JOINT REDUCE OF METAL AND BEAM HARDENING ARTIFACTS USING MULTI-ENERGY MAP APPROACH IN X-RAY COMPUTED TOMOGRAPHY

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ABSTRACT

Metal and beam-hardening artifacts are tough issues in Computed Tomography (CT) images. This paper proposes an iterative Maximum A Posteriori (MAP) reconstruction algorithm aiming to reduce both of them. This algorithm is based on a multi-energy acquisition system, a Gaussian noise measuring model and a basis material decomposition formula. In the Multi-Energy Computed Tomography (MECT) system, an energy discriminant detector which can measure the flux of photons at different energies is supposed to be employed. Our method can reconstruct two separate base material density maps where the metal and beam-hardening artifacts are highly reduced.

Index Terms— metal artifact, beam-hardening, multi-energy, Computed Tomography, MAP.

1. INTRODUCTION

X-ray CT is one of the most promising techniques used in medical imaging modalities. It aims to characterize the object’s inside structure. The CT images are corrupted by three types of artifacts:

- Motion artifacts, the patient moves during the acquisition,
- Metal artifacts, due to metallic implants and dental fillings,
- Beam-hardening artifacts, caused by polychromatic X-ray source.

In this paper we focus only on the last two types of artifacts. Without any artifact reducing treatments, the artifacts shown in the FBP reconstruction results largely limit its applications. As two major artifacts, the metal artifact is caused by the appearance of the metal which has a high attenuation property in the low attenuating measuring object, while the beam-hardening artifact result from the different attenuation abilities of the materials upon different X-ray beam energies. The typical metal artifact is the anisotropic streaks. And one of the typical beam-hardening artifacts is the cup form effect.

Reducing the artifacts in CT images is a very active research field. As for the metal artifacts, there are two families of approaches: the correction approaches and inversion techniques. In the second family there are two major approaches: the first gives different weights to the elements of the data after the linearization of the forward model, the second inverses the non-linear problem by using a Poisson likelihood [1].

As for the beam-hardening artifacts, pre-filtering methods, correction methods [2], and DE (Dual-Energy) technique are employed in different applications. In pre-filtering method, a filter is used to narrow down the acquisition spectrum. And correction methods work on the polychromatic data. In this type of approaches, one family of approaches correct the reconstructed attenuation map at certain energy by estimating its beam-hardening distortion coefficient. And the other family of approaches take into account the polychromatic acquisition fact. And they try to reconstruct the attenuation map at certain energy or the mass density map. The DE technique shows its promising advantages in reducing beam-hardening artifacts, but most of DE approaches use analytical reconstruction algorithms, which has serious problems in noise management, limited projection number and metal artifacts. A statistical DE approaches was proposed in [3], but it employed a Penalized Weighted Least Square (PWLS) algorithm after the linearization of the problem.

Our approach combines both the advantages of using non-linear likelihood function and the advantages of employing multi-energy technique. In [1] and [4], they proposed polychromatic statistic methods which can reduce both the metal and beam-hardening artifacts. But, they are both based on the Poisson non-linear likelihood function. In our method, a more general Gaussian non-linear likelihood function is proposed. Moreover, in [1], the iterative optimization is done by using a gradient conjugate algorithm with a fixed updating step while in [4] they were looking for a replacement criterion function at each iteration. In our method, a more sophisticated gradient conjugate algorithm with optimum updating step is used.

This paper is organized as below, section 2 shows the notations and analyzes the direct problem. Then, section 3 introduces the Multi-Energy FBP (ME-FBP) reconstruction method and our statistical algorithm. And, section 4 shows the simulation results comparing with the FBP and ME-FBP methods. At the end, section 5 gives a conclusion and previews the future works.

2. PRELIMINARY

2.1. Notations:

In this paper, the operator ◦ denotes the element product between two vectors. The scalars are denoted by lowercase letters, the vectors by bold lowercase letters, the constant by uppercase letters and the matrix by the bold uppercase letters.

2.2. Object parametrization:

We generally suppose that all elements of the object can be estimated of composing two basis materials: water and bone, which has been justified in [5] to be acceptable for human body. So, the map of attenuation $\mu(\varepsilon)$ at the beam energy $\varepsilon$ numericalized on a Cartesian grid composed of $N$ pixels can be written as:

$$\mu(\varepsilon) = m_w(\varepsilon)f_w \circ \rho + m_b(\varepsilon)f_b \circ \rho$$

where $\rho$ is the mass density map of the measuring object, $f_w$, $f_b$ are the ratio maps of water and bone. And $m_w(\varepsilon)$, $m_b(\varepsilon)$ are the
Mass Attenuation Coefficient (MAC) at the energy $\epsilon$ for the two basis materials. They are supposed to be known or can be pre-estimated.

