EFFICIENT CONVEX OPTIMIZATION FOR REAL-TIME ROBUST BEAMFORMING WITH MICROPHONE ARRAYS

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ABSTRACT

This paper presents an efficient implementation of a robust adaptive beamforming algorithm based on convex optimization for applications in the processing-constrained environment of a digital hearing aid. Several modifications of the standard interior point barrier method are introduced for use where the array data covariance matrix is changing rapidly relative to the algorithm’s convergence rate. These efficiency improvements significantly simplify the computation without affecting the algorithm’s fast convergence, and are useful for real-time adaptive beamforming regardless of the rate of array correlation change. Simulation results show that this implementation is numerically stable and succeeds where many minimum-variance distortionless response (MVDR) solutions fail.

Index Terms— adaptive directionality, acoustic beamforming, second-order cone programming, barrier method, hearing aids

1. INTRODUCTION

Although adaptive beamforming algorithms can improve the signal-to-noise ratio at the output of a microphone array [1], they are not robust against any mismatch in the steering vector [2]. Several methods have been proposed in the literature to resolve the steering mismatch issue [3, 4, 5]. The first two papers [3, 4] estimate the steering vector in real-time as part of the adaptive beamforming algorithm, while the third paper [5] establishes a protected region around the steering vector where it allows no reduction.

For hearing aids, the estimation of the steering vector would be difficult, because the steering vector changes every time the wearer puts on the hearing aid and the steering vector can change when the wearer touches the hearing aid. Hence the method in [5] is the most promising solution to solve the robustness problem of adaptive beamformers. It minimizes the output of the microphone array while maintaining a distortionless response for the worst-case (mismatched) steering vector. Furthermore it admits a convex formulation for such a robust adaptive beamforming problem using second-order cone programming (SOCP) [5]. The paper has, however, not been written with a hearing-aid application in mind; it neither takes into account the hearing aid’s constraints on the computational complexity nor the ever-changing sound fields in which hearing aids are typically used (which result in time-varying data covariance matrices). This paper proposes an efficient real-time convex optimization algorithm to solve the robust adaptive beamforming problem in a rapidly changing environment. It uses the modified logarithmic barrier method to solve the time-varying SOCP problem. The focus is on the balance among robustness, real-time adaptivity, and computational efficiency.

The remainder of this paper is organized as follows. Section 2 introduces the logarithmic barrier algorithm and its real-time implementation. Section 3 presents the evaluation of the new robust adaptive beamformer and its comparison to a non-robust MVDR implementation. Section 4 gives conclusions of the paper.

2. REAL-TIME ROBUST MVDR

Consider an MVDR beamformer that is robust against an arbitrary signal steering vector mismatch. The beamformer can be obtained by solving the following optimization problem [5]

\[
\min_{w} \quad w^H R w \\
\text{subject to} \quad |w^H a| \geq 1, \text{ for all } a \in \mathcal{A}(\epsilon) \tag{1}
\]

where \(w\) is the beamformer, \(R\) is the data covariance matrix, \(a\) is the steering vector, and \(\mathcal{A}(\epsilon)\) is the uncertainty set of the steering vector. Assume that the mismatch between the actual steering vector and the nominal one can be bounded by some known constant \(\epsilon\). The uncertainty set can then be expressed as:

\[\mathcal{A}(\epsilon) = \{a \mid a = a_0 + \Delta, ||\Delta|| \leq \epsilon\} .\]

The problem in (1) is a nonconvex quadratic programming problem with infinitely many constraints and is thus compu-
tationally intractable. However, it has been shown in [5] that (1) can be rewritten in the following equivalent convex form:

$$\min_w w^H Rw$$

subject to

$$w^H a \geq \epsilon ||w|| + 1$$
$$\Im\{w^H a\} = 0$$

In (2), the objective is a convex quadratic form and $a$ is the nominal steering vector. One can apply the Cholesky factorization $R = U^H U$ to obtain $w^H Rw = ||Uw||^2$. Thus minimizing the output power $w^H Rw$ is equivalent to minimizing $||Uw||$. One can further introduce an additional variable, $\tau$, as an upper bound on $||Uw||$. The following minimization problem with equality constraints only:

$$\min_{\tau, w} \tau$$

subject to

$$w^H a \geq \epsilon ||w|| + 1$$
$$||Uw|| \leq \tau$$
$$\Im\{w^H a\} = 0$$

The logarithmic barrier method is used to solve the problem in (3) when $R$ is time-varying. Initialization consists of choosing large enough $\epsilon$. For fixed $R$, the optimal beamformer $w$ can be solved by choosing large enough $t$.

For each fixed $t$, the barrier method uses Newton’s method to solve (5). This requires both the gradient and the Hessian of the barrier function $\phi(\tau,w)$, which can be derived from the following proposition.

**Proposition:** Assume a logarithmic function of the form

$$\psi(x) = -\log \left( (c^T x + d)^2 - ||Ax + b||^2 \right)$$

(6)

Then its gradient, given in [7, Chapter 11], and its derivative, the Hessian, can be expressed as:

$$\nabla_x \psi(x) = -2 \frac{f(x)}{g(x)} \in \mathbb{R}^{N \times 1}$$
$$\nabla^2_x \psi(x) = -2g^{-2}(x) [g(x) (cc^T + AT A) - 2f(x)f^T(x)] \in \mathbb{R}^{N \times N}$$

(8)

As an example, to obtain $\nabla_w \phi(\tau,w)$ and $\nabla^2_w \phi(\tau,w)$ for the first term in $\phi(\tau,w)$, one can choose $A = \epsilon I$, $b = 0$, $c = a$, and $d = -1$, with the real and imaginary components separated as in [5].

