A CONSTRAINED OPTIMIZATION APPROACH FOR MULTI-ZONE SURROUND SOUND

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ABSTRACT

A recent approach to surround sound is to perform exact control of the sound field over a region of space. Here, the driving signals for an array of loudspeakers are chosen to create a desired sound field over an extended area. An interesting subtopic is multi-zone surround sound, where two or more listeners can experience totally independent sound fields. However, multi-zone surround sound is a challenge because implementation can be very non-robust. We formulate multi-zone sound reproduction as a convex optimization problem, where the sound energy leakage into other listener zones is limited to fixed levels, and a constraint is placed on the loudspeaker weights to improve the robustness. An interior point algorithm is devised for computing the loudspeaker weights, and its performance is compared with least squares approaches of multi-zone reproduction in typical two-zone cases.

1. INTRODUCTION

Multi-zone sound field reproduction aims to provide independent listening environments at a number of locations. A preliminary study was performed in [1] and investigated further in [2] using a spatial mode decomposition approach. Global control of the sound in all zones was proposed in [2]. Controlling the sound fields independently whilst suppressing inter-zone interference was proposed in [3]. Independent control is more convenient as the audio played into each zone can be processed separately. However none of the works provide a precise control on the sound leaked into other listening zones. A two-channel private audio system was proposed in [4], with the objective of maximizing the ratio of the sound energy in the control region to the sound leakage in other listeners’ zones, though the approach is not able to reproduce a spatial sound field.

Multi-zone is very difficult to implement in scenarios when one zone is obscured by another [1]. The solution can easily become very non-robust, where large loudspeaker powers may be required and large reproduction errors may result from small perturbations of the acoustic transfer functions (ATFs) from the loudspeakers to the listener zones. Robustness can be improved by limiting the total available power in a manner similar to that achieved by regularization [5].

We propose a method to control the sound in each zone independently, while also controlling the leakage into other listeners’ zones. We use a constrained optimization similar to [5], and derive an interior point algorithm [6, 7] for determining the loudspeaker weights that minimize the mean square error (MSE) of reproduction in the control region. To improve robustness, we incorporate a constraint on the summed square value of the loudspeaker weights. The approach is applied to design at a single frequency in a 2-D environment, but is equally applicable in 3-D and over a frequency range.

2. PROBLEM STATEMENT

In multi-zone surround sound, we seek to control the sound independently over a number $N$ of different regions of space or zones, each corresponding to a different listener. Each listener is able to move through their personal sound reproduction region $D_k$, whilst limiting sound leakage into other zones. For simplicity of notation, we choose $D_κ$ as $D_1$. We ensure small sound leakage into other zones $D_2 \ldots D_N$, using an array of $L$ loudspeakers surrounding the listeners.

We examine the 2-D case where all listeners lie in the same plane and all loudspeakers are vertical line sources. Let each listener’s region be a cylinder of radius $r$. The number of loudspeakers required to reproduce the sound at frequency $f$ is approximately $N(2kr + 1)$ where $k = 2πf/s_ν$ is the wave number and $s_ν$ is the speed of sound [8].

We produce the multi-zone sound field by pressure matching at a number of points inside each zone. We create a target sound field in vector $p_d$ over $M_κ = M_1$ points in $D_κ$ whilst constraining the sound energy over the $M_n$ points in each quiet zone $D_n$, $n = 2 \ldots N$. As illustrated in Fig.1, the task is to determine the weights $G_i(f)$ to apply to each loudspeaker. We represent the weight for each loudspeaker $ℓ$ by $G_ℓ(f)$ and summarize all $L$ weights into the vector $g = [G_1(f) \ldots G_L(f)]^T$. The reproduction is performed separately for each frequency $f$ but for brevity the functional dependence on the frequency parameter $f$ will be suppressed.
A robust implementation of multi-zone reproduction would require that the total energy of the loudspeaker weights \( g^T g \leq K_0 \) where \( K_0 \) is not large. The loudspeaker weight energy (LWE) is inversely proportional to the white noise gain, which quantifies the ability of an array to suppress spatially uncorrelated noise in the loudspeaker signals. Major sources of error in the surround sound system, such as amplitude and phase errors in the ATFs and loudspeaker position errors are known to be nearly uncorrelated [9, p. 69] and affect the signal processing in a manner similar to spatially white noise [10]. The LWE provides a measure of the reaction to such errors, and we hence bound the LWE.