Denote

$$x_w = f_w \circ \rho$$
$$x_b = f_b \circ \rho$$

the density maps of the water and bone as the parameters that we are looking for.

2.3. Direct problem

Rewrite the Beer-Lambert law with each element decomposed in two materials.

$$y(\epsilon) = I_0(\epsilon) \exp \left[ -m_w(\epsilon) R x_w - m_b(\epsilon) R x_b \right] + \eta_M$$

where $y(\epsilon)$ is the expected measuring vector of $M$ values, $I_0(\epsilon)$ is the spectrum at the energy $\epsilon$, $\eta_M$ is an iid Gaussian noise of $M$ values with a variance $\sigma^2$, and $R$ is a matrix $M \times N$ corresponding to the Radon transform.

In this paper, the measures are done at $K$ energies $\epsilon_1, \ldots, \epsilon_K$. We collate all the expectations of the measures in a big vector $y = (y(\epsilon_1), \ldots, y(\epsilon_K))^T$, and the direct problem can be rewritten:

$$y = s \circ \exp \left[ -M_w R x_w - M_b R x_b \right] + \eta$$

where $\eta$ is the iid Gaussian noise of $K \times M$ values,

$$s = \begin{pmatrix} 1_M I_0(\epsilon_1) \\ 1_M I_0(\epsilon_2) \\ \vdots \\ 1_M I_0(\epsilon_K) \end{pmatrix}, \quad M_w = \begin{pmatrix} \text{Id}_M * m_w(\epsilon_1) \\ \text{Id}_M * m_w(\epsilon_2) \\ \vdots \\ \text{Id}_M * m_w(\epsilon_K) \end{pmatrix},$$

$1_M$ is a column vector composed of $M$ 1 and $\text{Id}_M$ is the identity matrix $M \times M$. As we see, the energy dependent information is only contained in the spectrum vector $s$ and the base material MAC matrices $M_w, M_b$.

3. METHOD

3.1. FBP

The beam-hardening artifacts can be shown by applying FBP algorithm directly on the polychromatic data. Here, we sum up the measures at $K$ different energies to estimate the polychromatic data and FBP with a Hanning filter is used to reconstruct the attenuation map.

$$\mu_{FBP}(\epsilon_m) = FBP \{ -\log(\sum_{i=1}^{K} y(\epsilon_i)) \}$$

where $\epsilon_m$ is the average energy of the $K$ energies, so that,

$$\epsilon_m = \sum_{i=1}^{K} \epsilon_i I_0(\epsilon_i)$$

3.2. ME - FBP

In our work, ME-FBP method is implemented as well to reconstruct two separate basis-material fraction maps $x_w^{ME-FBP}$ and $x_b^{ME-FBP}$. They are got by the following steps:

Step 1. Log-linearization,

$$q = -\log|y|_s$$

where $q$ is the sinogram of the measuring object with the same size of the observation $y$.

Step 2. Separation of the sinograms. This step aims to remove the energy dependence combined on the sinogram $q$ and separate it into two sinograms of water and bone. Least square estimations are used.

$$\begin{bmatrix} q_w \\ q_b \end{bmatrix} = (M' M)^{-1} M' y$$

Step 3. Inverse of the basis material density maps, FBP with Hanning filter is used to inverse the density maps of water and bone,

$$\begin{bmatrix} x_w^{ME-FBP} \\ x_b^{ME-FBP} \end{bmatrix} = FBP \{ q_w \}$$

$$\begin{bmatrix} x_w^{ME-FBP} \\ x_b^{ME-FBP} \end{bmatrix} = FBP \{ q_b \}$$

Step 4. Attenuation map at energy $\epsilon$. Besides of the density maps of water and bone, this ME-FBP approach enables us to get the attenuation map $\mu_{ME-FBP}(\epsilon)$ at any given energy based on eq (1),

$$\mu_{ME-FBP}(\epsilon) = m_w(\epsilon)x_w^{ME-FBP} + m_b(\epsilon)x_b^{ME-FBP}$$

3.3. Multi-Energy MAP

Based on the forward model given in (5), we have the likelihood of our model,

$$\Pr(y|x_w, x_b, \sigma) = (2\pi\sigma^2)^{-MK/2} \times \exp \left[ -\frac{||y - s \circ \exp \left[ -M_w R x_w - M_b R x_b \right]||^2}{2\sigma^2} \right]$$

Convex edge-preserving Huber priori information is used for the density maps of water and bone.

