EXPLOITING MULTIPATH FOR BLIND SOURCE SEPARATION WITH SENSOR ARRAYS

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ABSTRACT

The problem of blind source separation by spatial processing is considered for convolutive mixtures of narrowband sources received by a sensor array. Multipath propagation is due to time-varying doubly-spread finite impulse response channels where the signal modes are Doppler shifted and arise from diffuse scattering rather than specular reflection. This paper introduces a new blind spatial filtering technique, referred to as the Generalized Estimation of Multipath Signals (GEMS) algorithm, which exploits multipath as the basis for signal separation under relatively mild assumptions regarding the signal spatial signatures and source waveform properties. The performance of GEMS is compared with a benchmark signal-copy procedure using real data acquired by an experimental high frequency (HF) antenna array system.1

Index Terms— Radar Applications, Beamforming

1. INTRODUCTION

Blind source separation by spatial-only array processing is a topic that has received enormous attention in the literature. The vast majority of spatial processing methods for signal separation fall into two main categories: 1) manifold-based methods which rely on channel and system assumptions to characterize the signal spatial signatures, e.g. plane-waves parameterized by the signal direction-of-arrival (DOA) [1] or generalized array manifold (GAM) models for spatially spread sources [2], and 2) source-based methods which are not based on manifold models but utilize a-priori knowledge about some deterministic or statistical properties of the source waveforms, e.g. constant modulus, finite-alphabet, CDMA codes, cyclostationarity, or second and higher-order statistics. Many techniques for both categories are described in [3],[4].

The combined influence of diffuse multipath scattering and unreliable array calibration can significantly complicate the modelling of signal spatial signatures, while on the other hand, uncooperative waveforms from natural or anthropogenic sources may not have well known deterministic or statistical properties that can be utilized for source separation. Taking into account more recent work in this area, it is evident that there is a lack of techniques for situations where neither of the premises 1) or 2) apply. Despite this absence of knowledge, the FIR-MIMO system in question can nevertheless satisfy the conditions required for identifiability, which means that unambiguous estimation of the source input sequences (and FIR system functions) remains possible. The authors are not aware of any existing works which demonstrate the practical capability of blind spatial processing to “untangle” diffusely scattered multipath signals in a real-world application when the signal spatial signatures and source waveform properties are arbitrary subject to satisfying conditions on identifiability.

This article describes a blind spatial processing technique, referred to as Generalized Estimation of Multipath Signals (GEMS), for narrowband source waveform recovery in FIR-MIMO systems. The operation of GEMS under relatively mild assumptions within this framework is a feature of the algorithm which broadens its scope for practical applications. The format of the paper is as follows: section 2 describes the data model, section 3 introduces the GEMS algorithm and section 4 presents experimental results from a high frequency (HF) system. Concluding remarks are given in section 5.

2. DATA MODEL

The spatial snapshot data vectors \( x_k \in \mathbb{C}^N \) acquired by \( N \) receivers of the sensor array at time samples \( k = 1, \ldots, K \) are modelled as (1). Here, \( Q \) is the number of narrowband sources, \( M_q \) is the number of propagation modes received from source \( q \) along distinct paths and \( s_q(k) \) for \( q = 1, \ldots, Q \) are the transmitted baseband scalar waveforms. In (1), \( \ell_{mq} \) are the mode time-delays expressed in sample bins\(^2\), \( \nu_{mq} \) are the mode Doppler shifts normalized by the sampling frequency and \( a_{mq} \in \mathbb{C}^N \) are the mode wavefronts. The term \( \mathbf{n}_k \in \mathbb{C}^N \) is full-rank additive noise uncorrelated with all sources. The number of modes \( M = \sum_{q=1}^{Q} M_q \) for \( M_q > 1 \) and \( M < N \).

\[
\mathbf{x}_k = \sum_{q=1}^{Q} \sum_{m=1}^{M_q} s_q(k - \ell_{mq}) \exp \{ j2\pi k \nu_{mq} \} a_{mq} + \mathbf{n}_k \tag{1}
\]

The \( M \) mode wavefronts \( a_{mq} \) for all \( (m,q) \) are assumed to be linearly independent but otherwise arbitrary. A multi-ray in-

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1The experimental data is from the HF Radar branch, DSTO Australia.

