P AND T WAVE DELINEATION AND WAVEFORM ESTIMATION IN ECG SIGNALS USING A BLOCK GIBBS SAMPLER

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ABSTRACT
The delineation of P and T waves is important for the interpretation of ECG signals. We propose a Bayesian detection-estimation algorithm for simultaneous detection, delineation, and estimation of P and T waves. A block Gibbs sampler exploits the strong local dependencies in ECG signals by imposing block constraints on the P and T wave locations. The proposed algorithm is evaluated on the annotated QT database and compared with two classical algorithms.

Index Terms—ECG, P and T wave delineation, Bayesian analysis, Gibbs sampling, MCMC method.

1. INTRODUCTION
In electrocardiograms (ECGs), most of the clinically useful information can be found in the wave intervals, amplitudes, or morphology. Therefore, efficient and robust methods for automated ECG delineation are of great importance. The QRS complex is relatively easy to detect and is thus generally used as a reference within the cardiac cycle. For P and T wave detection and delineation (i.e., determination of peaks and boundaries of the P and T waves), most existing methods perform QRS detection first. They then define temporal search windows before and after the detected QRS location to search for the P and T waves using filtering [1], basis expansions [2], or thresholding [3]. Because of the low slope and magnitude of the P and T waves, as well as the presence of noise, interference, and baseline fluctuation, P and T wave delineation remains a difficult task. Furthermore, in addition to delineation, accurate estimation of the waveform itself may be important, e.g., for T wave alternans detection [4].

In this paper, we propose a Bayesian model and a detection/estimation method for simultaneous P and T wave delineation and waveform estimation. Our model takes into account prior distributions for the unknown parameters (wave locations and amplitudes as well as waveform and local baseline coefficients). A block Gibbs sampler [5] is used because the Bayesian detector/estimator cannot be calculated in closed form. While a Gibbs-type sampler for P and T wave delineation was recently proposed in [6], a new feature of our sampler is a block constraint on the wave locations that improves convergence and computational efficiency. Further novel contributions are the consideration of a local baseline in each block (which was assumed to be filtered out in [6]) and a dimensionality-reducing expansion of the P and T waveforms into Hermite basis functions.

This paper is organized as follows. Section 2 describes a model for the non-QRS signal components. In Section 3, the prior and posterior distributions of all unknown parameters are discussed. A block Gibbs sampler is proposed in Section 4. Simulation results using the QT database [7] and a comparison with two alternative algorithms are presented in Section 5.

2. SIGNAL MODEL FOR THE NON-QRS INTERVALS
Non-QRS intervals in an ECG signal are located between a QRS end and the subsequent QRS onset. The non-QRS interval \( \mathcal{J}_n \) associated with the \( n \)th heartbeat consists of a T wave interval \( \mathcal{J}_{T,n} \), which may contain a T wave, and a P wave interval \( \mathcal{J}_{P,n} \), which may contain a P wave. The temporal lengths of \( \mathcal{J}_n \), \( \mathcal{J}_{T,n} \), and \( \mathcal{J}_{P,n} \) will be denoted by \( N_n \), \( N_{T,n} \), and \( N_{P,n} \), respectively. Here, \( N_{T,n} \) and \( N_{P,n} \) can be determined by a cardiologist or simply as fixed percentages of \( N_n \). Note that \( N_{T,n} + N_{P,n} = N_n \).

Our goal is to estimate the waveforms and amplitudes of the P and T waves and their locations within their respective intervals. The locations of the non-QRS intervals are provided by a prior QRS detection step, which will not be considered here. Due to the nonstationary nature of ECGs, we perform detection and estimation only for a limited set of consecutive beats at once. More specifically, we will consider the beats \( n \in \{1, \ldots, D\} \) located within a D-beat processing window of length \( M \) (see Fig. 1(a)).