$$\Pr(x_w|x_b, \sigma_w) \propto \exp \left[ -\frac{\Phi_\delta(D x_w)}{2\sigma^2_w} \right]$$

$$\Pr(x_b|x_w, \sigma_b) \propto \exp \left[ -\frac{\Phi_\delta(D x_b)}{2\sigma^2_b} \right]$$

where $\sigma^2_w$ is the variance of the basis material density map. Be simple, we suppose the water and bone maps have the same variance.

$$\Phi_\delta(t) = \sum_{n=1}^{N} \phi_\delta(t_n), \quad \phi_\delta(t) = \left\{ \begin{array}{ll} t^2 & \text{if } |t| \leq \delta \\ 2\delta|t| - \delta^2 & \text{if } |t| > \delta \end{array} \right\}$$

and $D$ is the matrix of the finite difference. We get the posterior law,

$$\Pr(x_w, x_b|y, \sigma, \sigma_w) \propto \exp \left[ -\frac{\Phi_\delta(D x_w) + \Phi_\delta(D x_b)}{2\sigma^2} \right]$$

$$\times \exp \left[ -\frac{||y - s \circ \exp \left[ -M_w R x_w - M_b R x_b \right]||^2}{2\sigma^2} \right]$$
We estimate jointly $x_w$ and $x_b$ using the MAP technique.

$$ (\hat{x}_w, \hat{x}_b) = \arg \max \text{Pr}(x_w, x_b | y, \sigma, \sigma_x) \quad (18) $$

We use an alternate optimization. So our algorithm has the following structure.

1. Initialization of $x_{w}^{0}$;
2. $x_{w}^{k+1} = \arg \max \text{Pr}(x_w | x_{w}^{k}, y, \sigma, \sigma_x)$;
3. $x_{b}^{k+1} = \arg \max \text{Pr}(x_b | x_{b}^{k+1}, y, \sigma, \sigma_x)$;
4. return to 2 until convergence.

As the posterior law given in (17) has the symmetric form for $x_w$ and $x_b$, in the following part, we discuss only how to get an estimator of $x_w$ knowing $x_b$.

To estimate $x_{w}^{k+1}$ knowing $x_{b}^{k}$ and $y$, a conjugate gradient algorithm is used. The problem of MAP given in (18) is equivalent to the minimization of the criterion $Q(x_w)$,

$$ Q(x_w) = \| y - s \circ \exp (- M_w R x_w - M_b R x_b) \|^2 + \zeta (\Phi_b(D x_w - x_{b}^{k}))) \quad (19) $$

where $\zeta = \sigma^2 / \sigma_x^2$ is an hyper-parameter which tunes the compromise between the likelihood and the prior information.

We derive eq (19) with respect to $x_w$ to get its gradient,

$$ g_w = \frac{\partial Q(x)}{\partial x} \bigg|_{x=x_{w}^{k-1}} = R_1 M_w^t \left( p_b \circ \exp(-M_w R x_w) \right) $$

$$ - ( y - p_b \circ \exp(-M_w R x_w)) + \zeta D^t \Phi_b(D x_w) \quad (20) $$

where

$$ p_b = s \circ \exp(-M_b R x_b) , \quad \text{and} \quad \Phi_b(t) = \sum_{n=1}^{N} \phi_b(t_n) $$

An Polak-Ribière conjugate gradient method is used for the above numerical optimization, where the updating equation is:

$$ x^n = x^{n-1} + \alpha^n d^n \quad (21) $$

The step is determinate by minimizing the criteria at the next iteration,

$$ \alpha^n = \arg \min \alpha Q(x_{w}^{n-1} + \alpha d_w) $$

The criterion $Q(x)$ is not quadratic, so we do not have an exact form of the $\alpha$ optimum. To obtain $\alpha^n$ we consider a second order Taylor approximation of $Q$

$$ Q(x_{w}^{n-1} + \alpha d_w) = Q(x_{w}^{n-1}) + \alpha d_w \frac{\partial Q(x)}{\partial x} \bigg|_{x=x_{w}^{n-1}} + \frac{\alpha^2}{2} d_w^t \frac{\partial^2 Q(x)}{\partial x^2} \bigg|_{x=x_{w}^{n-1}} d_w \quad (22) $$

Using eq.(20) and eq.(22), we obtain,

$$ \alpha^n = - \frac{d_w^t g_w}{d_w^t \frac{\partial^2 Q(x)}{\partial x^2} \bigg|_{x=x_{w}^{n-1}} d_w} \quad (23) $$

where

$$ \frac{\partial^2 Q(x)}{\partial x^2} \bigg|_{x=x_{w}^{n-1}} d_w = R_1 M_w^t \text{diag}\left[p_b \circ \exp(-M_w R x_w) \right] $$

$$ \circ (2 p_b \circ \exp(-M_w R x_w) - y) \circ M_w R + \zeta D^t \Phi_b(D x_w) \bigg|_{x=x_{w}^{n-1}} D $$