Define the set of matrices of acoustic transfer functions \( H_1, \ldots, H_N \), where each \( M_n \times L \) matrix \( H_n \) describes the acoustic transfer function from each loudspeaker to \( M_n \) points inside \( D_n \) at frequency \( f \). Summarizing, the task is a multi-constraint optimization with the objective of minimizing the MSE of reproduction over \( D_e \):

\[
\min_{\tilde{x}} \left\| H_i \tilde{x} - p_d \right\|^2
\quad \text{s.t.} \quad \left\| \tilde{x} \right\|^2 \leq K_0
\quad \left\| H_i \tilde{x} \right\|^2 \leq K_n, n = 2, \ldots, N.
\]

We choose \( K_n = \alpha M_n \left\| p_d \right\|^2 / M_n \) where \( \alpha \) is the acceptable level of sound energy leakage into another listener’s zone.

### 3. INTERIOR POINT ALGORITHM

This problem possesses a quadratic objective and quadratic constraints, so it classifies as a Quadratically Constrained Quadratic Program. It is solved as a convex optimization following the approach of [5, 6]. Converting the problem to one involving real-number arithmetic, we define:

\[
x \triangleq \begin{bmatrix} \text{Re}(g) \\ \text{Im}(g) \end{bmatrix}, \quad c \triangleq \begin{bmatrix} \text{Re}(H_{i}^T p_d) \\ \text{Im}(H_{i}^T p_d) \end{bmatrix},
\]

\[
A_n \triangleq \begin{bmatrix} \text{Re}(H_{i}^T H_n) & -\text{Im}(H_{i}^T H_n) \\ \text{Im}(H_{i}^T H_n) & \text{Re}(H_{i}^T H_n) \end{bmatrix}.
\]

The problem can then be reposed as:

\[
\min_{\tilde{x}} \tilde{x}^T A_n \tilde{x} - 2c^T \tilde{x}
\quad \text{s.t.} \quad \tilde{x}^T \tilde{x} \leq K_0
\quad \tilde{x}^T A_n \tilde{x} \leq K_n, n = 2 \ldots N
\]

We write the Lagrangian as

\[
\mathcal{L} = \tilde{x}^T A_n \tilde{x} - 2c^T \tilde{x} + \lambda_0 (\tilde{x}^T \tilde{x} - K_0) + \sum_{n=2}^{N} \lambda_n (\tilde{x}^T A_n \tilde{x} - K_n),
\]

where the multipliers \( \lambda_n \geq 0 \). The optimal solution is found by setting the derivative of \( \mathcal{L} \) with respect to \( \tilde{x} \) equal to zero:

\[
\frac{\partial \mathcal{L}}{\partial \tilde{x}} = 2[A_n \tilde{x} - c + \lambda_0 \tilde{x} + \sum_{n=2}^{N} \lambda_n A_n \tilde{x}] = 0
\]

and so

\[\hat{x} = (A_c + \sum_{n=2}^{N} \lambda_n A_n + \lambda_0 I)^{-1} c.\]

Given the functional form for the optimum point \( \hat{x} \) has been deduced, the task is then to determine the multipliers \( \lambda_n \). The standard primal form can be written with the use of slack variables \( w_n \):

\[
\min_{\tilde{x}} \tilde{x}^T A_n \tilde{x} - 2c^T \tilde{x}
\quad \text{s.t.} \quad \tilde{x}^T \tilde{x} + w_n - K_0 = 0
\quad \tilde{x}^T A_n \tilde{x} + w_n - K_n = 0
\quad w_n \geq 0, \quad n = 0, 2, \ldots, N
\]

The Wolfe dual problem is obtained by substituting (2) into (1) to obtain the objective:

\[
\max_{\lambda} -c^T \tilde{x} - w^T \lambda
\]

Summarizing the set of equations, along with the Karush-Kuhn-Tucker (complementarity) condition \( w_n \lambda_n = 0 \), the solution vector \( x \) will satisfy:

\[
M(\lambda) x - c = 0
\quad Q(x) x + w - k = 0
\quad W \lambda = 0.
\]

where \( M(\lambda) = A_c + \lambda_0 I + \sum_{n=2}^{N} \lambda_n A_n \), \( Q^T(x) = [x, A_2^T x, \ldots, A_N^T x] \), \( k = [K_0, K_2, \ldots, K_N]^T \), \( \lambda = [\lambda_0, \lambda_2, \ldots, \lambda_N]^T \), \( w = [w_0, w_2, \ldots, w_N]^T \), and \( W = \text{Diag}(w) \).

