A SPACE TIME ARRAY PROCESSING FOR PASSIVE GEOLOCALIZATION OF RADIO TRANSMITTERS

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ABSTRACT

The problem of passive localization is commonly solved by independently measuring intermediate parameters (such as angles of arrival (AOA), times of arrival (TOA)...) on several multiple sensors base stations in a first step. In a second step, the transmitted parameters are then used to estimate the position. Recently, studies proposed new promising one step algorithms based on stacked multiple station observations vectors gathering all received signals of all base stations. This strategy leads naturally to a wideband estimation problem, solved in this paper by an alternative space-time processing. The proposed algorithm is compared to existing techniques and the corresponding Cramer-rao bound.

Index Terms—Array processing, Emitter geolocalization, AOA estimation

1. INTRODUCTION

The problem of radio-transmitters localization has always received great attention in the signal processing community. By means of several multiple sensors base stations, the problem lies in estimating the position of radio emitters. We focus here on line-of-sight (LOS) conditions.

Traditionally [1], localization algorithms consists in two steps. On the first step, intermediate parameters (often called measures) are independently estimated on each base station, for example it can be angles of arrival (AOA), times of arrival (TOA) or frequencies of arrival (FOA). On the second step, the emitters location is computed thanks to the previous available set of measures [2]. We focus in this article on angle of arrival (AOA) estimation based approach, widely discussed [3] for passive localization.

But such two steps approaches suffer from limitations[1]. First, solving the problem by means of a multiple step strategy is here suboptimal, as the first step does not take into account the fact that the received signals at the stations come from the same emitters. Secondly, in passive multi-emitter context, such approach require a “data association” procedure that aim to identify among all the whole set of measures, which subset characterizes each source [4]. Lastly, the performances and the number of resolvable emitters on each station are often locally limited by the number of sensors of the station.

These are the reasons why an approach that simultaneously exploits the received signals of all available stations, providing the emitter location directly, in a possibly “one step” procedure, appears of great interest. Nevertheless, such an approach assumes that each base station is able to transfer all the signals to a central process unit. Each station is seen as a subarray of a global array composed of all base stations leading naturally to a wideband estimation problem.


Based on a space-time observation vector [8], first introduced for the AOA estimation in wideband context, we propose an original geolocation algorithm that treats coherently and simultaneously all signals received on the global array. The outline of the paper lies as follows : we first propose the modeling of the signal and formulate the problem. In the third section we provide a new subspace based algorithm. Finally numerical results show the improvement of the proposed approach compared to the existing techniques.

2. PROBLEM FORMULATION

We focus on the problem of locating multiple emitters on L stations composed by \( N_l \) \((1 \leq l \leq L)\) sensors. Let us denote \( N = \sum_l N_l \) the total amount of sensors. Let \( x_l(t) \) be the observation vectors observed on the \( lth \) station, and let us consider \( Q \) emitters (\( Q \) is assumed to be known) whose unknown signals (complex envelopes) are denoted \( s_q(t) \) \((1 \leq q \leq Q)\). Assuming classically that the signals are narrowband on each
station (subarray), we have
\[
x_l(t) = \sum_{q=1}^{Q} \rho_{l,q} a_l(\theta_l(p_q)) s_q(t - \tau_l(p_q)) + n_l(t),
\]
where \(p_q\) denotes the \(D \times 1\) coordinates vector of the \(q\)th emitter. \(f_0\) stands for the carrier frequency. The attenuation \(\rho_{l,q}\) is an unknown complex parameter standing for the channel effect. The steering vector \(a_l(\theta_l(p))\) is the sensor response of the station \(l\) which norm is \(\sqrt{N_l}\). It depends on the angle of arrival on the \(l\)th station \(\theta_l\), seen here as a function of the position \(p\) directly. For notational convenience we will consider \(a_l(p) \equiv a_l(\theta_l(p))\). The delay \(\tau_l(p)\) is the relative time of arrival of the \(m\)th signal on the \(l\)th station, the origin being arbitrary chosen on the first station
\[
\tau_l(p_q) \equiv \frac{\|p_q - p(l)\| - \|p_q - p(1)\|}{c},
\]
where \(p(l)(1 \leq l \leq L)\) is the coordinates vector of the \(l\)th station. \(n_l(t)\) is additive noise, assumed temporally and spatially white, which covariance matrix is \(\sigma^2 I_{N_l}\) where \(I_{N_l}\) is the \(N_l \times N_l\) identity matrix.