As shown in Fig. 1(b), the signal in each non-QRS interval can be approximated by two pulses representing the P and T waves plus a local baseline. The waveforms of all T pulses within a D-beat processing window are assumed to be equal, whereas their amplitudes

![Fig. 1. ECG signal within a D-beat window: (a) QRS and non-QRS intervals (in this example, \( N_{T,n} = N_{P,n} = N_n / 2 \)), (b) signal in the non-QRS intervals, (c) impulse sequences \( \mu_{T,k} \) and \( \mu_{P,k} \).](image)

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and locations vary with the beat index \( n \). Thus, the T waves within a window can be modeled by the convolution of the unknown T waveform \( h_T = (h_{T,0}, \ldots, h_{T,l})^T \) with an unknown “impulse” sequence \( u_T = (u_{T,1}, \ldots, u_{T,l})^T \) indicating the T wave locations and amplitudes (see Fig. 1(c)). Note that at most D entries of \( u_T \) are nonzero; each of these “impulses” corresponds to one T wave. Similarly, the P waves within a window are modeled by the convolution of \( h_P = (h_{P,0}, \ldots, h_{P,l})^T \) with \( u_P = (u_{P,1}, \ldots, u_{P,l})^T \). Let \( J_T \) be the union of all T and P wave intervals \( J_{T,n} \) and \( J_{P,n} \) within the window considered, and let \( K \triangleq |J_T| \) be the corresponding signal length. The non-QRS signal component can then be written as

\[
x_k = \sum_{i=0}^{k} h_{T,i} u_{T,k-i} + \sum_{i=0}^{k} h_{P,i} u_{P,k-i} + c_k + w_k, \quad k \in J_T \text{, (1)}
\]

where \( c_k \) denotes the baseline sequence and \( u_k \) denotes white Gaussian noise with unknown variance \( \sigma_w^2 \). Note that we assume that \( u_{T,k} = u_{P,k} = 0 \) for \( k \notin J_T \).

We propose to represent the P and T waveforms by a basis expansion using discrete-time versions of the first G Hermite functions (cf. [8–10]). Thus, the waveform vectors can be written as

\[
h_T = H \alpha_T, \quad h_P = H \alpha_P, \quad \text{where } H \text{ is an } (L+1) \times G \text{ matrix whose columns are the first } G \text{ Hermite functions (with } G \leq L+1), \text{ suitably sampled and truncated to length } L+1, \text{ and } \alpha_T \text{ and } \alpha_P \text{ are unknown coefficient vectors of length } G. \text{ Furthermore, the local baseline within the } n \text{th non-QRS interval } J_{T,n} \text{ is modeled by a } 4\text{-degree polynomial, i.e.,}
\]

\[
c_{n,k} = \sum_{i=1}^{5} \gamma_{n,i} k^{i-1}, \quad k = 1, \ldots, N_n, \text{ (3)}
\]

for each \( n \in \{1, \ldots, D\} \). This local model extends that of [11], which assumes that the baseline constant is in \( J_{T,n} \) (i.e., \( \gamma_{n,i} = 0 \) for \( i \geq 2 \)). In vector-matrix form, (3) reads as \( c_n = M_n \gamma_n \), with the known \( N_n \times 5 \) Vandermonde matrix \( M_n \) and the unknown coefficient vector \( \gamma_n = (\gamma_{n,1}, \ldots, \gamma_{n,5})^T \). The baseline sequence for the entire D-beat window can then be written as

\[
c = M \gamma, \text{ (4)}
\]

where \( c, M, \) and \( \gamma \) are obtained by suitably stacking the \( c_n, M_n, \) and \( \gamma_n \), respectively, for \( n = 1, \ldots, D \). Using (2) and (4), we obtain the following vector representation of the non-QRS signal in (1):

\[
x = F_T u_T + F_P u_P + M \gamma + w. \text{ (5)}
\]

Here, \( F_T \) is the \( K \times M \) Toeplitz matrix with first row \( (h_{T,0}^T, 0_{M-1}^T) \), where \( h_{T,0}^T \) is the first row of the matrix \( H \) and \( 0_{M-1}^T \) is the zero vector of length \( M - 1 \), and with first column \( ((h_{T,0}^T)^T, 0_{M-1}^T)^T \). \( M \) is defined similarly, with \( \alpha_T \) replaced by \( \alpha_P \).

