BLIND BEAMFORMER FOR CONSTANT MODULUS SIGNALS BASED ON RELEVANCE VECTOR MACHINE

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ABSTRACT

The blind beamforming method for constant modulus (CM) signals based on relevance vector machine (RVM) is proposed. The proposed beamforming method is obtained by incorporating the constant modulus algorithm (CMA)-like error function into the conventional RVM framework. The RVM framework formulates the parameters of beamformer by exploiting a probabilistic Bayesian learning procedure with assumption of Gaussian prior for parameters. The simulation results show that the proposed blind beamforming method can restore the desired signals with crowded interference signals.

Index Terms—Blind beamforming, Constant modulus algorithm, Relevance vector machine

1. INTRODUCTION

In realizing performance improvement without additional power consumption, adaptive beamforming techniques are frequently considered for the systems equipped with antenna arrays [1-3]. However, for the nonblind beamforming algorithms, it is necessary to estimate the channel in advance by exploiting training sequences. Consequently, the corresponding systems result in reduction of spectral efficiency. Accordingly, blind beamforming algorithms without using training sequences are preferable.

Among various blind beamformers, the so-called constant modulus algorithm (CMA) based beamformers have been widely considered due to their structural simplicity [4-6]. Since CMA belongs to the class of stochastic gradient (SG) approaches, the following error function and weight update function are used [7].

$$E_p = \frac{1}{2}(|y(k)|^2 - R_p)^2$$  \hspace{1cm} (1)

$$e_{ci} = e_k - \mu_e \sigma_k |y(k)|^{p-2} y(k) x^*(k),$$ \hspace{1cm} (2)

where $R_p = E[|b(k)|^p]/E[|b(k)|^2]$, $p = 2$, $\mu_e$ is the learning rate parameter, $x(k)$ is the input vector, $y(k)$ is the output and $b(k)$ is the transmitted signal. $x^*(k)$ is the complex conjugate of the input vector $x(k)$. For constant modulus signals, i.e., $|b(k)|^2 = R_2$, the CMA penalizes the output samples $y(k)$ that do not have the desired constant modulus characteristics. Despite the CMA’s simple structure, it has some inherent drawbacks of instability in convergence period, requirement of long sequence for convergence period, and sensitiveness to learning rate [8].

In this paper, we propose a novel adaptive blind beamformer for CM signals based on relevance vector machine (RVM), which will be denoted as the blind RVM-CM beamformer. This is built by incorporating the CMA-like error function into the conventional RVM framework. The proposed blind RVM-CM beamformer will be compared with the blind beamformer of CM signals based on the support vector machine (SVM), which will be denoted as the blind SVM-CM beamformer [9].

2. SYSTEM MODEL

Consider the system with a linear array of $L$ uniformly spaced antennas as shown in Fig. 1. It is assumed that user $i$, $1 \leq i \leq N$, transmits the signal $b_i(k)$ with power amplification of $A_i$ on the carrier frequency $\omega = 2\pi f$. Without loss of generality, the user 1 is assumed to be the desired user and transmits the CM signal of $|b_1(k)|^2 = R_2$ with unit power amplification of $A_1^2 = 1$. The other incident signals are assumed to be constant modulus or Gaussian distributed with power of $A_i^2 R_2$, and act as interferers. Then, the...
received signals of the $l$th antenna is given by

$$x_i(k) = \sum_{k=1}^{K} b_i(k) \exp(i\theta_i) + n_i(k), \quad 1 \leq l \leq L,$$

where $\theta_i$ is the angle of arrival of the signal of the user $i$. The complex-valued additive white Gaussian noise (AWGN), $n(k)$, is assumed to have zero mean and variance of $E[n_l(k)^2] = 2\sigma_n^2$. Accordingly, the desired user’s signal to noise ratio (SNR) is defined as $SNR = A_s^2 / 2\sigma_n^2$, and the desired signal to interference ratio (SIR) with respect to the user $i$ is defined as $SIR_i = A_i^2 / A_s^2$, for $2 \leq i \leq N$. The array input $x(k) = [x_1(k) \cdots x_L(k)]^T$ can be rewritten in vector form as

$$x(k) = Pb(k) + n(k),$$

where the noise vector $n(k) = [n_1(k) \cdots n_L(k)]^T$, the transmitted symbol vector $b(k) = [b_1(k) \cdots b_N(k)]^T$, and the system matrix $P$ is given by $[A_1 \cdots A_N \times N]$. Now it is required to define the modified target vector, error vector, and weights vector, respectively, as $s_i = [\exp(i\theta_i) \cdots \exp(i\theta_i)]^T$. The beamformer output can be described as

$$y(k) = x^T(k)c,$$

where $c = [c_1 \cdots c_L]^T$ is the complex-valued beamforming vector.

