IMPROVED TRANSIENT OSCILLATION DETECTION WITH MULTIWAVELETS

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ABSTRACT

Multiwavelets have recently been used for analysis of transient oscillations. Here we use them for transient oscillation detection by incorporating them into two existing Morlet-based self-normalizing detectors, one that operates on a single time series and another on two correlated time series. The Morlet-based detectors are effective in detecting weak, transient oscillations, especially in strong clutter where FFT based methods fail. Simulated receiver operating curves reveal that the new multiwavelet detectors show marked improvement when applied to complex, transient oscillations in white Gaussian noise and 60 dB clutter. At 0.1% false alarm rate for a (signal on) signal to noise ratio of 0.25, detection rates are: 95.5% and 99.6% for the single and dual time series detectors, respectively. These compare to 75% and 95.5% for the Morlet wavelet detectors.

Index Terms— signal detection, wavelet transforms, statistics, time-varying, multiwavelets

1. INTRODUCTION

The analysis and detection of transient oscillations is currently of interest in many diverse applications such fluids, physiology and geophysics. Many researchers are using wavelets for this analysis, and most recently, multiwavelets (e.g., [1]). This paper modifies a set of detectors developed previously that were based on a single Morlet wavelet to now utilize multiwavelets. Since the first Morse multiwavelet is identical to the Morlet wavelet for certain parameter choices [2], the Morse wavelets are a natural extension of the single Morlet wavelet.

We focus specifically on the detection of short duration, complex, tapered oscillations with (signal on) signal to noise ratios (SNR) less than 1 (0 dB) in white, Gaussian noise and in strong, low frequency clutter. The transient’s amplitude, phase, frequency, duration and bandwidth are unknown a priori, and multiple transients may occur simultaneously. The bandpass nature of the Morlet and Morse wavelets ensures their usefulness in the strong clutter that renders other detector approaches ineffective [3, 4].

The set of existing detectors under modification include one detector that applies to a single time series [3] and another detector that uses two correlated time series [4]. The detectors employ the continuous Morlet wavelet as a signal conditioner, then compute a self-normalization on the wavelet power spectrum or wavelet cross spectrum, as appropriate, and finally apply a binary hypothesis test for noise only. For brevity, we will refer to the existing single time series detector as NSM, the self normalized dual time series detector as NCM and their multiwavelet modifications as NSMW and NCMW, respectively.

The next section will introduce the multiwavelet detectors. Subsequent sections describe the means of measuring the effectiveness of the detectors via receiver operating curves (ROCs) and present a comparison of the single time series and dual time series detectors applied to vibrations recorded from a bleeding porcine artery. The paper concludes with a discussion of the relative advantages and disadvantages of the detectors.

2. THE DETECTORS

2.1. NSM and NCM detectors

The NSM detector involves the following steps. 1) Compute the continuous wavelet power spectrum of a time series with the Morlet mother wavelet. 2) Compute adaptive noise estimates at each time, t, and wavelet scale, s. 3) Normalize the resulting wavelet power spectrum, resulting in N(t,s). 4) Compare the resulting values with the distribution under the null hypothesis of a time series consisting of independent Gaussian noise. Declare a detected oscillation when N(t,s) exceeds the distribution at the desired confidence level. See [3] for more details.

The NCM detector involves similar steps. 1) Using the Morlet mother wavelet, compute the continuous wavelet power spectrum of each of two time series corresponding to the same suspected oscillation, assuming independent noise. 2) Compute adaptive noise estimates at each time and wavelet scale. 3) Normalize the two resulting wavelet power spectra, and multiply them to form the magnitude squared of the normalized cross wavelet spectrum, N(t,s). 4) Same as for the NSM. See [4] for more details.
2.2. NSMW and NCMW Detectors

The multiwavelet versions of the NSM and NCM detectors follow most of the same steps, with the modifications described below.

**Step 0:** Determine the number, K, of Morse multiwavelets to use. (Use gamma = 3 and beta = 24 so that the first of the multiwavelets matches the Morlet wavelet of [3] and [4].)

**Step 1:** Compute the K multiwavelet transforms and corresponding wavelet power spectra for each time series. Find the average (at each time and wavelet scale) of the K wavelet power spectra for each time series.

