ABSTRACT

Hyperspectral imaging analysis aims at the estimation of the number of constituent substances, known as endmembers, their spectral signatures as well as their abundance fractions. Due to the nature of hyperspectral sensors, output data is mostly associated with correlated noise rather than with the white Gaussian noise considered in most of the analysis. In the presence of correlated noise, estimation of dimensionality with the assumption of white noise is associated with considerable error. This error in the very first step will be propagated to the next steps and fully invalidate the unmixing process. On the other hand, existing methods which consider a correlated noise are lacking in robustness to noise. A Whitened Noiseless Code-length method (WNCLM) is presented for hyperspectral signals dimension estimation and unmixing in the presence of spectrally or spatially correlated noise. Variance and correlation coefficients are calculated to estimate the noise correlation matrix. This matrix is further used to whiten the noise. New processed hyperspectral data then goes through a simultaneous denoising and Least Square Error (LSE) based unmixing process that leads to the estimation of data dimensionality. Some numerical simulations are provided to illustrate the effectiveness of our proposed method.

1. INTRODUCTION

Hyperspectral unmixing is in fact a blind source separation problem that consists of recovering the independent signals from their linear mixtures without any apriori knowledge of the mixtures. Estimation of the number of constituent substances which represents the dimensionality of hyperspectral data is a key point factor in an unmixing problem. The structure of the noise in hyperspectral data affects the accuracy of this estimation. Illumination variation effect, atmospheric characteristics and tunable filter spectral efficiency in a hyperspectral system generally lead to a correlated noise. The assumption of white Gaussian additive noise causes a considerable error in the estimation of hyperspectral data dimensionality. In [11] a Neyman-Pearson detection theory-based thresholding method (named HFC) has been developed to determine the number of spectral endmembers in hyperspectral data (referred to as virtual dimensionality in [6]). The whitening method in [6] uses the decomposition of the sample covariance matrix to equalize the noise variance of each band in the whitened signal. In [2] a whitening procedure has been proposed by eigenvalue decomposition of a positive definite linear combination of a set of correlation matrices taken at nonzero lag. In [12] a whitening transformation has been proposed to enhance the Signal-to-Noise Ratio (SNR) in colored structural noise. Approaches proposed in [7] and [3] estimate the signal source by pre-whitening the signal followed by a unitary transformation, which jointly diagonalize a set of correlation matrices. In [5] diagonalization is applied to cumulant matrices while in [4] a special time frequency distribution matrix is considered. Even though noise whitening seems to be a well known topic in the literature, a complete and accurate solution for hyperspectral signal dimension estimation in the presence of a strong correlated noise has not yet been proposed. The method proposed in [6] is lacking the accuracy for noisy applications where SNR is below 15 db, and yet the method is still not providing the estimation of endmembers and abundance fractions. In this paper, we propose WNCLM as a complete solution for hyperspectral unmixing in the presence of strong correlated spectral or spatial noise. This method is an extension of our NCLM method (presented in ICASSP09) for the case of spectrally or spatially correlated noise.

2. MATHEMATICAL MODEL

We consider a linear mixture model due to its effectiveness and simplicity. Assuming that the hyperspectral image has $K$ spectral bands, the noiseless data in this model for the pixel $(i,j)$, is formulated as

$$\tilde{y}_{i,j} = A_p S_{i,j} = \sum_{i=1}^{p} a_i s_{i,j},$$

where the elements of $\tilde{y}_{i,j} \in R^K$ are the original noiseless hyperspectral signal at the kth spectral band. We consider $c$ endmembers as the constituent materials in the scene. Therefore, $A_p = [a_1 \ a_2 \ ... \ a_p]$ is a $K \times p$ source matrix, with each column $a_\theta$ being the spectral signature of endmember $\theta$. We assume that $\tilde{y}_{i,j}$ has been originally formed by a linear combination of $p$ endmembers which belong to a spectral library including $P$ spectral signatures ($p < P$). Parameter $p$ is also denoted as the dimension of hyperspectral data. The abundance
vector for the pixel \(i,j\), \(S_{i,j} = [s_{ij1}, s_{ij2}, \ldots, s_{ijp}]^T \in \mathbb{R}^p\), consists of the mixing coefficients for the pixel \((i,j)\) satisfying two physical constraints \(s_{ij} \geq 0\) (non-negative) and \(\sum_{j=1}^{p} s_{ij} = 1\) (sum-to-one). The noisy measured hyperspectral signal, \(y_{i,j,k}\), for pixel \((i,j)\) in \(k\)th band is given as

\[y_{i,j,k} = \tilde{y}_{i,j,k} + n_{i,j,k}\]  

(2)

In (2) the noise term \(n_{i,j,k}\) is considered as a fully stochastic process. Noise is regarded as a zero-mean Gaussian process independent of \(\tilde{y}\), stationary along \((i,j)\) but not along \(k\), and correlated spectrally and spatially. Spatial stationarity of noise implies that the noise variance remains constant in each spectral band for different regions. Noise correlation coefficients are \(\rho_x, \rho_y, \rho_\lambda\) in spatial and spectral directions respectively.