3. BLIND BEAMFORMING FOR CM SIGNALS BASED ON RVM

In this section, we formulate the blind beamformer for CM signals through incorporating a CMA-like error function into a general RVM regression framework, and then we present the associated learning algorithm.

3.1. Relevance Vector Machine [10]

The RVM regression task can be regarded as supervised learning of a model which defines the dependency of the targets $\{t_i\}_{i=1}^K$ on the given set of input samples $\{x_i\}_{i=1}^K$. For notational simplicity, we substitute the subscript $k$ for the sample index $k$ in parentheses. Prediction of targets for the previously unseen inputs is made through this established regressor described as follows:

$$y_k = \sum_{m=1}^{M} w_m \Phi_m(x_k) = \Phi(x_k)w,$$  

where $y_k$ is an intended approximation of a real-valued function of interest (i.e., regression), $\Phi_m(\cdot)$ is the basis vector function, and $\Phi(x_k) = [\Phi_1(x_k) \cdots \Phi_M(x_k)]$. The $K \times M$ matrix $\Phi = [\Phi_1 \cdots \Phi_M]$ is called the design matrix whose columns comprise the complete set of $M$ basis vectors. Although the model is linear in the weights (or parameters), it may still be highly flexible as the size of the basis set, $M$, may be very large. Given a set of $K$ corresponding training pairs $\{x_i, t_i\}_{i=1}^K$, the objective is to find values for the weights $w = [w_1 \cdots w_M]^T$ such that $y_k$ well generalizes new data.

3.2. Blind Beamformer for CM Signals Based on RVM

For blind beamforming of the CM signals, the target values are the constant squared modulus of signals, i.e., $t_k = |b_k|^2 = R_k$. Therefore, if we set $R_k = 1$, the target value $t_k = 1$ can be expressed as the sum of the squared modulus of beamformer output and error $e_k$:

$$1 = |y_k|^2 + e_k = |\Phi(x_k)w|^2 + e_k.$$  

As seen in (7), the target value is expressed by the formula which is nonlinear in weights $w$. This nonlinearity does not allow (7) to be inserted into the conventional RVM framework which can only treat regression problems linear in weights. We can linearize (7) by approximating the squared modulus of the beamformer output as

$$|y_k|^2 = |\Phi(x_k)w|^2 \approx \tilde{y}_k^2 |\Phi(x_k)w|,$$  

where $\tilde{y}_k$ is the beamformer output of the previous iteration, i.e., it is not a variable, but a constant [12]. For the first beamforming iteration, the raw received signals at first antenna are exploited, i.e., $\tilde{y}_k = x_1(k)$.

In addition to linearity in weights, the RVM has one more inherent restriction, which is the unavailability of direct injection of complex values. Since the mixing matrix $P$ in (4) and user signals are complex-valued, we need some tricks to accommodate the conventional RVM framework into the given system. In this context, it is necessary to modify (8) as [12]

$$\begin{bmatrix}
\text{Re}[y_k^2] \\
\text{Im}[y_k^2]
\end{bmatrix} \approx \begin{bmatrix}
\text{Re}(\tilde{y}_k \Phi(x_k)^*) & \text{Im}(\tilde{y}_k \Phi(x_k)^*) \\
-\text{Im}(\tilde{y}_k \Phi(x_k)^*) & \text{Re}(\tilde{y}_k \Phi(x_k)^*)
\end{bmatrix} \begin{bmatrix}
\text{Re}(w) \\
\text{Im}(w)
\end{bmatrix}.$$  