**Step 2:** Compute noise estimates, h(t,s), for each of the averaged wavelet power spectra at each time, t, and wavelet scale, s. As discussed in [3] and [4], to accomplish this, first a maximum possible oscillation time duration must be assumed. To make the adaptive estimates, each averaged wavelet power spectrum is considered one wavelet scale at a time. Then, for each sample of interest, R(t,s), two intermediate noise estimates are made, one prior to (h_{prior}(t,s)) and one subsequent to (h_{following}(t,s)) the time sample of interest. Each intermediate noise estimate is computed as the mean of the wavelet power spectrum over a selected time duration. The adaptive noise estimate, h(t,s), is set equal to the minimum of h_{prior}(t,s) and h_{following}(t,s).

**Step 3:** Compute the normalized wavelet power spectrum, N_i(t,s), corresponding to the ith original time series according to:

\[ N_i(t,s) = \frac{R_i(t,s)}{\frac{h_i(t,s)}{c(s)v(s)}}, \]  

where c(s) and v(s) are the scale-dependent values that cause the noise estimates, h_i(t,s), to be closely approximated as a random variable with a chi-square distribution (with v degrees of freedom) after it has been multiplied by a scale-dependent constant, c(s): h_i(t,s) ~ c(s)\chi^2_v. Further, m is the scale-independent value that causes the wavelet power spectrum corresponding to the ith multiwavelet, R_i(t,s), to have the distribution of a chi-square random variable with 2K degrees of freedom after it has been multiplied by the scale-independent constant (m): R_i(t,s) ~ m\chi^2_{2K}. (Averaging K multiwavelet power spectra changes the number of degrees of freedom to 2K [1].) Determine the value of m by first computing a time averaged estimate of the first moment and then dividing it by the number of degrees of freedom. To determine values for c(s) and v(s), estimate the first and second moments of h_i(t,s) via averaging over time, and then solve for c(s) and v(s) as follows [5]:

\[ v(s) = 2 \frac{E^2\{h_i(t,s)\}}{\text{Var}\{h_i(t,s)\}}, \quad \text{and} \]

\[ c(s) = \text{Var}\{h_i(t,s)\}/(2E\{h_i(t,s)\}), \]

where E represents the expected value and Var{} represents the variance operator.

For the NSMW detector, set N(t,s) equal to N_1(t,s). For the NCMW detector, compute the magnitude squared of the normalized cross wavelet spectrum, N_i(t,s), as the product of N_i(t,s) and N_2(t,s).

**Step 4:** Compare the value of N(t,s) from step 3 for each combination of t and s with a scale dependent threshold for a given confidence level of the appropriate distribution. For the NSMW, it is the F distribution with 2K and v(s) degrees of freedom [3]. For the NCMW detector, it is the distribution whose density is defined by: ([4])

\[ f_z(z) = \int_0^\infty \frac{dy}{y\left[1 + \left(\frac{2}{v}\right)^2 z + \left(\frac{2}{v}\right)^4 z + \left(\frac{2}{v}\right)^6 y\right]^{1+\sqrt{z}}}. \]

3. EVALUATING THE DETECTORS

The effectiveness of the detectors is compared here in the same manner as in [3] and [4] for simplicity in comparing the results. Receiver operating curves (ROCs) have been generated from simulated time series. Further, the detectors have been applied to received ultrasound echoes from a vibrating, bleeding porcine artery. In all cases, the detectors used wavelets with center frequencies ranging between 275 Hz and 2000 Hz in factors of 1.4, except where noted, and with the first multiwavelet having a quality factor of 3.

Simulated oscillations had center frequencies ranging from 500 to 1500 Hz, durations ranging from 10 to 150ms and quality factors ranging from 1 to 10. ROCs were generated for (signal on) SNRs of 0.125, 0.178, 0.25 and 0.5. Detection confidence levels for the NSMW and NCMW detectors were 0.9 (1-10^{-1}), 0.99 (1-10^{-2}), 0.999 (1-10^{-3}), ... 0.999999999 (1-10^{-9}). See [3] and [4] for more details on the simulated realizations. The NSMW and NCMW were implemented for a K of 3 and a K of 5 to examine the advantages of increasing K. A value of 3 was chosen because it spans roughly the same frequency range as when K is 1, though more evenly. Since Morse multiwavelets with a K of 5 span a larger frequency range, increasing K beyond 5 is not expected to yield better results. Morse multiwavelets were generated using JLab [6].
To further examine the value of using multiwavelets compared to single wavelets, the NSM and NCM detectors were also implemented for wavelet center frequencies ranging between 275 Hz and 2000 Hz in factors of sqrt(1.4) to double the number of wavelet scales in the same frequency range.