Considering narrowband signals on the global array [6] exploits the following stacked steering vector
\[
x(t) = \begin{bmatrix} x_1^T(t) & \cdots & x_Q^T(t) \end{bmatrix}^T.
\]
The purpose of this paper is to extend this approach to the case of signals that are wideband on the global array. For that, each source is now a linear combination of \(r_q\) (assumed to be known) signals \(s_{qr}(t)\), centered on \(f_{qr}\) and assumed narrowband on the the global array
\[
s_q(t) = \sum_{r=1}^{r_q} s_{qr}(t)
\]
so that we have
\[
s_{qr}(t - \tau_l(p_q)) \approx s_{qr}(t) e^{-j2\pi f_{qr}\tau_l(p_q)},
\]
leading to
\[
x_l(t) = \sum_{q=1}^{Q} \sum_{r=1}^{r_q} \rho_{l,q} a_l(p_q) e^{-j2\pi f_{qr}\tau_l(p_q)} s_q(t) + n_l(t).
\]

So, the observation vector (3) writes
\[
x(t) = \sum_{q=1}^{Q} \sum_{r=1}^{r_q} u(p_q, \rho_q, f_{qr}) s_{qr}(t) + n(t),
\]
where
\[
u(p, \rho, f) = \frac{1}{\sqrt{\sum_{l=1}^{L} |\rho_l|^{2}}} \begin{bmatrix} \rho_1 a_1(p) e^{-j2\pi f_{1}\tau_1(p)} \\ \vdots \\ \rho_L a_L(p) e^{-j2\pi f_{L}\tau_L(p)} \end{bmatrix},
\]
where \(u(p, \rho, f)\) has a constant norm equal to \(\sqrt{N}\), we define the following signal to noise ratio
\[
SNR_{qr} = 10\log_{10} \left( \frac{E|s_{qr}(t)|^2}{\sigma^2} \right).
\]

We choose without loss of generality \(\rho_{1,q} = 1\) (origin of the relative nuisance complex parameter). We can write
\[
x(t) = \sum_{q=1}^{Q} \sum_{r=1}^{r_q} u(p_q, \rho_q, f_{qr}) s_{qr}(t) + n(t).
\]

The signal subspace spanned by the steering vectors \(\{u(p_q, \rho_q, f_{qr}), 1 \leq q \leq Q, 1 \leq r \leq r_q\}\) in (12) is of dimension \(M = \sum_{q=1}^{Q} r_q\) which can become quickly large, so that the size of the observation vector \(x(t)\), equal to the number of sensors \(N\), might not be large enough to identify all the parameters. It appears of great interest to increase the size of the observation vector without increasing the number of sensors. For that, we consider the following space-time observation vector, as [8] did for the wideband AOA estimation problem
\[
y(t) = \begin{bmatrix} x^T(t) & x^T(t - T_c) & \cdots & x^T(t - (K-1)T_c) \end{bmatrix}^T.
\]

Where \(T_c\) is the sampling period. We can show [8] that
\[
y(t) = \sum_{q=1}^{Q} \sum_{r=1}^{r_q} b(p_q, \rho_q, f_{qr}) s_{qr}(t) + n_K(t),
\]
where
\[
b(p_q, \rho_q, f_{qr}) = z(f_{qr}) \otimes u(p_q, \rho_q, f_{qr}),
\]
\[
z(f) = \begin{bmatrix} 1 & e^{-j2\pi f T_c} & \cdots & e^{-j2\pi f (K-1)T_c} \end{bmatrix}^T,
\]
\[
n_K(t) = \begin{bmatrix} n^T(t) & n^T(t - T_c) & \cdots & n^T(t - (K-1)T_c) \end{bmatrix}^T,
\]
where \(\otimes\) denotes the Kronecker product. We aim to estimate the set of positions \(\{p_q, 1 \leq q \leq Q\}\) in (14), which is well suited to the use of high resolution methods.

### 3. A Space Time Processing

As suggests (14) the total number of parameters to be estimated can be large. This is the reason why a subspace approach is here preferred to a maximum likelihood approach, from a computational point of view. According to Schmidt [7] the set of positions corresponds to the zeros of the criterion
\[
C(p, \rho, f) = \frac{b_H^T(p, \rho, f) \hat{N} b(p, \rho, f)}{b_H^T(p, \rho, f) b(p, \rho, f)},
\]
where $\hat{\Pi}_n = I - \hat{U}_n \hat{U}_n^H$ is the noise projector of the following estimated spatio-temporal correlation matrix

$$\hat{R} = \sum_{n=1}^{T-K+1} y(nT_e)y^H(nT_e).$$

(19)

The column of $\hat{U}_n$ are the $M$ dominant eigenvectors of $\hat{R}$. Note that

$$b(p, \rho, f) = V(p)\eta(\rho, f, p),$$

(20)

where

$$V(p) = I_K \otimes U(p),$$

(21)

$$U(p) = \text{diag}(a_1(p), \ldots, a_L(p)),$$

(22)

$$\eta(\rho, f, p) = z(f) \otimes \left( \text{diag}(e^{-j2\pi f \tau_1(p)}, \ldots, e^{-j2\pi f \tau_L(p)}) \right) \rho,$$

