DESIGN AND ANALYSIS OF A NARROWBAND FILTER FOR OPTICAL PLATFORM

Yujia Wang, Andrew Grieco, Boris Slutsky, Bhaskar Rao, Yeshaiahu Fainman, Truong Nguyen

University of California, San Diego
Department of Electrical and Computer Engineering

ABSTRACT

This paper presents an approach to designing narrowband digital filters that are realizable using optical allpass building blocks. We describe a top-down design method by explicitly examining the derivation of an Infinite Impulse Response (IIR) architecture. Our result demonstrates a design that can achieve a 0.0025π passband edge while providing 60dB stopband attenuation. The design is aimed to reduce filter pole magnitudes, providing tolerance for waveguide losses and fabrication errors. The narrowband filter is based on the foundation of latticed allpass sections, which makes it naturally realizable using basic photonic components. Furthermore, analysis is performed on delay length variations that can result from the fabrication process.

Index Terms— Allpass filters, eigenfilters, optical waveguide filters, delay effects, interpolation

1. INTRODUCTION

With the increasing bandwidth demand of modern communication systems, a renewed interest in all optical signal processing has started in recent years. Developments in Wavelength Division Multiplexing (WDM) and Silicon Photonics are creating a new platform for the development of digital signal processors [1, 2]. To support data communication in all optical signal environment, digital filter designs that can be realistically implemented using photonic components are required.

In this paper, we focus on discrete-time coherent optical processing (DTCOP) devices [3]. In DTCOP, the RF signal of interest \( f \approx 2 \cdot 10^{10} \) Hz is imposed as amplitude modulation on the optical carrier \( f \approx 2 \cdot 10^{14} \) Hz, and filtering is carried out in the optical domain. Photonic components such as couplers and partially transmissive mirrors perform arithmetic operations. An optical waveguide loop with traversal time on the order of the RF period serves as the discrete delay element \( e^{-\frac{2\pi}{T}z} \). Fine adjustment to the traversal time on the order of the period of the optical carrier multiplies the signal by a complex phase factor.

Although IIR design techniques for photonic filters have been presented [4], no realizable narrowband design with high attenuation has been studied. In this paper, we consider the design approach to a narrowband IIR digital filter that achieves 0.0025π passband edge and 60dB stopband attenuation. To meet the given requirements, a Finite Impulse Response (FIR) design would require filter orders higher than 200. A realizable photonic structure requires low complexity, therefore we aim to obtain a filter design with an order less than 64.

Classical IIR filters are only able to achieve these requirements with pole magnitudes larger than 0.999, marginally satisfying the system stability criterion in the traditional DSP implementation. In a photonic system, however, design parameters are further restricted as a result of fabrication error and waveguide loss [5]. Our challenge is therefore to derive a filter design method that yields a narrow passband with high stopband attenuation, while maintaining small pole magnitudes.

Traditional design approaches typically aim to minimize the approximation error or to achieve maximally flat passband/stopband. The consideration in filter designs for photonic platforms are drastically different in that the pole magnitudes are the critical constraints. We describe an approach to obtain the desired cutoff frequency and attenuation while lowering the pole magnitudes to be suitable for the optical platform. A number of papers have shown that an allpass structure can naturally be realized in a photonic scheme [4, 6]. To be consistent with the fabrication of the narrowband filter, our structure is based on allpass sections throughout the design process. We demonstrate the novelty in combining the design method for lowpass through allpass [7], and technique for linear phase allpass [8] to achieve narrowband filter suitable for photonic filtering.

Most studies in digital filter design analyze the performance of the structure under error by examining finite word length effects on the weights [9]. Photonic filters need to be analyzed differently because the tap weights are not controlled by finite memory. Although waveguide losses and errors in fabrication will affect the coefficients, the effects are minimal so long as the pole magnitudes are within bound. Instead, phase sensitivity that results from delay length variations in the form of \( H(e^{-\frac{2\pi}{T}z}) \) must be carefully studied. In this paper, we consider the phase error for two different implementations: cascaded direct form and cascaded lattice structures.

The rest of the paper is organized as follows: Section 2 presents the photonic allpass structure that can be used as the basic building block. This section also discusses the related sources of error and the emergence of the constraint on pole magnitude. Section 3 describes in detail our approach to designing the narrowband filter through interpolation and eigenfilter design, followed by results in Section 4. Analysis on the most prominent source of error is presented in Section 5, and Section 6 concludes the paper.

2. PHOTONIC STRUCTURE

An allpass filter can be realized using a variety of photonic components [4]. In this paper, we consider the pure passive unit cell shown in Figure 1. The unit cell includes a pair of Bragg mirrors as its main components, one with an amplitude reflection coefficient \( \rho \) and one

Fig. 1. Photonic Allpass Unit Cell. \( L \) indicates the optical path length, \( s \) represents phase delays and \( \rho \) is the amplitude reflection coefficient of the first Bragg mirror.