We solve system (3) using the predictor-corrector method of [7], where we set \( W = \mu I \) and substitute \( x + \Delta x \) for \( x \), \( \mu + \Delta \mu \) for \( \lambda \) and \( w + \Delta w \) for \( w \) yield:

\[
\begin{bmatrix} M(\lambda) & Q^T(x) & 0' \\ 2Q(x) & 0 & I \\ 0 & A^T W^{-1} & I \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta w \end{bmatrix} = \begin{bmatrix} \alpha \\ \gamma \\ \delta \end{bmatrix}
\]

where

\[\begin{bmatrix} \alpha \\ \gamma \\ \delta \end{bmatrix} = \begin{bmatrix} c - M(\lambda) x - M(\Delta \lambda) \Delta x \\ k - w - Q(x) x - Q(\Delta \lambda) \Delta x \\ \mu \Delta \lambda^{-1} w - \lambda^{-1} \Delta \lambda \Delta w \lambda \end{bmatrix}\]

and \( A = \text{Diag}(\lambda) \). We can eliminate \( \Delta w \) and \( \Delta \lambda \) by noting that \( \Delta w = \gamma - 2Q \Delta x \) and \( \Delta \lambda = W^{-1}(\Delta \lambda \Delta w \lambda) \) and thus solve for \( \Delta x \) using

\[
\begin{bmatrix} M(\lambda) + 2Q^T W^{-1} A Q \\ \Delta x = \alpha - Q^T W^{-1} A \delta + Q^T W^{-1} A \gamma \end{bmatrix}
\]

At each iteration, the update \( x \leftarrow x + \Delta x \) is applied, after the step size \( \zeta \) has first been adjusted as suggested in [6] to ensure that the slack variables and Lagrange multipliers remain non-negative.

The algorithm terminates when the difference between the primal and dual objective functions is small. It was discovered that omission of the corrector step reduces the algorithm to a Newton’s method.

### 4. RELATION TO WEIGHTED LEAST SQUARES

An equivalence exists between performing the reproduction with the constrained approach and weighted least squares. The multi-zone problem has been solved using standard least squares in [1] where an acoustic monopole field was created in \( D_e \) and heavily attenuated form of the same field in \( D_2, \ldots, D_N \). We pose the similar problem of reproducing the sound field pressure vector \( p_d \) in \( D_e \) and a null field in the \( N - 1 \) quiet zones.
Fig. 2. Reproduction of a plane wave in $D_1$ (red) whilst the sound is deadened in $D_2$ (blue). (a) and (b) show single-zone reproduction using standard least squares and two-zone reproduction using convex optimization respectively for $\phi_d = 45^\circ$. (c) and (d) show single-zone and two-zone reproduction respectively for $\phi_d = 15^\circ$.

The desired sound pressure over all $M = \sum_{n=1}^{N} M_n$ points in the $N$ zones and the stacked ATF matrix are:

$$\tilde{p}_d \triangleq \begin{bmatrix} p_d \ 0 \end{bmatrix}, \ H \triangleq \begin{bmatrix} H_1 \\ \vdots \\ H_N \end{bmatrix},$$

respectively where $0$ is the zero vector of length $M - M_1$. We weight the importance of nullying the field in each quiet zone $D_n$ with weight factor $\lambda_n (n = 2, \ldots, N)$. The diagonal weighting matrix is given by $W = \text{Diag}(1, \lambda_2 I_{M_2}, \ldots, \lambda_N I_{M_N})$ where $I_m$ is a length $m$ row vector of ones. [1] is then a specific case, where the weights $\lambda_n = 1$. The weighted least squares solution with regularization is $\hat{g} = (H^H W H + \lambda_0 I)^{-1} H^H W \tilde{p}_d$ where $\lambda_0$ can be chosen to ensure the LWE constraint $\hat{g}^H g \leq K_0$ is satisfied. Substituting for $\tilde{p}_d$, $H$ and $W$:

$$\hat{g} = \left( H^H H + \sum_{n=2}^{N} \lambda_n H_n^H H_n + \lambda_0 I \right)^{-1} H^H p_d$$

which is equivalent to the minimum MSE equation in (2). Whilst in the proposed approach (2), $\lambda_n$ are chosen to meet design constraints, in (4) they are chosen to weight the relative importance of (i) small sound leakages into the other listener zones and (ii) accurately reproducing sound in the control region. However, the actual choice of $\lambda_n$ here is not clear, whereas design parameters $K_0$ for the proposed approach have a straightforward and intuitive physical interpretation.