The proposed detection-estimation method is based on writing the impulse sequences as products \( u_{T,k} = b_{T,k} a_{T,k} \) and \( u_{P,k} = b_{P,k} a_{P,k} \) of binary indicator sequences \( b_{T,k} \in \{0, 1\} \) and \( b_{P,k} \in \{0, 1\} \) and amplitude factors \( a_{T,k} \in \mathbb{R} \) and \( a_{P,k} \in \mathbb{R} \). Each \( b_{T,k} = 1 \) (\( b_{P,k} = 1 \)) indicates the location of a T (P) wave, and the corresponding \( a_{T,k} \) (\( a_{P,k} \)) is the respective amplitude. Note that the \( a_{T,k} \) (\( a_{P,k} \)) are undefined for all \( k \) where \( b_{T,k} = 0 \) (\( b_{P,k} = 0 \)). Let \( b_{T,k}, b_{P,k}, \alpha_T, \) and \( \alpha_P \) denote the length-\( M \) vectors corresponding to \( b_{T,k}, b_{P,k}, \alpha_T, \) and \( \alpha_P,k, \) respectively. Then (5) can be rewritten as

\[
x = F_T B_T \alpha_T + F_P B_P \alpha_P + M \gamma + w, \text{ (6)}
\]

with the diagonal \( M \times M \) matrices \( B_T \triangleq \text{diag}(b_T) \) and \( B_P \triangleq \text{diag}(b_P) \).
vague prior [12]. Our choice of conjugate priors for \( \alpha_T, \alpha_P, \gamma, \) and \( \sigma_w^2 \) considerably simplifies our algorithm since the resulting posterior distributions are also Gaussian and inverse gamma, respectively.

**Posterior distribution.** The posterior distribution of the parameter vector \( \theta \) is given by

\[
p(\theta | x) \propto p(x | \theta) p(\theta) = p(x | \theta_T)p(\theta_T)p(\theta_P)p(\theta_{cw}),
\]

with \( p(\theta_T) = p(\alpha_T|b_T)p(b_T|\alpha_T), p(\theta_P) = p(\alpha_P|b_P)p(b_P) p(\alpha_P), \) and \( p(\theta_{cw}) = p(\gamma)p(\sigma_w^2). \) Due to the complexity of this distribution, we propose to use a block Gibbs sampler that generates samples asymptotically distributed according to \( p(\theta | x). \) From these samples, the discrete parameters \( b_T \) and \( b_P \) are then detected by means of the sample-based maximum a posteriori detector, and the continuous parameters \( \alpha_T, \alpha_P, \alpha_T, \alpha_P, \gamma, \) and \( \sigma_w^2 \) are estimated by means of the sample-based minimum mean square error estimator.

4. BLOCK GIBBS SAMPLER

The proposed block Gibbs sampler is summarized in Algorithm 1. The sampling distributions for the P wave parameters \( \theta_P \) are similar to those of \( \theta_T; \) they are omitted because of space limitations. We note that \( b_{\sim J_T,n} \) denotes \( b_T \) without the entries indexed by \( J_T,n, \) and similarly for \( a_{\sim J_T,n}. \) To see that the proposed algorithm is valid, note that the sampling steps for \( b_{\sim J_T,n} \) and \( a_{\sim J_T,n} \) are equivalent to jointly sampling \( (b_{J_T,n}, a_{\sim J_T,n}) \) from \( p(b_{J_T,n}, a_{\sim J_T,n}|\theta_{\sim J_T,n}, \theta_P, \theta_{cw}, x), \) where \( \theta_{\sim J_T,n} \triangleq \{b_{\sim J_T,n}a_{\sim J_T,n}\alpha_T\}^T. \)