### 3. NOISE ESTIMATION

Since the noise variance is only a function of the spectral band index, the variance of (2) on a general region \(\xi\) on the \(k\)th spectral layer is calculated as

\[\sigma^2_y(\xi, k) = \sigma^2_\theta(\xi, k) + \sigma^2_n(k)\]  

(3)

where \(\sigma^2_\theta(\xi, k)\) is the noise variance in the \(k\)th spectral band and \(\sigma^2_n(\xi, k)\) is calculated over all pixels belonging to the region \(\xi\). We assume a first-order Markov model for the noise \([1]\):

\[n(k+1) = \rho_n \cdot n(k) + \varepsilon_n(k+1)\]  

(4)

where \(\varepsilon_n(k+1)\) is a white Gaussian random process having a variance \(\sigma^2_n(1-\rho_n^2)\). From (3), by considering spatial stationarity, it appears that \(\sigma^2_n(\xi, k)\) can be estimated by averaging the \(\sigma^2_n(\xi, k)\) over a homogenous area where \(\sigma^2_\theta(\xi, k) \equiv 0\). In fact, a homogenous area is defined as a scanned region related to the same material on all pixels in that region. Thus, we have \(\sigma_n(\xi, k) = \sigma_n(k)\) in a general spectral band in homogenous areas. Therefore, an estimate of \(\sigma_n(k)\) namely \(\hat{\sigma}_n(k)\), is the \(y\)-intercept of the horizontal regression line drawn on the scatterplot of estimated variance \(\sigma_n(y)\) versus estimated mean \(\hat{\mu}_y\). To avoid the drawback of a supervised method to find the homogenous areas, we adopt a similar automatic method presented in [1] where a homogenous area originates a cluster of scatterpoints aligned along a horizontal line having \(y\)-intercept equal to \(\hat{\sigma}_n(k)\). We can now consider the covariance of unity lag along either of the coordinate directions, say \(k\). For simplicity, we eliminate the index \(i,j\):

\[C_y(k;1) = E[(y(k) - \bar{y}(k))(y(k+1) - \bar{y}(k+1))]\]  

(5)

Replacing (2) in (5) and using the independency between noise and reflectance signal to eliminate the expected value of the noise multiplicative terms, we have

\[C_y(k;1) = C_y(k;1) + E[n(k).n(k+1)]\]  

(6)

Multiplying both sides of (4) by \(n(k)\) and taking the expected value we can obtain

\[E[n(k).n(k+1)] = \rho_n.\sigma_n^2(k)\]  

(7)

Replacing (7) in (6) will result in to the unity-lag noise covariance of the noisy signal:

\[C_y(k;1) = C_y(k;1) + \rho_n.\sigma_n^2(k)\]  

(8)

In a homogenous area, the term \(C_y(k;1)\) is identically zero such that

\[C_y(k;1) = \rho_n.\sigma_n^2(k)\]  

(9)

Hence \(\rho_n\) can be estimated from the slope of the covariance-to-variance scatterplots in those areas. Knowing the \(\rho_n\) for each band, we can provide the noise correlation matrix in spectral direction as \(R_n \in \mathbb{R}^{K+K} (K\) is the total number of spectral bands in the data cube) such that

\[R_n(i,j) = \begin{cases} 1 & i = j \\ \prod_{k=1}^{K+1} \rho_k & i \neq j \end{cases}\]

Since \(R_n\) is available, the whitened noisy signal \(y_w\) would be calculated as

\[y_w = R_n^{-1/2}y = (V_n\Lambda_n^{-1/2}V_n')y\]  

(10)

The term in parentheses is provided based on the eigenvalue decomposition of \(R_n\).

### 4. UNMIXING THE HYPERSPECTRAL DATA

We have proposed an iterative LSE-based unmixing algorithm in [9]. To save space, we briefly explain the method and will leave the details for [9]. Our proposed algorithm starts with certain initial endmembers, then finds the most possible abundance distributions of all image pixels based on the constrained least squares. At each step, we define a subset including the estimated endmembers in the last \(m\) steps as \(\hat{A}_m = [\hat{a}_1 \hat{a}_2 \ldots \hat{a}_m]\). The LSE solution to the abundance vectors from (1) is

\[S_{i,j,m} = T_m y(i,j)\]

\[T_m = (\hat{A}_m' \hat{A}_m)^{-1} \hat{A}_m'\]

(11)

