We present a new image denoising method based on the nonlocal means filtering in the wavelet domain. A noisy image is first decomposed into subbands by wavelet transform and the nonlocal means filter is applied to each subband. It is also noted that the performance of the nonlocal means filter depends on the kernel bandwidth (size of the filter) and the image properties. Hence we propose a method to adjust the kernel bandwidth for each of the subband images, based on the estimation of noise statistics. This filtering method preserves the wavelet coefficients corresponding to the structures, while effectively suppressing noisy ones. Experimental results show that the proposed method provides comparable or sometimes higher peak signal-to-noise ratio (PSNR) than the state-of-the-art wavelet denoising methods and the spatial nonlocal means filter. Subjective comparison also shows that the proposed method provides better contrast than the spatial nonlocal means filter, and less ringing artifacts that commonly arise in the conventional wavelet denoising.

Index Terms— image denoising, wavelet, bandwidth, nonlocal means filter

1. INTRODUCTION

Image denoising has been extensively studied and thus there is a large amount of literature on denoising. Among these numerous works, we will briefly mention only a few of recently developed methods that are related with our method, specifically the wavelet domain coefficient thresholding and modeling [1, 2, 3] and nonlocal means filter [4].

In the case of wavelet domain thresholding methods, an image is decomposed into subbands and noisy coefficients are suppressed by hard or soft thresholding. These methods are shown to provide pleasing results while requiring not much computational complexity. The most widely used thresholding techniques may be the VisuShrink [1] and BayesShrink [2]. The probabilistic coefficient modeling method [3] fits the neighborhood of a coefficient as the Gaussian Scale Mixture (GSM) model and applies the Bayesian Least Squares (BLS) technique to adjust the coefficients. Although the wavelet domain denoising provides relatively high PSNR improvement, shrinking or modifying wavelet coefficients sometimes bring ringing or wavelet-like noise. Specifically, if a signal with a step edge is wavelet transformed, the coefficients consist of larger ones around the edge position and smaller ones around it. When the smaller ones are removed by thresholding and inverse transformed, then there arise ringing artifacts due to loss of high frequencies. In the case of probabilistic wavelet coefficient modeling, wrong coefficients can be generated in the flat area, which results in the wavelet-like noise in the spatial domain.

From the aspect of kernel density estimation, the nonlocal means filter can be considered as a Nadaraya-Watson estimator, which is a local constant regression [5]. The smooth kernel estimate in the nonlocal means approach is a sum of bumps placed at the data points. The kernel function determines the shape of the bumps, and the "smoothing parameter" or "bandwidth" denoted as \( h \) controls the degree of smoothness. In [6], an automatic bandwidth selection method was proposed based on the reduction of the entropy of the image pattern, while the global bandwidth was applied to the overall areas of images. However, narrower kernels are prone to be used in the regions with more available samples, whereas larger kernels are more suitable for the more sparsely sampled areas of the image. Hence it is important to find an appropriate bandwidth according to the local characteristics, which is not an easy task.

The problem with the conventional wavelet domain filtering is the removal of small but important coefficients while thresholding or the generation of unwanted coefficients in the probabilistic modeling approach as stated above. In this paper, it is expected that the nonlocal means filtering of the coefficients can alleviate these problems while effectively removing noisy coefficients. Specifically we propose a wavelet domain image denoising method where the nonlocal means filtering is applied to each of the subbands. In this process, it is also noted that the bandwidth of the filter affects the performance, and thus we propose an adjustable bandwidth depending on the subband and its property.

The rest of this paper is organized as follows. In the sec-
ond section, we review the nonlocal means filter and its bandwidth parameter estimation. In the third section, we propose the extension of nonlocal means filter to the wavelet domain denoising and a criterion for the bandwidth selection for the wavelet subbands. Then, we show some experimental results on synthetic and real images.

2. RELATED WORKS

2.1. Nonlocal Means Filter

Let us denote a noisy observation of an image as \( y(i) = u(i) + n(i) \), i.e., \( y(i) \), \( u(i) \) and \( n(i) \) are the intensities of the pixel \( i \) of the noisy observation, the original image, and the noise. We assume that \( n(i) \) is the signal-independent additive white Gaussian noise of zero mean and variance \( \sigma^2 \). Also we define \( N_i \) and \( S_i \) as a square neighborhood and a square search-window centered at the pixel \( i \) respectively. Then the nonlocal means filter proposed by [4] can be described as

\[
\hat{u}(i) = \frac{1}{Z(i)} \sum_{j \in S_i} e^{-\frac{|y_j - y|^2}{2\sigma^2}} y(i)
\]

where \( Y_i \) represents the vector of pixel intensities in \( N_i \), \( Z(i) = \sum_{j \in S_i} e^{-\frac{|y_j - y|^2}{2\sigma^2}} \) is a normalizing factor, and \( h \) is the smoothing kernel width which controls the degree of averaging. From eq.(1), it can be seen that a small \( h \) shrinks the area of averaging and thus noise is not likely to be suppressed enough. Conversely, if \( h \) is too large, the weights at the boundary of \( S_i \) are also very large, which results in blurry output. In the conventional work [4], \( h \) is set between 10\( \sigma \) or 15\( \sigma \) and the noise standard deviation \( \sigma \) is estimated from the image statistics.