Now it is required to define the modified target vector, error vector, design matrix, and weights vector, respectively, as follows:

$$\tilde{t} = [\text{Re}(t_1) \cdots \text{Re}(t_K), \text{Im}(t_1) \cdots \text{Im}(t_K)]^T = [1, 0, \cdots, 0]^T,$$

$$\tilde{e} = [\text{Re}(e_1) \cdots \text{Re}(e_K), \text{Im}(e_1) \cdots \text{Im}(e_K)]^T,$$

$$\tilde{\Phi} = \begin{bmatrix}
\text{Re}(\tilde{y}_1 \Phi^*) & \text{Im}(\tilde{y}_1 \Phi^*) \\
-\text{Im}(\tilde{y}_1 \Phi^*) & \text{Re}(\tilde{y}_1 \Phi^*)
\end{bmatrix},$$

$$\tilde{w} = \begin{bmatrix}
\text{Re}(w) \\
\text{Im}(w)
\end{bmatrix}.$$  

where $\tilde{Y} = \text{diag}(\tilde{y}_1, \cdots, \tilde{y}_K)$.

The sparse Bayesian framework has the conventional assumption that the errors are modeled probabilistically as independent zero-mean Gaussian with variance $\sigma^2$. Therefore, $p(\tilde{e}) = \prod_{k=1}^{K} \mathcal{N}(\tilde{e}_k | 0, \sigma^2)$. This is equivalent to a mean-squared-error criterion. This error model thus implies a multivariate Gaussian likelihood for the target vector $\tilde{t}$.
\[ p(\mathbf{\tilde{y}|}\mathbf{w}, \sigma^2) = \prod_{k=1}^{2K} (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma^2} \left[ \mathbf{1}_k - \mathbf{\Phi}_k \mathbf{w}^T \right]^2 \right\} , \quad (11) \]

where \( \mathbf{\Phi}_k \) is the \( k \)th row of \( \mathbf{\Phi} \).

It can be expected that the maximum likelihood estimation of parameters from (11), with the same number of parameters in the model as training examples, results in severe over-fitting. By imposing an additional constraint on the parameters, such as the margin term in the SVM, the over-fitting problem can be solved. In this context, we can complement the likelihood function in (11) by introducing a prior over the parameters \( \mathbf{\alpha} \) as follows:

\[ p(\mathbf{w}|\mathbf{\alpha}, \sigma^2) = \frac{p(\mathbf{w}|\mathbf{\alpha}, \sigma^2)p(\mathbf{\tilde{y}|}\mathbf{w}, \sigma^2)}{p(\mathbf{\tilde{y}|}\mathbf{\alpha}, \sigma^2)} \quad (13) \]

and is Gaussian \( N(\mathbf{w}|\mathbf{\alpha}, \Sigma) \) with

\[ \Sigma = (\Lambda + \sigma^{-2} \mathbf{\Phi}^T \mathbf{\Phi})^{-1} , \quad (14) \]

\[ \mathbf{\mu} = \sigma^{-2} \mathbf{\Phi}^T \mathbf{\tilde{y}} , \quad (15) \]

where \( \Lambda = \text{diag}(\alpha_1, \cdots, \alpha_{2M}) \). By the product rule of probability, we can rewrite the ideal full posterior as

\[ p(\mathbf{\tilde{y}|}\mathbf{\tilde{y}}, \mathbf{\alpha}, \sigma^2) = p(\mathbf{\tilde{y}|}\mathbf{\tilde{y}}, \mathbf{\alpha}, \sigma^2) p(\mathbf{\tilde{y}|}\mathbf{\alpha}, \sigma^2) \]

The first term is the parameter posterior which is defined as \( N(\mathbf{\tilde{y}|}\mathbf{\alpha}, \Sigma) \) in (13). The second term can be divided as

\[ p(\mathbf{\tilde{y}|}\mathbf{\tilde{y}}, \mathbf{\alpha}, \sigma^2) = p(\mathbf{\tilde{y}|}\mathbf{\alpha}, \sigma^2) p(\sigma^2) / p(\mathbf{\tilde{y}}). \]