**T wave indicators.** The sampling distribution for \( b_{\sim J_T,n} \) is

\[
p(b_{\sim J_T,n}|\theta_{\sim J_T,n}, \theta_P, \theta_{cw}, x) \propto \sigma_1 \exp\left( \frac{\mu_1^2}{2\sigma_1^2} \right) p(b_{J_T,n}),
\]

with

\[
\mu_1 = \frac{\sigma_1^2 b^T_{J_T,n} F_{J_T,n}^T \left(x_T - F_{\sim J_T,n} B_{\sim J_T,n} a_{\sim J_T,n} \right)}{\sigma_w^2},
\]

\[
\sigma_1^2 = \left( \frac{\|F_{J_T,n} b_{J_T,n}\|^2}{\sigma_w^2} + \frac{1}{\sigma_2^2} \right)^{-1},
\]

where \( F_{J_T,n} \) contains the columns of \( F_T \) indexed by \( J_T,n, \) \( F_{\sim J_T,n} \) is \( F_T \) without those columns, \( x_T = x - F_T B_T a_T - M \gamma, \) and \( B_{\sim J_T,n} = \text{diag}(b_{\sim J_T,n}). \) The sampler evaluates all hypotheses of \( b_{\sim J_T,n} \) conditioned on the samples of all other parameters. There are \( N_{T,n} + 1 \) such hypotheses, because \( b_{\sim J_T,n} \) has either no 1-entry or exactly one 1-entry at one of \( N_{T,n} \) possible locations (cf. the prior in (7)).

**T wave amplitudes.** The sampling distribution for the \( \alpha_T \) is

\[
p(\alpha_T | b_T, \alpha_P, \theta_T, \theta_{cw}, x) = \mathcal{N}(\alpha_T; \mu_2, \Sigma_2),
\]

with

\[
\mu_2 = \frac{\Sigma_2 H^T U_T^T}{\sigma_w^2} x_T, \quad \Sigma_2 = \left( \frac{H^T U_T^T U_T H}{\sigma_w^2} + \frac{I_{D+1}}{\sigma_2^2} \right)^{-1}.
\]

**Algorithm 1 Block Gibbs sampler**

For \( n = 1, \ldots, D \) do

Sample the block \( b_{J_T,n} \sim p(b_{J_T,n}|\theta_{\sim J_T,n}, \theta_P, \theta_{cw}, x) \) (see (9))

for \( k \in J_{T,n} \) do

if \( b_T,k = 1 \) then

Sample \( \alpha_T,k \sim p(\alpha_T,k | b_T,k = 1, \theta_{\sim J_T,n}, \theta_P, \theta_{cw}, x) \) (see (10))

end if

end for

Sample the block \( b_{J_{P,n}} \) from \( p(b_{J_{P,n}}|\theta_{\sim J_{P,n}}, \theta_P, \theta_{cw}, x) \) (see (11))

Sample \( \alpha_P \sim p(\alpha_P | b_P, \alpha_T, \theta_P, \theta_{cw}, x) \) (see (12))

Sample \( \sigma_w^2 \sim p(\sigma_w^2 | \theta_T, \theta_P, \gamma, x) \) (see (13))

Here, \( U_T \) is the Toeplitz matrix of size \( K \times (L+1) \) with first row \( (u_T, 0^T) \) and first column \( (a_T^T 0^T)^T \).

**Local baseline coefficients.** The sampling distribution for \( \gamma \) is

\[
p(\gamma | \theta_T, \theta_P, \sigma_w^2) = \mathcal{N}(\gamma; \mu_3, \Sigma_3),
\]

with

\[
\mu_3 = \frac{\Sigma_3 M^T}{\sigma_w^2} x_T, \quad \Sigma_3 = \left( M^T M + \frac{I_{D+1}}{\sigma_w^2} \right)^{-1},
\]

where \( x_T = x - F_T B_T a_T - F_T B_P a_P \).