2.2. Bandwidth Selection

Choosing an appropriate bandwidth is thus very important for the balanced nonlocal means filtering. Traditionally, the bandwidth \( h \) is selected to minimize the error between the estimate of density and the true density. For this purpose, the mean square error (MSE) at a point \( x \) is defined as [5]

\[
MSE_x(p_{KDE}) = E[(p_{KDE}(x) - p(x))^2]
\]

where \( p_{KDE}(x) \) is the estimate of density and \( p(x) \) is the true density at \( x \), respectively. This shows that there is a trade-off between bias and variance, which also means that a large bandwidth is likely to reduce the variance of the estimator but increase the bias and vice versa.

3. WAVELET DOMAIN NONLOCAL MEANS FILTER WITH ADAPTIVE BANDWIDTH

For exploiting the advantages of the wavelet-domain signal processing and the nonlocal means filter, simultaneously we attempt to apply the nonlocal means filtering to the wavelet domain. The main idea behind this approach is to exploit the excellent localization property of the wavelet transform as demonstrated in the conventional wavelet domain denoising, while keeping the main coefficients and its neighbors(structures) which might have been shrunk in the conventional wavelet denoising. That is, by averaging structures similar as the current significant coefficient and its neighbors by the nonlocal means filtering, the structures are kept while the noisy coefficients are averaged out. Thus it is expected that the ringing artifacts would be alleviated compared to the conventional wavelet denoising while keeping the structures very well like the spatial domain nonlocal means filter.

Another novelty of the proposed method is the subband adaptive bandwidth selection. As already mentioned, the fixed bandwidth does not well reflect the local variability of the data set. The motivation of bandwidth adjustment is that the subbands of the wavelet domain have their unique characteristics and thus the bandwidth of the filter should be optimally controlled accordingly. Moreover image noise may have spatially varying properties which are reflected to the variation in the subband characteristics.

Typically, the selection of optimal bandwidth \( h \) is based on the minimization of the mean integrated squared error (MISE) between the kernel with \( h \) and the true but unknown density \( f(x) \) as [5]

\[
MISE(f_h) = \int E[\hat{f}_h(x) - f(x)]^2 dx.
\]

Since the plug-in approach [7] is currently known one of the best data-driven methods for bandwidth selection, we employ this method to minimize \( MISE \) and thus obtain a bandwidth, \( h \). According to the plug-in method, the optimal bandwidth is

\[
h_{\text{opt}} = \left( \frac{\|K\|^2}{\|f''\|^2 \mu_2(K)^2 N} \right)^{1/5}
\]

where \( \|K\|^2 = \int K(x)^2 dx \) and \( \mu_2(K) = \int x^2 K(x) dx \) are constants depending on the kernel function \( K \), and \( N \) is the number of the sample data. Note that \( \|f''\|^2 \) is the only unknown term in (4) and the idea behind the plug-in estimate is to replace \( f'' \) by an estimate from the data. Silverman’s rule of thumb[8] computes \( f'' \) as if \( f \) were the density of the normal distribution \( N(\mu, \sigma^2) \) and then the optimal bandwidth can be approximated value as

\[
h_\alpha = \left( \frac{4\sigma^5}{3N} \right)^{1/5} \approx 1.06\sigma N^{-1/5}
\]

where the kernel function \( K \) is also assumed to be the Gaussian kernel.

The result shows that the variation of the bandwidth depends on the noise density characteristic. In this paper, we introduce the preliminary estimation of the noise standard deviation \( \sigma \) empirically from each subband wavelet coefficients as [9]
Table 1. The PNSR results of the nonlocal means filter, the BayesShrink method, the multiresolution bilateral, and the proposed method for the simulated additive white Gaussian noise