(23)

where $\text{diag}(v_1, \ldots, v_N)$ stands for the block diagonal matrix formed with $v_1, \ldots, v_N$. According to a well-known mathematical result, the minimization of (18) with respect to $\eta(\rho, f, p)$ reduces to

$$C(p) = \lambda_{\min}\left\{ (V^H(p)V(p))^{-1} V(p)^H \hat{\Pi}_n V(p) \right\},$$

(24)

where $\lambda_{\min}\{\cdot\}$ denotes the minimal eigenvalue. [9] enables us to estimate the set of positions by searching the $Q$ zeros of the following reduced criterion

$$D(p) = \frac{|V(p)^H \hat{\Pi}_n V(p)|}{|V^H(p)V(p)|},$$

(25)

where $|.|$ denotes the determinant. In the sequel we call this approach LOCAlization by Space-Time (LOST) processing.

We can remark that contrary to [5], all signals are treated coherently through the spatio-temporal covariance matrix (19).

### 4. NUMERICAL RESULTS

In this section we consider four base stations A, B, C and D, all composed of a uniform circular array, with a radius equal to 0.5 wavelength and a number of sensor equal to three. The carrier frequency is $f_0 = 100$ MHz. We consider two emitters in $S_1$ and $S_2$ illustrated by figure 1. The signals are $B = 100kHz$ Bandwidth white complex Gaussian random processes, resulting from the discrete sum of $K$ Gaussian random process of bandwidth $\frac{F_c}{K}$ centered on $f_{qr} = \frac{F_c}{K} r (1 \leq r \leq K)$. The sampling frequency is $F_c = B = 100kHz$. The signal to noise ratio, defined on equation (11), are the same for every frequency channel indexed by $r$. In this section, we choose all $\rho_{i,m}$ equal to 1. The noise $n_K(t)$ is circular white Gaussian. The performances of our algorithm are studied through the root mean square error (RMSE) of the miss distance defined in meters:

$$RMSE = \sqrt{\frac{1}{I} \sum_{i=1}^{I} (x - \hat{x}_i)^2 + (y - \hat{y}_i)^2},$$

(26)

where $I$ is the number of Monte-Carlo runs and $\hat{x}_i$ and $\hat{y}_i$ denote the $ith$ estimation of the true position $(x, y)$. The experimental RMSE will be compared to the Cramer-Rao Bound (CRB) of the space-time model (14), but due to the lack of space, its expression will not be provided in this article.

![Fig. 1. Four base stations A, B, C and D on the corner of a 2 x 2 km square. The sources are in $S_1 (0,1000)$ and $S_2 (0,-1000)$, respectively.](image)

![Fig. 2. RMSE of the source in $S_1$ in presence of a second source in $S_2$, T=150, Monte-Carlo runs = 100, K = 4](image)
corresponding CRB.

In order to examine the robustness of the DPD and LOST technique, we now consider the case of a wideband signal for the global array in $S_1$, generated according to (1). The product “emitter bandwidth $\times$ time delay across the global array ($\tau$)” (called $B\tau$) can be large, so that (4) and (5) do not hold. The SNR is now $SNR_q = 10\log_{10}\left(\frac{E[|s_q(t)|^2]}{\sigma^2}\right)$.

On Figure 3, we compare the DPD technique when $B\tau$ grows. As we can see the proposed approach appears to be more robust than the DPD technique.

Figure 4 shows the role played by the number of shifts $K$ in the proposed approach when $B\tau$ grows. Above a particular value of $B\tau$ a performance breakdown occurs due to the high modeling error. As we can see when $B\tau$ is large the number of shifts $K$ should be increased.

![Fig. 3. RMSE of one source in $S_1$, K=4, SNR=10 dB, T=150, Monte-Carlo runs = 100](image)

![Fig. 4. RMSE of one source in $S_1$ for the proposed algorithm, RSB=10, T=800, Monte-Carlo runs = 1000](image)

5. CONCLUSION

Based on a space time observation vector, gathering all received signals from all stations we proposed an original location algorithm that provides an estimation of the position in wideband context directly. Simulations underline the improvement compared to a classical location algorithm where the parameter are estimated independently from each other on each station and then used to estimate the position. Moreover the proposed alternative coherent processing is shown to outperform the recent DPD technique. The proposed approach is compared to the CRB and we finally underline the role played by the number of shifts $K$ when the product $B\tau$ becomes large. The performance analysis of the proposed method and the proper choice of the number of shifts are ongoing works.

6. REFERENCES


