearly, when considering only a single polarization channel, the above problem reverts to the traditional TWRI using CS. Note that the matrix $\Psi$ remains the same when collecting different polarization data since it is only influenced by the physical scene and the configurations of the antennas (including the location and the transmitted frequency). In addition, we can define the selection matrix $\Phi_i$ to be identical for all polarization channels, i.e., $\Phi_i = \Phi \ (\forall i \in [1, L])$. Consequently, all polarization channels possess the same dictionary $A = \Phi \Psi$.

A simple method is to combine all the measurements together into a composite average measurement vector and solve the resulting problem using the SMV model:

$$\min \|s\|_1 \quad \text{subject to} \quad \sum_{i=1}^{L} y_i / L = \sum_{i=1}^{L} A_i s_i / L = A s$$

(10)

Another approach is to use all individual polarization data simultaneously. This is done by arranging the collected data from each polarization in a column vector of the measurement matrix $Y = [y_1, y_2, \ldots, y_L, \sum_{i=1}^{L} y_i / L]$, where the last column contains the average of the polarization data. Accordingly, the scene is represented as $S = [s_1, s_2, \ldots, s_L, s]$. The matrix $S$ can be recovered using the MMV model:

$$\min R(S) \quad \text{subject to} \quad Y = AS$$

(11)

The MMV problem in (11) not only takes each polarization observation $y_i$ into account, but it also calculates a sparse solution for the composite measurement vector from all the channels. By doing so, it is expected that the MMV method will achieve a better performance, since more information or constraints are considered in the solution.

4. EXPERIMENT RESULTS AND ANALYSIS

The proposed MMV-based TWRI method is tested on simulated data. The radar system is assumed to be placed in front of a wall with a thickness of 0.3m and a dielectric constant $\eta = 6$. The array aperture consists of 41 antennas spaced at 0.04m intervals. The transmitted stepped-frequency signals have frequencies ranging from 1 to 3 GHz, with 50-MHz frequency step. The coverage of the down-range and cross-range of the scene are $[-4.0545m, 4.0545m]$ and $[0m, 3.825m]$, respectively. The centre of the array is 1.05m away from the centre point of the wall. We assume that there are three targets located behind the wall, placed at the following positions: $(-1.7531, 0.6563), (-0.4781, 3.0938), \text{and } (0.7969, 2.5313)$. To measure the quality of the reconstructed image, we use the Peak Signal-to-Noise Ratio (PSNR):

$$PSNR = 20 \log_{10} \left( \frac{I_{\text{max}}}{RMSE} \right)$$

(12)

where $I_{\text{max}}$ denotes the maximum pixel value and $RMSE$ is the root-mean-square error between the reconstructed image and the true synthesized image. Simulated data are generated for three polarization channels, where each target has different complex reflectivity across the channels. To form a radar image from these measurements, we apply both the SVM and MMV models as given in (10) and (11).

Figure 1 shows the images produced by these two models when only 12 measurements are considered. The proposed MMV-based TWRI method gives superior result in terms of the number of correct and false detections. For example, the proposed method detects all three targets while the SMV-based TWRI method only detects one target correctly with many false detections.

Fig. 1. TWR images reconstructed from multiple-polarization data using (a) the SMV and (b) the MMV models.

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To test the robustness of the proposed MMV-based TWRI method, we generate several sets of simulated data with different noise levels. White Gaussian noise is added to the received signals, with noise variance ranging from 0 to 15. Furthermore, the number of measurements is reduced from 20 to 10. In each experiment, the SMV and MMV models are applied to generate the radar image, and the image quality is calculated in terms of PSNR. For each set of measurements and noise variance, the experiment is repeated 15 times and the average PSNR and detection accuracy are computed. Figure 2 shows the performances of the SMV-based and MMV-based TWRI methods in terms of average PSNR and detection accuracy as a function of the number of measurements $K$ and the noise variance $\sigma^2$. The solid curves represent MMV, whereas the dotted curves are for SMV. It is evident from the figure that the MMV-based TWRI method outperforms the SMV-based method in terms of PSNR; for example, when the scene is recovered in the absence of noise, the average PSNR of SMV (93.48 dB) is much lower than that of MMV (112.86 dB). Moreover, the experimental results show that the MMV-based TWRI method improves the detection accuracy compared to the SMV. This is due to the fact that the MMV model considers all the measurements from different polarization channels simultaneously to generate a sparse matrix with fewer number of non-zeros rows. As a result, the last column of the sparse matrix produces a sparse scene with all the correct target locations and minimum false detections.

5. CONCLUSION

In this paper, a new through-the-wall radar imaging method was proposed based on the MMV model. The proposed MMV-based TWRI method processes all the received antenna signals from several multi-polarization channels simultaneously, thereby producing high quality sparse solutions. The proposed method was tested on synthetic data. The experimental results showed that the MMV model is more noise tolerant and achieves better image quality and detection accuracy compared to the traditional SMV model.

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6. REFERENCES