Using this alternative optimization algorithm, we finally arrive at two density maps of water and bone. Based on our basis material decomposition model given in eq.(1), the attenuation map at any give energy $\varepsilon$ can be estimated by the sum of the two density maps $\hat{x}_w$ and $\hat{x}_b$ weighted by their MACs at energy $\varepsilon$.

$$ \hat{\mu}(\varepsilon) = m_w(\varepsilon) x_w + m_b(\varepsilon) x_b \quad (24) $$

4. RESULTS

4.1. Phantoms and configurations

Phantoms of water and bone shown in Fig.1a and Fig.1b respectively are used in the simulation. In this pair of phantoms, a high attenuating point, with its density as high as 150, is added on a low attenuation map. It’s estimated to be a metal implant. As we know, it doesn’t exist such a kind of material with such high density. But with the decomposition equation given in (1), the attenuation coefficient becomes acceptable. All the phantoms are normalized into 256 x 256 pixels.

An experimental 250kV X-ray tube spectrum is used for our simulation. The energy number $K = 4$ and detection energies $\varepsilon = 20, 40, 80, 120$ keV. All the data used in the simulation are acquired with a parallel-beam geometry at 180 angle positions uniformly distributed on $[1^\circ, 180^\circ]$ for 367 detection cells. The exposure time is $T = 1$ s and the variance of the Gaussian noise is $\sigma^2 = 90000$. The multi-energy data is simulated by the eq.(5) using the above phantoms and configurations.

4.2. Density maps of water and bone

Both ME-FBP and our multi-energy MAP approaches are applied on the multi-energy data to get two separate density maps of water and bone. The ME-FBP reconstructed water and (resp. bone) density maps $x_w^{ME-FBP}$ Fig.1c, (resp. $x_b^{ME-FBP}$ Fig.1d). The results of our method are shown in Fig.1e and Fig.1f for the water and bone density maps respectively. Clearly, the ME-FBP can’t reduce the metal artifacts. Meanwhile, it fails to identify the metal implant both in the water and bone maps. Because ME-FBP removes the energy dependent part in the sinogram by using an separation matrix, so it can remove the beam-hardening artifacts. But, it gives the same weights to all the elements of the sinogram including the useless information, such as the measures done at the positions where nearly no X-ray beam are transmitted as a result of the metal implants. So it can not manage the metallic artifacts.

In contrast, our reconstruction approach introduced in the section3.3 reconstruct correctly in the both density maps. Moreover, our method greatly reduces the streak artifact caused by the metal implant.

4.3. Attenuation map

In order to visualize both the metal and beam-hardening artifacts, FBP reconstruction method (introduced in sec.3.1) has been used to get the attenuation map at energy $\varepsilon_m = 45$keV. Fig.2a shows the
original attenuation map. Fig.2b shows the FBP reconstructed attenuation map based on eq.(6). Fig.2c shows the reconstructed result of our method based on eq.(24). And Fig.2d shows their profiles at the position $y = 76$ (marked with red lines). From the profiles, we see that the FBP reconstructed attenuation map (green line) shows upward cup form effect while the original (black line) is flat. That is one of the typical beam-hardening artifacts. Moreover, FBP method has black-white streak metal artifacts (the large up-down aliasing). In contrast, the result of our method (red line) is nearly coincide with the original profile. It justifies that our method can greatly reduce both the metal and beam-hardening artifacts.

5. CONCLUSION

Our method uses a statistical MAP on a non-linear Gaussian problem, it takes in account the energy dependence of the observations and gives low weight for the useless measures which might introduce metal artifacts. So it can reduce both the beam-hardening and metal artifacts. As a promising reconstruction approach, our method switches the problem to reconstructing two separate basis material density maps. Meanwhile, our method can also estimate the attenuation map at any given energy. According to the simulation results, we can conclude that our iterative method gives good reconstruction results where both the metal and beam-hardening artifacts are greatly reduced.

This method can be used in medical CT imaging for clinical diagnosis where metal implants might be appeared. With different basis materials, it can also be used in security control and industry applications. As to improve the algorithm, employing a joint gradient conjugate direct may accelerate the calculate. For future work, we can test multi-energy MAP reconstruction method with the experimental data acquired by an actual MECT system.

6. REFERENCES