ABSTRACT

It is known that the proportionate normalized least mean square (PNLMS) algorithm outperforms traditional normalized least mean square (NLMS) algorithm, in terms of fast initial convergence rate. However, the PNLMS has been widely observed to not be optimal. This study presents a new perspective into the “proportionate” gain (step-size) allocation scheme. A relative proportionate scheme is established and shows better performance than the original absolute proportionate scheme. Although the correspondingly derived relative proportional LMS (R-PNLMS) algorithm is similar to PNLMS, it differs greatly in terms of conception and convergence behavior. Simulation results for the problem of acoustic channels identification, show improved performance over existing methods.

Index Terms—Adaptive filtering, proportionate normalized least-mean-square (PNLMS) algorithm, room impulse response, acoustic channel identification

1. INTRODUCTION

Acoustic channel identification is one of the most active topics in adaptive filtering, with wide applications for room impulse response (RIR) identification, relative transfer function (RTF) identification, acoustic echo cancelation (AEC), as well as network echo cancelation (NEC). Conventional adaptive filters, such as the normalized least-mean-square (NLMS) algorithm, often fail in terms of convergence rate for these applications, because of the time-varying, long and sparse responses of many acoustic channels.

The proportionate NLMS (PNLMS) algorithm [1], represents a set of schemes that exploit the sparseness of acoustic channel responses to achieve a significantly faster convergence rate than that of the conventional NLMS algorithm. The idea in PNLMS is to “proportionally” allocate gains (e.g., step-size) to update filter weights, according to their optimal values: larger weights result in larger updating gains. As summarized in [1], the allocation of gains is, intuitively, according to the magnitude of the optimal filter weights. Other algorithms specifically explore better gain allocation strategies includes the Exponentially Weighted NLMS [2], PNLMS++ [3], or IPNLMS [4]. An interesting recent study [5] theoretically determines that the optimal gain should be applied according the $\mu$-law compression of the absolute value of the optimal filter weights. This version of PNLMS was named MPNLMS or $\mu$-law PNLMS, which shows a faster convergence rate than PNLMS.

In this study, we mainly address the gain allocation problem in the proportionate NLMS. A new gain allocation scheme, termed to be relative proportionate, is proposed, which achieves both a fast convergence rate and low misalignment errors. Simulations show that the resulting algorithm performs better than NLMS, PNLMS and MPNLMS for the problem of acoustic channel identification.

This paper is organized as follows: in Section 2, the gain allocation method for proportionate NLMS is reviewed. This motives the proposed relative proportionate NLMS (R-PNLMS) approach presented in Section 3, followed by Section 4 which considers implementation trade-offs. The proposed scheme is evaluated in Section 5, followed conclusion in Section 6.

Fig. 1. General Model for Acoustic Channel Identification
2. BACKGROUND: GAIN ALLOCATION SCHEME IN PROPORTIONATE NLMS

Fig. 1 illustrates the general configuration for the problem of acoustic system identification, where the goal is to estimate an unknown system $h_{\text{opt}}$ using an adaptive finite impulse response (FIR) filter $\hat{h}(n)$, where $n$ represents the iteration index. The PNLMS iteratively updates the estimated filter weights by the following set of equations,

$$
e(n) = y(n) - \hat{h}^T(n)x(n),$$
$$\hat{h}(n+1) = \hat{h}(n) + \frac{\mu G(n)x(n)e(n)}{\hat{x}^T(n)G(n)x(n)+\xi},$$

where $\mu$ is the step-size parameter, and $\xi > 0$ is a regularization parameter (preventing division by zero and stabilizing the solution). The gain distribution matrix $G(n) = \text{diag}(g_1(n), g_2(n), \ldots, g_L(n))$, with $\sum_{l=1}^{L} g_l(n) = 1$, governs the gain (step-size) adjustment for individual filter tap weights. In the standard PNLMS, gains are allocated based on the magnitude of the optimal filter weights, for the intuition that assigning large step-sizes to large coefficients, the overall convergence time will be decreased.

The MPNLMS (or $\mu$-law PNLMS) algorithm was derived such that all coefficients converge to a value within a vicinity of their optimal values in the same number of iterations. Here, we briefly review the derivation of MPNLMS algorithm as originally proposed in [5]. From the framework in Fig. 1, the updating equation of individual filter coefficients can be written as,

$$\hat{h}_l(n) - h_{l_{\text{opt}}}' = (1 - g_l)^n(\hat{h}_l(0) - h_{l_{\text{opt}}}')$$

where $\hat{h}_l(0)$ is the initial filter response and is usually set to zeros for the acoustic channel. (2) can therefore be written as,

$$\hat{h}_l(n) = (1 - (1 - g_l)^n)h_{l_{\text{opt}}}'.$$

The coefficient convergence time of the $l_{\text{th}}$ coefficient is defined as

$$n_l = \frac{\ln |h_{l_{\text{opt}}}'|}{\ln \frac{1}{|1 - g_l|}}.$$  (4)