2Providing the sources are not under-sampled, the GEMS algorithm does not require the mode time-delays to coincide exactly with a sampling instant.
terpretation of $a_{mq}$ due to diffuse scattering from a localized region is given by (2). Here, $R_{mq}$ is the number of "mini-rays" with time-dispersion much less than the reciprocal of the signal bandwidth, $\beta_{mq}$ are complex amplitudes, $\psi_{mq}$ are directions-of-arrival (DOA) and $v(\psi_{mq})$ is the array manifold response vector. The resultant waveform $a_{mq}$ is a complex scale $\alpha_{mq}$ multiplied by a unit-length spatial signature vector $v_{mq} = v(\psi_{mq}) \odot d_{mq}$ expressed as the element-wise product of $v(\psi_{mq})$ at the nominal mode DOA $\psi_{mq}$ and a distortion vector $d_{mq} \in \mathbb{C}^N$. The total number of plane waveform rays $R = \sum_{q=1}^{Q} \sum_{m=1}^{M_q} R_{mq}$ is considered to be greater than $N$.

$$a_{mq} = \sum_{r=1}^{R_{mq}} \beta_{mq}^r v(\psi_{mq})^r = \alpha_{mq} v(\psi_{mq}) \odot d_{mq} \quad (2)$$

Identifiability requires the source waveforms to have linear complexity $P_q > 2L_q$, where $L_q = \max\{\ell_{mq}\} m=1$ is the channel impulse response duration for source $q$ [5]. For finite length deterministic sequences $s_q(k)$, linear complexity is the smallest integer $P_q$ for which there exist coefficients $\{\lambda_p\}_{p=1}^P$ that satisfy (3). Besides meeting this identifiability condition, no other information is assumed about the source waveforms $\{s_q(k)\}_{q=1}^Q$. The number of samples required for identifying source $q$ from the data is $K_q > 3L_q$ according to [6]. A total of $K > \max\{K_q\}_{q=1}^Q$ samples are assumed available in the collection interval such that all sources can be identified.

$$s_q(k) = - \sum_{p=1}^{P_q} \lambda_p s_q(k-p) \quad (3)$$

Apart from the relatively mild conditions assumed for the mode wavefronts $a_{mq}$ and the source waveforms $\{s_q(k)\}_{q=1}^Q$, both $Q$ and $M$ are also unknown along with the mode time-delays $\ell_{mq}$ and Doppler-shifts $\nu_{mq}$. The differences between the mode parameter tuples $\ell_{mq}, \nu_{mq}$ are assumed to be distinct (4). Except for in certain contrived scenarios, the differential delay and Doppler between two propagation modes ($i \neq j$) of a particular source $q$ will in general not be identical to that of another pair of modes ($i' \neq j'$) from the same or different source $q'$ when attention is restricted to the relatively small number of dominant signal components. Clearly, $(i, j) \neq (i', j')$ is imposed for the case $q = q'$.

$$\{\rho_{iq} - \rho_{jq}\}_{i,j=1}^{M_q} \neq \{\rho_{i'q'} - \rho_{j'q'}\}_{i',j'=1}^{M_{q'}} \quad (4)$$

3. GEMS ALGORITHM

GEMS is based on a deterministic optimization problem with an algebraic solution that can identify the source waveforms exactly using a finite amount of data when noise is absent. In the presence of additive noise, a least squares criterion is adopted to derive the spatial filters which estimate the source waveforms. For brevity, the GEMS algorithm is described directly here with reference to the flow-chart in Fig. 1. In essence, the underlying rationale is that the GEMS exploits the multipath delays and Doppler-shifts to form two weight vectors that isolate a pair of modes from each source in turn.

Define the data matrix $X = [x_1, \ldots, x_K]$ and an auxiliary data matrix $U(\ell, \nu) = [u_1, \ldots, u_K]$, where $u_k = x_k - e^{j2\pi\nu k}$. The GEMS algorithm is based on solving the optimization problem in (5) for an input setting of $\{\ell, \nu\}$ by finding the minimizing reference and auxiliary weight vectors $w \in \mathbb{C}^N$ and $r \in \mathbb{C}^N$ respectively ($\dagger$ denotes Hermitian). Note that $U$ is implicitly a function of $\{\ell, \nu\}$ in (5) but this dependence is dropped momentarily for notational convenience.

$$\epsilon(\ell, \nu) = \min_{w, r} \|w^\dagger X - r^\dagger U\|^2 \quad \text{s.t. } \|w^\dagger X\|^2 = 1 \quad (5)$$