This work is supported by DARPA under grant HR0011-09-1-00012.
with perfect amplitude reflection. The unit cell transfer function is
\[ H_{UC}(z) = e^{j\rho} - \alpha z^{-1} \left( 1 - \alpha e^{j\rho} z^{-1} \right)^{-1} \]  
where \( \alpha \) is the waveguide amplitude propagation loss, and \( s \) represents the phase delay that can be controlled by small changes in optical path length [6]. The optical path length \( L \) between the mirrors is related to the filter free spectral range (FSR) by \( L = \frac{2\pi c}{n_{eff} \lambda} \), where \( n_{eff} \) is the waveguide effective index and \( \lambda \) is the free space wavelength. For a typical Si waveguide with effective index 2.55 at 1550nm wavelength, an optical path length deviation of 10nm will cause a phase deviation 0.033\( \pi \). The mechanism behind this type of path length deviation is fabrication errors in the physical dimensions of the waveguide. As a result, it may be considered uncorrelated over long sections of waveguide.

Additionally, the magnitude of the pole is determined by the value of \( \alpha \rho \). Thus both the waveguide loss and mirror reflectivity limit how close the pole can be placed to the unit circle. State-of-the-art Si waveguides exhibit propagation loss of 0.21dB/cm [5]. For a typical 3.8mm waveguide with a group index of 4.1 and a FSR of 20GHz, the power loss coefficient \( \alpha^2 = 0.98 \). Sidewall modulated Bragg mirrors [10] exhibit loss of approximately 3% per reflection (by power) even when implemented on relatively lossy (~7dB/cm) waveguides; it is expected that similar structures in low-loss waveguides could achieve reflectivity \( \rho > 0.99 \). Thus, poles of magnitude \( \alpha \rho \) as high as 0.97 should be realizable with current technology.

### 3. DESIGN APPROACH

Traditional filter design approaches are unable to yield results with pole magnitudes less than 0.97 for the given requirements. Our design method yields a transfer function in the form:
\[ H(z) = (H(z)^{M})^L I(z) \]  

By interpolation through a factor of \( M \), we ease the design constraint on passband edge and thus reduce the complexity of the problem. We first design a prototype filter \( H(z) \) with a 0.01π passband edge and 15dB attenuation. The prototype filter can then be cascaded to reach high attenuation, and interpolated to meet the required passband frequency. The images that arise from interpolation are removed by a subsequent lowpass filter \( I(z) \). For our specific design presented in this paper, the interpolation factor \( M \) and the cascade factors \( L \) are \( M = L = 4 \). The design method is detailed in the following subsections.

#### 3.1. Prototype Lowpass Filter

Relaxing the design specifications allows for a slower roll off in the passband and transition band, thus allowing for lower magnitude poles. We increase the cutoff from 0.0025π to 0.01π, and reduce the attenuation from 60dB to 15dB. Since photonic filters are best realized as allpass sections, we use the method described in [7] to obtain the lowpass filter as sum of two allpasses:
\[ H(z) = \frac{1}{2} \left( z^{-(N-1)} + A_1(z) \right) \]  
where \( A_1(z) \) is an allpass filter of order \( N \). The design of the lowpass prototype filter thus reduces to designing the allpass filter \( A_1(z) \). It can be shown that a \( 7^{th} \) order allpass filter will be able to achieve the reduced specifications. To obtain the allpass section, we employ the iterative method presented in [8]. As outlined in [8], we first calculate the eigenvector corresponding to the smallest eigenvalue of \( P_2 \) in the error measure
\[ E_{LS}^{(2)} = a^T P_2 a \]  

where
\[ P_2 = \alpha \sum_{k=1}^{K} \int_{\omega_{k,1}}^{\omega_{k,2}} \left[ c(\omega_k) - c(\omega) \right]^2 d\omega \]  

and
\[ E_{LS}^{(3)} = 4 \int_{R} \frac{|a(\omega)|^2 |s_\beta(\omega) s_\beta(\omega) a(\omega)|}{|a(\omega)|^2 |c_\beta(\omega) c_\beta(\omega) a(\omega)|} d\omega \]  

The \( \Theta_{pre} \) is the desired response, and in our case corresponds to two flat sections in the passband and stopband. \( N \) is order of the filter, which we have determined to be 7. \( \alpha \) is the control parameter that distributes the weighting to each of the error terms, and is set at 0.099 to give large weight to the phase error term. \( K = 2 \) is the number of frequency bands of interest, signifying the passband and stopband regions. The \( \omega_{k,1} \) and \( \omega_{k,2} \) terms are the start and stop frequencies of band \( k \); \( \omega_k \) is the reference frequency when the magnitude response of \( H(z) \) is expected to reach maximum.