### 2.2. Real-time Implementation

This section presents an efficient real-time implementation for solving (3) when $R$ is time-varying. Initialization consists of the following steps:

- $R$ is initialized to the first estimate given to the system.
- $w$ is initialized to be feasible; that is, it slightly exceeds the robustness constraint for the given $\epsilon$ and $a$.
- $\tau$ is initialized to meet the SOCC involving it from (3).
- $t$ is initialized a small value that provides a gentle slope throughout the feasible region.

At each iteration, which might be much less often than the sampling period, the following steps are taken:

1. Track environment change

   - Update $R$ using a one-pole averaging filter.
   - Adjust $\tau$ upward if needed to ensure the solution is feasible (meets all SOCCs).

2. Update the multiplier $t$

   - If $t$ is high enough to yield the desired solution precision of $(\tau,w)$, go to step 3.
   - Otherwise, if the root mean square change in $(\tau,w)$ in the last iteration was small, increase $t$ by a fixed percentage.

3. For fixed $t$, Newton’s update method is adopted to proceed toward the optimum solution of (5)

   - Calculate the gradient and Hessian of $\phi(\tau,w)$. 

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- Obtain the Newton step by solving a linear system [7, Eqn. 11.14] using the conjugate gradient (CG) method.
- Update \((\tau, w)\) by adding the Newton step.

### 2.3. Efficiency Improvement

A few efficiency improvements are obtained in the proposed algorithm when compared to the standard SOCP solver:

- **Eliminating the Cholesky factorization:** The problem formulation (3) requires the Cholesky factor \(U\) of \(R\). But, the form (6) squares \(|Uw|\), so calculating \(w^HRw\) directly suffices as suggested by (2), removing the computationally expensive Cholesky factorization.

- **Iteration number reduction per update:** Instead of solving a new SOCP per update of \(R\), the method above requires very few iterations to track changes in the environment. Also, the solution \((\tau, w)\) from the previous \(R\) provides an excellent basis for taking the next step.

- **Truncating the CG method:** The CG method is efficient for solving the linear systems in the barrier method. It iterates to the exact solution through a number of steps equal to the system order, with earlier steps making the most progress. Convergence is accelerated when eigenvalues are clustered [8]. With \(M = 3\) microphones and the resulting system of order 6, truncating the solution after 3 iterations results in a negligible performance degradation across a wide range of inputs.

- **Eliminating the linear constraint:** The linear constraint \(\text{Im}\{w^H a\} = 0\) is used to eliminate a variable from the solution vector, which contains \(\tau\) and the real and imaginary portions of \(w\), resulting in a system of order \(2M\), where \(M\) is the number of microphones. This also eliminates a rank deficiency in the Hessian caused by the linear constraint. The variable elimination can be done without division if \(a\) is properly normalized.

### 3. EVALUATION

Three simulations are provided to illustrate the performance of the algorithm. For all simulations, there are three microphones in a uniform linear array. The distance between two adjacent microphones is 7.5 mm. The simulation results are shown in the 2 kHz frequency band. The array output is simulated as follows:

- There is a point source with signal strength 10 dB coming from azimuth angle \(0^\circ\) and elevation angle \(0^\circ\). The robust region is \(5^\circ\) around the nominal source direction.
- There is a moving interference with strength 10 dB and elevation \(5^\circ\). Its azimuth angle changes from \(0^\circ\) to \(105^\circ\) in the first 5 s. During the remaining 5 s, the interference stays at \(105^\circ\). The output data covariance \(R\) is updated 20 times per second.
- The white noise in each microphone is taken to be \(-40\ dB\).

![Fig. 1. Output power vs. iteration.](image1)

![Fig. 2. Filter directional response vs. simulation iteration: robust case. The vertical white line at left shows the protected region. The white line with a bend shows the instantaneous position of the interferer, which the algorithm only sees through the averaging filter.](image2)
Fig. 2 shows the array response vs. time. The robustness constraint combined with the minimum power constraint keeps any null a sufficient angle away from the region that is guaranteed to have at least 0 dB gain. The null cannot move into the “protected region” around azimuth and elevation angles of 0°. It can also not move too close to this region because that would require large values for the beamformer $w$ resulting in a large amplification of the white noise and thereby it would be a non-minimum solution for the array output power. Once the interferer moves sufficiently far from the protected region, the null begins tracking the interferer. In more detail, for the early iterations, the maximum gain is at 180°, reaching a maximum of 15.5 dB at iteration 40 and surpassing 5.0 dB only between iterations 27 and 78. Regarding the robustness constraint, the gain at 5° never goes below 0 dB; it reaches a maximum of 1.2 dB at iteration 38. The final polar pattern in the azimuth plane is given in Fig. 3.

Fig. 4 shows a simulation of the non-robust MVDR with no protection region and a 5° steering vector mismatch. This allows signal nulling, which persists at −17 dB after iteration 20, −11 dB after iteration 40, and −7 dB after iteration 60, −3 dB after iteration 80, and −1 dB after iteration 100.

4. CONCLUSIONS

This paper has shown that the barrier method of solving an SOCP problem is well suited to adaptive acoustic beamforming with robustness to steering vector uncertainty. The method can be implemented with low computational complexity approaching the available processing power in current hearing aids. Furthermore, the barrier method has been adapted to solve a continually changing problem to sufficient precision instead of solving a static problem to great precision as is the common case. Several other techniques to minimize the computational complexity have been applied. Simulations show that the method can adapt quickly even when the interferer moves rapidly. Also, the results are robust to a user-specified level of steering vector mismatch.

5. REFERENCES