The objective function $J(w, r) = \|w^\dagger X - r^\dagger U\|^2$ may be expanded and expressed in the form (6), where $R = XX^\dagger$, $F = UU^\dagger$ and $G = XX^\dagger$. Similarly, the quadratic constraint $C(w) = \|w^\dagger X\|^2 = w^\dagger XX^\dagger w$ is expressed in terms of $R$.

$$\begin{cases}
J(w, r) = w^\dagger Rw - r^\dagger Gw - w^\dagger Gr + r^\dagger Fr
\quad (6)
\end{cases}$$

Minimizing $J(w, r)$ with respect to $r$ yields $\hat{r} = F^{-1}G^\dagger w$. Substituting in (6) leads to $J(w, \hat{r}) = w^\dagger (R - GF^{-1}G^\dagger)w$. Defining $Q = R - GF^{-1}G^\dagger$, the optimization problem (5) is equivalent to (7) for particular input parameter values $\{\ell, \nu\}$.

$$\hat{w} = \arg\min_w w^\dagger Qw \quad \text{s.t. } w^\dagger Rw = 1 \quad (7)$$

The vector $\hat{w}$ is the generalized eigenvector corresponding to the smallest generalized eigenvalue $\lambda_{\text{min}}$ of the matrix pencil $\{Q, R\}$. Defining $Z = Q^{-1}R$, $\hat{w}$ has the form (8) where the operator $P\{\cdot\}$ returns the principal eigenvector of a matrix.

$$Z\hat{w} = \frac{1}{\lambda_{\text{min}}} \hat{w} \Rightarrow \hat{w} \propto P\{Z\} = z_1 \quad (8)$$

The scale of $\hat{w}$ is determined by the constraint $C(\hat{w}) = 1$. The solution for a particular value of $\{\ell, \nu\}$ may be written in closed-form (9). Scaling by $(z_1^\dagger Rz_1)^{-1/2}$ is important to meaningfully compare the cost $\epsilon(\ell, \nu) = \hat{w}^\dagger Q\hat{w}$ as a function of $\{\ell, \nu\}$, recalling the implicit dependence of $\hat{w}$ on $\{\ell, \nu\}$.

$$\hat{w} = z_1 (z_1^\dagger Rz_1)^{-1/2} \quad (9)$$

Deep minima in $\epsilon(\ell, \nu)$ are expected at coordinates $\{\ell, \nu\}$ that match the delay-Doppler displacement between the strongest pair of modes for a given source $q$. The GEMS filter $w_G(q)$ estimating source $q$, which yields a deep local minimum at location $\{\ell_q, \nu_q\}$, is given by $w_G(q) = \hat{w}(\ell_q, \nu_q)$. The source waveform is estimated as $s_q(k)$ by spatial filtering in (10).

$$s_q(k) = w_G(q)^\dagger x_k \quad (10)$$

Since the coordinates of the minima are not known a-priori, a simple option is to perform a search over a bank of delay
and Doppler bins that cover the domain of uncertainty. Pre-
processing methods may also be proposed to cue GEMS more
rapidly to the coordinates of interest. Computational aspects
of the algorithm will not be discussed further here as this is
left as a topic for future work.

4. EXPERIMENTAL RESULTS

Experimental data were collected by a calibrated L-shaped
HF antenna array near Darwin, Australia, containing \( N = 16 \)
vertical monopoles with a digital receiver per element [7].
Two sources acquired in the 62.5 kHz receiver bandwidth (at
carrier frequency 21.639 MHz) were superimposed, namely a
linear frequency modulated (FM) continuous waveform from
a HF radar (bearing 134\(^\circ\), range 1850 km) and an amplitude
modulated (AM) radio broadcast from a BBC station (bear-
ing 295\(^\circ\), range 3400 km). Both sources propagated to the
array via the ionosphere [8]. The receiving system acquired
in-phase and quadrature (I/Q) components of the baseband
signals at a sampling rate of \( f_s = 62.5 \) kHz. The data was
collected in coherent intervals of 2 seconds duration.