The resulting eigenvector \( c(\omega) \) contains the weights for the allpass filter that approximately minimize the least square error. \( a \) is then used as the initial value for the cost function
\[ E_{LS}^{(3)} = 4 \int_{R} \frac{|a(\omega)|^2 |s_\beta(\omega) s_\beta(\omega) a(\omega)|}{|a(\omega)|^2 |c_\beta(\omega) c_\beta(\omega) a(\omega)|} d\omega \]  

where
\[ c_\beta(\omega) = \left[ \cos(\beta(\omega)) \cdots \cos(\beta(\omega) - N\omega) \right]^T, \]  

with \( \beta(\omega) = \frac{1}{2} (\Theta_{pre}(\omega) + N\omega) \).

#### 3.2. Interpolation

The passband edge is 4 times that of the design requirement. To obtain a final filter with 0.0025π passband edge, we upsample the prototype filter by a factor of 4. The poles of the interpolated filter \( H_1(z) \) are the solutions to
\[ \prod_{i=1}^{M} (1 - k_i z^{-1}) \prod_{j=1}^{N-M} (1 - k_j z^{-1}) (1 - k_j^* z^{-1}) = 0 \]  

where \( M \) corresponds to the number of real valued poles and \( k_i, k_j \) are the filter taps of the original prototype filter. From (10), it can be seen that upsampling by a factor of 4 also magnifies the pole magnitudes by the same amount. The prototype lowpass filter is designed to counter this effect, allowing for a resulting filter with pole magnitudes less than 0.97. Upsampling also creates images of the desired filter throughout the spectrum, which can be suppressed using a lowpass filter \( I(z) \). Since the first image occurs at \( \frac{\pi}{2} \), the interpolation lowpass can be designed with lenient specifications and poses no concerns for pole magnitudes. The final designs for (2) are
\[ H_1(z) = \frac{1}{2} (z^{-6} + A_1(z)) \]  

and
\[ I(z) = \frac{1}{2} (z^{-3} + A_1(z)). \]
4. DESIGN RESULTS

The prototype lowpass filter \( H_l(z) \) has a maximum pole magnitude of 0.882 and an attenuation of \(-13.73\)dB. The resulting narrowband filter \( H_f(z) \) has a passband of 0.0026\(\pi\) and an attenuation of \(-57.762\)dB with maximum pole magnitude of 0.962. Figure 2 displays the magnitude response of \( H_l(z) \) and the corresponding pole and zero placements. The interpolated version of \( H_l(z) \) and the filter used to remove images, \( I(z) \) are shown in Figure 3. A close up version of the magnitude and phase response for the final design is presented in Figure 4.

5. ERROR ANALYSIS

As discussed in Section 2, the most prominent error in a photonic implementation of the design arises from delay length variations. We examine how this error manifests in two different configurations: a cascaded lattice allpass structure, and a straightforward Direct Form II implementation. The narrowband filter \( H_f(z) \) can be implemented as a lattice allpass structure using a cascade of the unit cells as shown in Figure 5. Since fabrication error affects each of the cascaded lowpass sections independently, we can examine the effect on \( H_l(z) \) individually. While both the delay component and the allpass in \( H_l(z) \) are affected, we focus our study on \( A_l(z) \) because it contains the critical elements for generating the filter response. We can investigate the effect by first studying a second order allpass filter and then extending the analysis. Imperfection in fabrication can lead to differences in the lengths of the lattice unit cells, leading to the transfer function of the form

\[
H_{UC}(z) = e^{i(\delta + \beta)z} \frac{\rho e^{-j(\delta + \beta)z} - \alpha z^{-1}}{1 - \alpha e^{j(\delta + \beta)z} - 1} 
\]  

The lattice allpass sections in our design have real coefficients, indicating that \( \delta = 2n\pi \) or \( H_{UC}(z) = e^{-j\theta}z^{-1} \). The transfer function of a unit cell under error (11) then directly corresponds to the response of single section in the lattice structure of the form \( H_{UC}(e^{-j\beta}z) \). The overall effect on the allpass structure \( A_l(z) \) implemented using the lattice structure would thus be the result of [11]:

\[
A_l^{lat}(e^{-j\beta}z) = S_l([e^{j\beta}z^{-1}S_2(e^{-j\beta}z)]^{-1}) 
\]  

where \( S_l(z) = \frac{k_1 + z^{-1}}{1 + k_1k_2z^{-1}} \). Expanding the equation, the transfer function \( A_l^{lat}(e^{-j\beta}z) \) becomes:

\[
k_1 + (k_2e^{j\beta} + k_1k_2e^{j\beta})z^{-1} + e^{j(\beta_1 + \beta_2)z^{-2}} 
\]

\[
1 + (k_2e^{j\beta} + k_1k_2e^{j\beta})z^{-1} + k_3e^{j(\beta_1 + \beta_2)z^{-2}} 
\]