In addition to the simulated ROC curves, we also applied these detectors to vibrations recorded via ultrasound in a bleeding porcine artery (see Fig. 1). (These were collected using a protocol approved by the University of Washington’s Animal Care department. See [3] for more details.) The data were collected after an induced bleed in anticipation that the bleeding might induce arterial vibration. This particular example was chosen to highlight the ability of the detectors for variable oscillation duration, oscillation strength, and both simultaneous and consecutive oscillations. The detectors were applied to the porcine bleed data seeking vibrations in the 300 to 2000 Hz range in up to two neighboring depth locations. Since vibrations appear symmetrically as both positive and negative frequency components in the power spectrum, the detectors were modified to only indicate a vibration if a detection is made on both sides of the frequency axis.

4. RESULTS AND DISCUSSION

Fig. 2 shows the ROC results for the single time series multiwavelet detectors. In white noise, the NSMW detector has slightly higher detection rates when 5 multiwavelets are used rather than 3, but when 60dB clutter is present with a corner frequency of 80Hz, the detection rates drop significantly for the detector that uses 5 multiwavelets. This response to clutter is due to the fact that Morse multiwavelets with a K value of 3 span roughly the same frequency range as the Morlet wavelet, but a K value of 5 spans a significantly larger frequency range, thus removing the ability to filter out all of the clutter. Comparing Fig. 2 with the similar figure in [3], reveals that the NSMW has much higher detection rates than the NSM when they are implemented with the same factor between scales (1.4). However, when the NSM utilizes double the number of scales (NSMdbl), its results approach, but do not reach, those of the NSMW (results not shown). For an SNR of 0.25 and 0.1% false alarm rate (10^{-3}), detection rates were 95.5% (Fig. 2), 75% [3] and 90.61% (not shown) for the NSMW, NSM and NSMdbl, respectively.

Fig. 3 shows the ROC results for the dual time series multiwavelet detectors, which are consistently higher than for the NSMW. (Note that this figure uses lower SNR values than Fig. 2.) Interestingly, the NCMW yields slightly higher detection rates with a K value of 3 than with a K value of 5, unlike the NSMW detector. Again, though, the detection rates do not change when clutter is added if K is 3, but drop significantly when K is 5, for the same reasons as stated above. As in the single time series case, in white noise the NCMW detector has significantly higher detection rates than the NCM when it is implemented with the same factor between scales (1.4). However, the NCMW yields comparable rates to the NCM when the NCM utilizes double the number of scales (NCMdbl) (not shown). For an SNR of 0.25 and 0.1% false alarm rate (10^{-3}), detection rates were 99.6% (Fig. 3), 95.5% [4] and 99.37% (not shown) for the NCMW, NCM and NCMdbl, respectively.

The detectors’ effectiveness in an application can be examined from Fig. 4, which reveals the detectors applied to a bleeding porcine artery. The confidence levels/thresholds were selected to maximize the detection of the vibrations labeled in Fig. 1 while minimizing apparent false alarms. Here we see that the NSMW detector detected all labeled potential vibrations except the two labeled ‘g’ and ‘i’. The NCMW detected all except the one labeled ‘i’, and it did so without any apparent false alarms, unlike the NCM [4]. Notably, the single wavelet based detectors did not detect potential vibration ‘i’ either [4], so it is likely not truly a vibration, just noise. Also noteworthy is that vibration ‘g’ is
detected by the NSMW if the threshold is lowered to 0.99, but that also yields many more apparent false alarms.

5. CONCLUSION

The NSMW and NCMW detectors are significantly more effective plug-in detectors than their single wavelet counterparts. Further, the NSMW and NCMW detection rates surpass that of the FFT based detectors even in white noise [3] and [4], unlike the NSM and NCM detectors. Since the higher number of multiwavelets, 5, does not consistently yield higher detection rates, and since it yields significantly worse clutter rejection, 3 multiwavelets are preferable for the implementation described here. The NSMW and NCMW have two disadvantages. First, they have a higher computation cost than their single wavelet counterparts when computed with the same number of wavelet scales. Second, the normalizing values, $c(s)$ and $\nu(s)$, must be computed experimentally because the statistics are not yet known for the average of the K power spectra.

8. REFERENCES