\(T_m\) is the transition matrix that is calculated only once for each subset. This makes WNCLM considerably faster compared to the other unmixing approaches. The next endmember is selected as the pixel that generates the largest residual:

\[(i,j)^* = \arg \max_{i,j} \| y(i,j) - \hat{A}_{m-1}S_{i,j,m-1} \|\]  

(12)

\[\hat{a}_m = y(i,j)\]  

(13)
5. HYPERSPECTRAL DATA DIMENSION ESTIMATION

In [10] we presented an information theoretic approach to find the optimum dimension based on our defined reconstruction error $z_{S_m}$ for different subsets $S_m$ created by our method. Based on this error we have defined a description length for each subset

$$DL(\hat{y}; \hat{y}_{S_m}) = \log_2 \sqrt{2\pi e} + \log_2 e z_{S_m}$$

Due to the extensive mathematics in the method, we are leaving the detail to [10] to the reader. We have proved that the optimum subset is the one that holds

$$S_{m^*} = \arg\min_{S_m} DL(\hat{y}; \hat{y}_{S_m})$$

Index $m$ associated with the optimum subset represents the dimensionality of the hyperspectral data. As an advantage of WNCLM, the denoising and dimension estimation are performed simultaneously leading to more robustness to noise.

6. SIMULATION RESULTS

The spectral reflectance used in the subsequent experiments was selected from the USGS digital spectral library [8] which contains 224 spectral bands covering wavelengths ranging from 0.38 to 2.5 $\mu$m. A set of spectral profiles was selected as the endmembers to generate the mixture. Abundance fractions, $s_{ij}$, were generated according to a Dirichlet distribution. The data cube was 64 $\times$ 64 pixels in 188 bands. In the first step, we selected five endmembers from the USGS library to form the synthetic data. We added a colored noise with a correlation coefficient, $\rho_\lambda = 0.5$, along the spectral bands with a unity variance. We concatenated all rows ($i$ direction) in the data cube to create a 4096 $\times$ 188 data matrix and divided the new matrix into 9 $\times$ 9 sliding blocks in the direction of the spectral bands (sliding one pixel at a time). For each sliding block, variance and covariance were calculated. Figure 1 shows the scatter plot of the variance and covariance for all blocks. We fitted a horizontal line into the variance scatterplot based on the LSE of the distance to the line. The $y$-interception of the line is equal to 1.02 which matches the variance of the original colored noise. We then fitted a line that passes the origin (since we selected a zero-mean noise) into the covariance-variance scatter plot. The slope of the line as shown in the figure is 4.9 which is identical to the correlation coefficient of 0.5 selected originally in the synthetic signal. Table 1 shows the estimated values versus true values of different variance and correlation coefficients associated with the synthetic hyperspectral data. We selected different number of endmembers and SNR values to consider a variety of mixed scenarios. It can be seen that the estimation provided by the scatter plot method is fairly accurate in different ranges of $\rho_\lambda$ and $\sigma_\lambda$ or $p$ and SNRs. Figure 2 shows the result of the data dimension estimation. In two different scenarios, we used 9 and 12 endmembers from the USGS library to generate the hyperspectral data. In each figure, $z_{S_m}$ was plotted for each subset. It can be seen that the reconstruction error has a minimum value at a subset index which is identical to the number of original endmembers used in the generation of the synthetic data. Thus the data dimension has been estimated precisely for two different dimensionalities. Further simulation results showed that our method was quite robust for the number of endmembers in the scene. Figure 3 shows the result of data dimension estimation for different SNR values. We selected 10 endmembers from the USGS library and considered two different SNRs (2db and 5db) to generate the signals. The minimum value of the reconstruction error $z_{S_m}$ occurs at $m=10$ in both cases. It can be seen that our method is able to estimate the dimensionality accurately when the SNR is as low as 2db. This SNR is lower than any SNR in a real hyperspectral application, indicating that our method is extremely robust to the noise.

Table 1. True and estimated value of variance and correlation coefficients

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<th>$\sigma_\lambda$</th>
<th>$\rho_\lambda$</th>
<th>$\rho_\lambda$</th>
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<td>5</td>
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A robust unmixing method to process the hyperspectral signal in the presence of correlated noise has been presented. Most of the existing hyperspectral unmixing methods with the assumption of white Gaussian noise are just a simplification of the real scenario and therefore do not accurately estimate the dimensionality. On the other hand, in noisy applications such an assumption would lead to an estimation of data dimension which is quite inaccurate. Simulation results showed that our proposed method (WNCLM) is robust to the noise as well as number of the endmembers in the hyperspectral signal. Our proposed method outperformed the existing approaches by providing an accurate dimension estimation in very low SNR applications. Owing our use of a transition matrix in the unmixing stage, WNCLM is also substantially faster than many existing approaches.

7. CONCLUDING REMARKS

8. REFERENCES