**Signal-Copy Method:** Fig. 3 shows the MUSIC spatial
spectrum for azimuth \( \theta \) and elevation \( \phi \) or DOA \( \vec{\psi} = [\theta, \phi] \)
computed from the data matrix \( \mathbf{R} \) assuming two signals. The
two peaks occur at \( \vec{\psi}_1 = [134^\circ, 19^\circ] \) and \( \vec{\psi}_2 = [295^\circ, 16^\circ] \).
The azimuth estimates match the known FM and AM source
bearings. However, different propagation modes could not be
resolved in elevation when more signals were assumed (not
shown). Letting \( \mathbf{w}_1^\dagger \mathbf{A} = \mathbf{f}_1 \), where \( \mathbf{A} = [\mathbf{v}(\vec{\psi}_1), \mathbf{v}(\vec{\psi}_2)] \)
and \( \mathbf{f}_1 = [1, 0] \), leads to \( \mathbf{w}_1 = \mathbf{A}(\mathbf{A}^\dagger \mathbf{A})^{-1} \mathbf{f}_1 \)
as the classic null-steering signal-copy weight vector for the FM source. The
resulting estimate \( \hat{s}_1(k) = \mathbf{w}_1^\dagger \mathbf{x}_k \) is compared with the true
source waveform and the output of a single receiver in Fig. 4.
While the null has diminished the contaminating effects of
the AM signal, significant distortion remains due to multipath
interference and residual AM signal energy.

**GEMS Technique:** Fig. 2 shows the mean square error cost
function \( \epsilon(\ell, \nu) \) computed for the same data. Two deep loca-
local minima \( \{ \ell_q, \nu_q \}_{q=1,2} \) are indicated on the display. The
GEMS weight vectors \( \mathbf{w}_G(q) \) corresponding to these minima
were used to form the estimates \( \hat{s}_q(k) \) for \( q = 1, 2 \) in (10).
Fig. 5 compares the GEMS estimate \( \hat{s}_1(k) \) with the signal-
modep copy method \( \hat{s}_1(k) \) and the true source waveform. Clearly,
the GEMS procedure provides a much higher quality estimate
of the true waveform. The waveform recovered by GEMS
exhibits minimal contamination and practically overlays the
known waveform\(^3\). It is emphasized that the known FM source
waveform was only used as ground-truth to compare perfor-
ance. Both estimation techniques operated in a strictly blind
manner. No ground-truth is available for the GEMS AM wave-
form estimate \( \hat{s}_2(k) \) shown as the solid line in Fig. 6. By

\[ \hat{s}_2(k) = \mathbf{r}_G(2)^\dagger \mathbf{x}_k \], where \( \mathbf{r}_G(2) \)
is the auxiliary weight vector at \( \{ \ell_2, \nu_2 \} \), and delaying it by
\( \ell_2 \), it is possible to compare signals recovered from different
modes of the same source. The dashed line in Fig. 6 shows
a remarkable agreement between the amplitude envelopes of
\( \hat{s}_2(k) \) and \( \hat{s}_2(k - \ell_2) \), as expected for correct identification.
This result provides great confidence that the AM source has
also been separated and estimated accurately.

5. CONCLUSIONS

GEMS is a blind source separation technique for narrowband
waveform recovery in doubly-spread multipath channels with
diffuse localized scattering. The main contribution resides in
the mathematical formulation of the GEMS criterion which
allows a problem of significant practical interest to be solved
under mild assumptions regarding the source waveforms and
the mode wavefronts. Specifically, the authors are not aware
of any spatial processing techniques designed to tackle this
particular problem under the same assumptions as stated for
GEMS. In this respect, the introduced approach provides a
new and unique capability for solving the problem at hand
with respect to previous spatial processing methods. The set
of experimental results confirm the validity of GEMS in a
practical HF system and demonstrate its superior performance
relative to a benchmark signal-copy procedure.

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\(^3\)The transmitted FM signal is tapered in amplitude at the edges to reduce
out of band spectral emissions
GEMS optimization
• Time-delay
• Doppler-shift

\( k \), \( x \)

GEMS waveform estimate
(at minimum MSE)

\( k \), \( x \)

Inner product

Auxiliary output

Auxiliary weights

Reference weights

Mean Square Error (MSE)

Error Signal

\( \hat{E} \), \( \hat{F} \)

GEMS waveform estimate
(at minimum MSE)

Fig. 1. Flow-chart illustrating the GEMS scheme.

Fig. 2. Cost function resulting for experimental data.

Fig. 3. Two-dimensional MUSIC spectrum.

Fig. 4. Waveform estimates for FM source

Fig. 5. GEMS estimate for FM source

Fig. 6. GEMS estimate for AM source