**Noise variance.** The sampling distribution for \( \sigma_w^2 \) is

\[
p(\sigma_w^2 | \theta_T, \theta_P, \gamma) = \mathcal{IG}(\sigma_w^2; \xi', \eta'),
\]

with \( \xi' = \xi + \frac{N}{2} \) and \( \eta' = \eta + \frac{1}{2} \|x - F_T B_T a_T - F_T B_P a_P - M \gamma\|^2. \)

5. SIMULATION RESULTS

We evaluated the performance of the proposed algorithm on the QT database (QTDB) [7]. The QTDB contains 105 15-minute excerpts of two-channel Holter recordings from several widely used ECG databases, chosen to include a variety of P and T wave morphologies. In a preprocessing step, the QRS complexes were detected using the algorithm proposed in [13]. Based on the detected QRS-complex locations, P and T wave search intervals were then defined.

Fig. 2(a) shows an ECG signal segment from the QTDB dataset sele0136. Corresponding estimates—produced by our algorithm—of the marginal posterior probabilities of having a P or T wave at a given location are shown in Fig. 2(b). (We note that the maximum a posteriori detector used in our algorithm detects a wave at a given location if the respective posterior probability estimate is above 1/2.)

Fig. 2(c) shows the estimated T and P waveforms for the 10-beat window considered (i.e., \( D = 10 \)). The wave onsets and ends were determined by searching for the first negative local minimum on ei-
Fig. 2. (a) Segment of an ECG signal. (b) Posterior probabilities of the occurrence of P waves (black solid line) and T waves (red dotted line). (c) Estimated T and P waveforms (normalized).

Fig. 3. (a) ECG signal segment (dashed blue), estimated local baseline (black), and estimated P and T waves (red). (b) Results of P and T wave delineation.

Table 1. Delineation and detection performance for QTDB

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<tr>
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<tbody>
<tr>
<td>$b_0$: $\text{Se} (%)$</td>
<td>99.60</td>
<td>97.70</td>
<td>98.87</td>
</tr>
<tr>
<td>Onset-P: $\mu \pm \sigma (\text{ms})$</td>
<td>1.7±10.8</td>
<td>14.0±13.3</td>
<td>2.0±14.8</td>
</tr>
<tr>
<td>Peak-P: $\mu \pm \sigma (\text{ms})$</td>
<td>2.7±8.1</td>
<td>4.8±10.6</td>
<td>3.6±13.2</td>
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<tr>
<td>End-P: $\mu \pm \sigma (\text{ms})$</td>
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<td>−0.1±12.3</td>
<td>1.9±12.8</td>
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<td>$b_1$: $\text{Se} (%)$</td>
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<td>97.74</td>
<td>99.77</td>
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<tr>
<td>Onset-T: $\mu \pm \sigma (\text{ms})$</td>
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<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Peak-T: $\mu \pm \sigma (\text{ms})$</td>
<td>0.7±9.6</td>
<td>−7.2±14.3</td>
<td>0.2±13.9</td>
</tr>
<tr>
<td>End-T: $\mu \pm \sigma (\text{ms})$</td>
<td>2.7±13.5</td>
<td>13.5±27.0</td>
<td>−1.6±18.1</td>
</tr>
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Waveform estimation of P and T waves. The block Gibbs sampler exploits the strong local dependencies in ECG signals. Validation using the annotated QT database showed that the proposed method provides reliable detection and accurate delineation for a wide variety of wave morphologies, and it outperforms two alternative methods. Our method also provides accurate waveform estimation, which makes it useful for T wave alternans analysis and other ECG analyses requiring wave morphology information.

7. REFERENCES