Fig. 3. Average sound energy leakage and loudspeaker weight energy versus MSE as parameters $\lambda_0$ and $K_0$ are varied. (a) and (b) are for the $\phi_d = 45^\circ$ case whilst (c) and (d) are for the $\phi_d = 15^\circ$ case. Marked on these curves are the performance of the examples in Fig. 2 for convex optimization ($\circ$), weighted least squares ($\times$) and single-zone least squares ($\triangle$).

5. RESULTS AND DISCUSSION

We consider multi-zone reproduction in $N = 2$ regions each of $r = 0.6m$ and $R = 1.2m$ (see Fig. 1) at $500$ Hz with $L = 30$ loudspeakers. The target zone is located at $\phi_1 = \phi_2 = 0^\circ$. We reproduce a plane wave at angle $\phi_d$ from the $x$-axis in control region $D_1$, whilst deadening the sound $D_2$ oriented at $\phi_2$, comparing:

1. Interior point algorithm using LWE constraint parameter $K_0 = 20$ and attenuation factor $\alpha = 0.001$.
2. Weighted least squares in (4) using $\lambda_0 = 1$ and $\lambda_1 = 5$.
3. Standard least squares which makes no effort to deaden the sound in $D_2$: $\hat{g} = (H^H H + \lambda_0 I)^{-1} H^H p_d$ using $\lambda_0 = 1$.

Fig. 2 illustrates several multi-zone reproduction examples. In Fig. 2(a) and Fig. (b), a plane wave is created at $\phi_d = 45^\circ$ with quiet zone at $\phi_d = 120^\circ$. In Fig. 2(a) reproduction with the single-zone method performs with small sound leakage because the quiet zone is not in-line with the control zone. However the interior point method shown in Fig. 2(b) still achieves an 18dB smaller sound energy leakage.

Fig. 2(c) and (d), $\phi_d = 15^\circ$ and $\phi_d = 180^\circ$. Here shows a multi-zone reproduction scenario which is more difficult. Since the plane wave is almost collinear with a line drawn through the centres of the zones, sound created in $D_1$ propagates straight into $D_2$ if not for multi-zone compensation. The interior point method (Fig. 2(d)) reduces the sound leakage by 27dB over the single zone approach (Fig. 2(c)), but at the expense of a 22dB drop in MSE. Weighted least squares creates a field visually similar to that of convex optimization. The MSE, sound leakage and LWE for the three approaches are marked onto Fig. 3.

Given any particular zone geometry, there is a design trade-off between the metrics of sound energy leakage, LWE and MSE. In
Fig. 4. Performance as the plane wave is panned, comparing convex optimization for $\alpha \in \{0.01, 0.001, 0.0001\}$ (solid curves), two-zone weighted least squares (dashed) and single-zone reproduction (dotted). Plotted are (a) the MSE in control region and (b) average sound energy leakage.

Fig. 3, multi-zone reproduction performance is studied plotting average sound leakage energy and LWE both against MSE using the $\phi_d = 180^\circ$ geometry. To produce these curves as a function of MSE, we vary the design parameters $\lambda_0$ (for least squares approaches) and $K_0$ (for convex optimization) over a range. Unlike the least squares approach, constrained optimization is shown able to hold the sound energy leakage constant. However, the constraints on sound energy leakage and LWE do impose more stringent limits to how accurate the sound field can be reproduced.

Fig. 4 studies performance of the three methods as the plane wave angle $\phi_d$ is panned. Worst performance is achieved when the plane wave is in-line with both zones. Here, whilst weighted least squares is shown to reproduce the plane wave with smaller MSE than convex optimization multi-zone for in-line cases, it does so at the expense of a 3.5 times larger LWE. Further, the least squares approaches are unable to keep the sound energy leakage constant with plane wave angle. Fig. 4 shows performance of convex optimization approach for several choices of $\alpha$.

6. CONCLUSION

We have devised a constrained optimization approach to multi-zone surround sound, where the sound leakage into other listener’s zones is upper-limited to a small fraction of the reproduced sound energy. As the robustness of multi-zone sound reproduction is sometimes poor, a robustness constraint has been incorporated into the algorithm, based upon limiting the summed squares of the loudspeaker weights. The approach is shown algebraically equivalent to a weighted least squares approach.

Though convex optimization is a non-linear approach for determining the loudspeaker weights, the algorithm leads to a linear technique for processing input audio signals for playback. After determining the loudspeaker weights, any audio signal can be played over the desired spatial sound field.

7. REFERENCES