<table>
<thead>
<tr>
<th>Denoising Method</th>
<th>( \sigma_n )</th>
<th>Barbara</th>
<th>Lena</th>
<th>Hill</th>
<th>Peppers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>512</td>
<td>512</td>
<td>512</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>26.65</td>
<td>29.22</td>
<td>27.26</td>
<td>27.53</td>
</tr>
<tr>
<td>Bayes Shrink[2]</td>
<td>20</td>
<td>26.76</td>
<td>28.09</td>
<td>25.97</td>
<td>27.05</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>24.28</td>
<td>25.01</td>
<td>24.58</td>
<td>25.07</td>
</tr>
<tr>
<td>Multi. Bi. [10]</td>
<td>20</td>
<td>27.25</td>
<td>30.10</td>
<td>28.82</td>
<td>28.52</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>25.20</td>
<td>27.60</td>
<td>26.78</td>
<td>26.20</td>
</tr>
<tr>
<td>BLS-GSM[3]</td>
<td>20</td>
<td>29.08</td>
<td>32.24</td>
<td>30.10</td>
<td>30.56</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>26.78</td>
<td>30.45</td>
<td>28.51</td>
<td>28.52</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>20</td>
<td>30.28</td>
<td>31.63</td>
<td>29.75</td>
<td>29.23</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>27.50</td>
<td>29.52</td>
<td>27.95</td>
<td>27.61</td>
</tr>
</tbody>
</table>

\[
\hat{\sigma} = 1.4826 \text{med}(\|r - \text{med} r\|) \quad (6)
\]

where \( r = \{ r_1, r_2, ..., r_{|G|} \} \) is the set of local residuals of the entire subband wavelet coefficients defined as

\[
r_i = \frac{2Y_{i_1,i_2} - (Y_{i_1+1,i_2} + Y_{i_1,i_2+1})}{\sqrt{6}} \quad (7)
\]

and \( Y_{i_1,i_2} \) denotes the observation of \( Y_i \) at the point \( i = (i_1, i_2) \).

In summary, our framework of bandwidth selection is based on the idea that the optimal bandwidth can be obtained from an initial value found by the plug-in method, and the initial choice of the bandwidth influences the whole bandwidth selection. To be specific, we adaptively regulate the bandwidth in each subband according to the relationship, \( h = kh_o \), where \( k \) is a scaling factor.

4. EXPERIMENTAL RESULTS

Daubecheie’s orthogonal wavelet is used for the subband decomposition, specifically \( db8 \) filters in MATLAB is used for one-level multiresolution analysis. In the experiments with the artificially added noise, it is assumed that the noise standard deviation \( \sigma \) is known. The noisy images are denoised using four state-of-art algorithms and the proposed method, where the parameters of the compared methods were adjusted considering the tradeoff between structure preservation and noise suppression. The neighborhood size in the nonlocal means method is set to 21 \( \times \) 21 window size, instead of searching through the whole image, in order to reduce the computing time. Specifically, it is the best in the case of Barbara image, and almost the second best next to BLS-GSM.

The results are compared subjectively and objectively, where the objective measure is the PSNR although it does not always faithfully reflect the visual quality. The objective comparison is first summarized in Table 1, which shows the PSNR values of each algorithm for the noise variance of \( \sigma = 20 \) and \( \sigma = 30 \). It can be seen that the proposed method shows comparable or higher PSNR values to the state-of-the-art methods.

Fig. 1 gives the comparison of the visual quality between the spatial nonlocal means filter and the proposed method. The details of the tassels at the bottom of the hat in (b) are better restored than those in (a). Fig. 2 shows the subjective comparison with the state-of-the-art wavelet denoising where the ringing artifacts are observed in the case of BLS-GSM. Although BLS-GSM gives higher PSNR values than other algorithms in many cases, that does not guarantee visual qualities as it can be seen in Fig. 2 that shows a part of the hat...
and the proposed method gives superior visual qualities on the edges and the flat regions. Fig. 3 shows the visual comparison of the denoised images from the fixed bandwidth and the subband adaptive bandwidth. As well as the lower MSE values of the subbands, the subband dependent bandwidth gives the much less blurred texture than the fixed $h$. Fig. 4 shows the subjective comparison for a real noisy image between BLS-GSM and the proposed method. The denoised result, (a) by BLS-GSM has more artifacts than the result by the proposed method. Especially the proposed method gives less ringing artifacts while the results of BLS-GSM have many artifacts.

5. CONCLUSION

We have proposed a new image denoising algorithm based on the nonlocal means filtering in the wavelet domain. By the nonlocal means filtering, the small wavelet coefficients which are part of important image structures are well kept while suppressing the noisy coefficients, whereas the conventional wavelet denoising methods sometimes suppress small but important coefficients as well. It is also noted that the kernel bandwidth in nonlocal means filter needs to be changed according to the properties of the images. Hence we have also proposed a method to find the appropriate kernel bandwidth to each of the subband images for their effective nonlocal means filtering. As a result, the proposed method provides comparable or sometimes higher PSNR than the conventional algorithms. Also subjective comparisons show that the proposed method keeps the structures of the images very well and gives less ringing artifacts compared to the conventional wavelet denoising methods.

6. REFERENCES