(13)

In a Direct Form II implementation, the error results in an overall transfer function \( A_l^{DF}(e^{-j\beta}z) \) of the form

\[
k_1 + (k_2e^{j\beta} + k_1k_2e^{j\beta})z^{-1} + e^{j2\beta z^{-2}} 
\]

\[
1 + (k_2e^{j\beta} + k_1k_2e^{j\beta})z^{-1} + k_3e^{j2\beta z^{-2}} 
\]

(14)

The term \( \delta \) is the result of the fabrication process, and can be viewed as a random variable with Gaussian distribution \( \mathcal{N}(0, \sigma) \). In \( A_l^{lat}(e^{-j\beta}z) \), the \( \delta \)'s can be assumed to be independent and identically distributed (i.i.d.) for the lattice sections since they will be identically fabricated. In a direct form implementation, the same error is coupled with each delay term \( z^{-2} \). To compare the performance of the two structures under error, we can examine the statistical measures of \( A_l(e^{-j\beta}e^{j\omega}) \). Due to length limitation, we skip the derivation here and present the final result. The responses of the two systems can be compared by first looking at the effect of error in the mean \( \mathbb{E}[A_l(e^{-j\beta}e^{j\omega})] \). The expectation involves \( \mathbb{E}[B(\delta)A^{-1}(\delta)] \), which can be approximated by \( \mathbb{E}[B(\delta)] \) for a small \( \sigma \) [12]. Consider \( \mathbb{E}[e^{j\delta}] \):

\[
\mathbb{E}[e^{j\delta}] = \Phi_3(n) = e^{j\mu n}e^{-\frac{1}{2}\sigma^2} = e^{-\frac{1}{2}\sigma^2} 
\]

(15)
where $\Phi_i(n)$ is the characteristic function of a Gaussian random variable. The mean of the lattice implementation under error, $E[A^\text{lat}_f(e^{-j\delta}e^{j\omega})]$ is therefore:

$$
E[A^\text{lat}_f(e^{-j\delta}e^{j\omega})] = \frac{k_1 + e^{-\frac{1}{2}\sigma^2} (k_2 + k_1 k_2) e^{-j\omega} + e^{-\sigma^2} e^{-j2\omega}}{1 + e^{-\frac{1}{2}\sigma^2} (k_2 + k_1 k_2) e^{-j\omega} + k_1 e^{-\sigma^2} e^{-j2\omega}} \tag{16}
$$

Similarly, $E[A^\text{DF}_f(e^{-j\delta}e^{j\omega})]$ is

$$
E[A^\text{DF}_f(e^{-j\delta}e^{j\omega})] \approx \frac{k_1 + e^{-\frac{1}{2}\sigma^2} (k_2 + k_1 k_2) e^{-j\omega} + e^{-2\sigma^2} e^{-j2\omega}}{1 + e^{-\frac{1}{2}\sigma^2} (k_2 + k_1 k_2) e^{-j\omega} + k_1 e^{-2\sigma^2} e^{-j2\omega}} \tag{17}
$$

An allpass structure requires the form $z^{-N} D(z^{-1})$. Comparing $E[e^{jN\delta} z^{-N} D(e^{j\delta} z^{-1})] - E[D(e^{j\delta} z)]$, the error measure for the lattice implementation is $|1 - e^{-\sigma^2}|$. For Direct Form II implementation, the resulting error is $|1 - e^{-\sigma^2}|$. This result indicates that the lattice realization is less susceptible to error in the mean than direct form. Consequently, it can also be readily shown that $\text{var}[A^\text{lat}_f(e^{-j\delta}e^{j\omega})] < \text{var}[A^\text{DF}_f(e^{-j\delta}e^{j\omega})]$.

The second order error analysis can be extended to our $l^{th}$ order $A_l(z)$ design. Since the fabrication error is in the form $A_l(e^{-j\delta}e^{j\omega})$, a direct form implementation would carry correlated errors in every term of the transfer function. From (15), it can be concluded that the i.i.d. error terms in the $l^{th}$ order lattice form yield a transfer function that is less sensitive to error as well. Figures 6 and 7 show the effect of the delay variation on the final structure using the two different implementations.

### 6. Conclusion

We present a method of designing narrowband filter that is realizable using photonic components. The approach emphasizes consideration of the physical limitations that arise as a result of the fabrication process of the photonic components. Our result shows a filter structure that achieves 0.0025 rad passband edge and 60 dB stopband attenuation. Since our design is purely based on allpass and delay components, it can be readily realized using all passive photonic building blocks. We also demonstrate the effect of a unique error to photonic filters in the form $H(e^{-j\delta}z)$. From statistical measures, it can be concluded that the lattice allpass implementation is more robust to photonic component error than a direct form realization.

### 7. References