As the denominator \( p(\mathbf{\tilde{y}}) \) is independent of \( \mathbf{\alpha} \) and \( \sigma^2 \), with approximation of uniform priors over \( \log \mathbf{\tilde{y}} \) and \( \log \sigma^2 \), we can find most probable estimate \( \mathbf{\alpha}_{MP} \) by maximizing marginal likelihood,

\[ p(\mathbf{\tilde{y}|}\mathbf{\alpha}, \sigma^2) \] . This approach is denoted as type-II maximum likelihood procedure [10, 13]. That is, the sparse Bayesian learning is formulated as the (local) maximization with respect to \( \mathbf{\alpha} \) of the marginal likelihood, or equivalently, its logarithm \( L(\mathbf{\alpha}) \) given by

\[ L(\mathbf{\alpha}) = \log p(\mathbf{\tilde{y}|}\mathbf{\alpha}, \sigma^2) = -\frac{1}{2} [ \log 2\pi + \log |\mathbf{C}| + \mathbf{\tilde{y}}^T \mathbf{C}^{-1} \mathbf{\tilde{y}} ] , \quad (16) \]

where \( \mathbf{C} = \sigma^{-2} \mathbf{I} + \mathbf{\Phi}^T \mathbf{\alpha}^{-1} \mathbf{\Phi} \). The most probable point estimate \( \mathbf{\tilde{y}}_{\alpha_{MP}} = \mathbf{\tilde{y}}_{\alpha_{MP}} \) for the parameters is then obtained by evaluating (15) with \( \mathbf{\alpha} = \mathbf{\alpha}_{MP} \). The final (posterior mean) approximator can be obtained by \( \mathbf{y} = \mathbf{\Phi} \mathbf{\tilde{y}}_{\alpha_{MP}} \). The summarized procedure is as follows:

**4. SIMULATION RESULTS**

In this section, simulation results of the BER performances for the proposed blind RVM-CM beamformer are presented and compared with the blind SVM-CM beamformer [9]. Two examples as in [14] are considered. We fix the antenna array element spacing at a half of the wavelength, i.e., \( \lambda/2 \). As the mixing matrix \( \mathbf{P} \) is linear, inner product is appropriate as a kernel function, i.e., \( \mathbf{\Phi}_{ij} = x_i^T x_j \).

**4.1. Example 1**

For the first example, we consider the system with six users and a three-element antenna array. The user 1, desired user, is assumed to transmit quadrature phase shift keying (QPSK) modulated signals with the incident angle of 0°. The other users, interfering users, are assumed to transmit complex Gaussian distributed random signals with the incident angles of 15°, -30°, 30°, -60°, 60°, respectively.

Fig. 2 shows the BER performances with respect to data size, \( K \), for convergence period at SNR = 10dB and SIR = 10dB. It can be seen from Fig. 2 that more than 100
symbols are enough to converge for the blind RVM-CM and SVM-CM beamformers. Fig. 3 shows the BER curves with various SIRs. 150 samples are used for the convergence period. Both of the beamformers can restore CM property of the desired signal when the SIR is higher than 10dB. In addition, the performance superiority of the proposed blind RVM-CM beamformer over the blind SVM-CM beamformer gets larger as SIR and SNR increase.

4.2. Example 2

For the second example, the system with five users and a two-element antenna array is considered. The user 1, desired user, is assumed to transmit QPSK modulated signals with the incident angle of 10°. The other users, interfering users, are assumed to transmit complex-valued Gaussian distributed random signals or QPSK signals with the incident angles of -30°, 50°, -55°, 70°, respectively.

Fig. 4 shows the BER performances with constant modulus (QPSK) or Gaussian distributed interference signals. When SIR is 0dB, both of the beamformers can hardly restore CM property of the desired signals regardless of the modulus properties of interferers. However, both of the beamformers perform successfully, when SIR = 10dB. Especially, the proposed blind RVM-CM beamformer under the CM interferers performs best.

5. CONCLUSION

In this paper, a novel adaptive blind beamformer for CM signals based on RVM (blind RVM-CM beamformer) is proposed by formulating the blind beamforming problem as a regression problem and solving it using RVM regressor. The proposed blind RVM-CM beamformer can restore CM property with moderate SIR. In comparison with the state-of-the-art blind SVM-CM beamformer, the proposed blind RVM-CM beamformer shows a better performance, and the performance gap gets larger as SIR and SNR increase.

6. ACKNOWLEDGEMENT

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7. REFERENCES