For $0 < g_l < 1$, $\ln(1 - g_l) \approx -g_l$, (4) can be approximated by,

$$n_l = \frac{\ln |h_{l_{\text{opt}}}'|}{g_l}. $$  (5)

To achieve the same convergence rate for each of the filter coefficients, the optimal gain factor is easily derived as,

$$g_l \approx \frac{\ln(|h_{l_{\text{opt}}}'|)}{\sum_{l=1}^{L} \ln(|h_{l_{\text{opt}}}'|)}.$$  (6)

Detailed implementations of PNMLS and MPNLS algorithms are summarized in Table 1.

3. RELATIVE PROPORTIONATE GAIN ALLOCATION

As stated in the previous section, MPNMLS converges fast since it allows all filter coefficients to attain a converged value to within a vicinity of their optimal value in the same number of iterations from the initial iteration. Some interesting questions could be considered: What is the optimal gain if the reference time is not at the beginning but, say, at the present or previous (non-initial) iteration? If we require filter coefficients to attain a converged value within a vicinity of their optimal value in the same number of the subsequent iterations from the present or previous (non-initial) iteration, rather than from the very beginning, is the convergence rate accelerated? To address these questions, a relative proportional NLMS algorithm is derived to pursue a better convergence behavior. The derivation can be conducted assuming the “initial” filter weights in (2) to be non-zero and equal to the estimated value from a reference point at a previous iterations (i.e., $n_{r} < n$), as

$$h_l(n) - h_{l_{\text{opt}}}' = (1 - g_l)^{n-n_{r}}(h_l(n_{r}) - h_{l_{\text{opt}}}').$$  (7)

Following a similar derivation of MPNMLS as that from previous section, we have,

$$n - n_{r} = \frac{\ln |\hat{h}_l(n_{r}) - h_{l_{\text{opt}}}'|}{\ln \frac{1}{|1 - g_l|}.}$$  (8)

Different from the absolute convergence time (e.g., $n$) obtained in (4), a relative convergence time (e.g., $n - n_{r}$) is introduced in (8) above. We require all the filter coefficients have the same relative convergence time with respect to a pre-select reference iteration point $n_{r}$. A relative gain allocation scheme

<table>
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<tr>
<th>Table 1. Proportionate NLMS Algorithms</th>
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<tbody>
<tr>
<td>Filter Weights Updating:</td>
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<tr>
<td>$x_{n} = [x(n)x(n-1) \ldots x(n-L+1)]^T$</td>
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<tr>
<td>$\hat{g}(n) = \hat{h}^T(n)x(n)$</td>
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<tr>
<td>$\epsilon(n) = y(n) - \hat{h}^T(n)x(n)$</td>
</tr>
<tr>
<td>$\hat{h}(n+1) = \hat{h}(n) + \frac{\mu G(n)x(n)e(n)}{\hat{x}^T(n)G(n)x(n)+\xi}$</td>
</tr>
<tr>
<td>$G(n) = \text{diag}(g_1(n), g_2(n), \ldots, g_L(n))$</td>
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<tr>
<td>Gain Allocation Scheme:</td>
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<tr>
<td>$\gamma_l(n) = \max {\rho \times \max {\delta, F(\hat{h}_l(n)), \ldots, F(\hat{h}_L(n))}, F(\hat{h}_l(n))}$</td>
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<tr>
<td>$g_l(n) = \frac{\gamma_l(n)}{\sum_{l=1}^{L} \gamma_l(n)}$</td>
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<tr>
<td>where, for</td>
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<tr>
<td>PNMLS: $F(\hat{h}_l(n)) =</td>
</tr>
<tr>
<td>MPNLMS: $F(\hat{h}_l(n)) = \ln(1 + \beta</td>
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</table>
for MPNLMS (R-MPNLMS), hereby, is obtained from the following equation,
\[ g_l \approx \frac{\ln(|\hat{h}_l(n_r) - h_{l_{opt}}|)}{\sum_{l=1}^L \ln(|h_l(n_r) - h_{l_{opt}}|)}. \] (9)

Also, the relative gain allocation scheme for PNLMS (R-PNLMS) is straightforward as shown below,
\[ g_l \approx \frac{|\hat{h}_l(n) - h_{l_{opt}}|}{\sum_{l=1}^L (|h_l(n_r) - h_{l_{opt}}|)}. \] (10)

Some interesting observations can be drawn here: the relative gain scheme incorporates the conventional counterpart by setting the reference to the initial iteration. If \( n_r \) is selected at the present iteration, the introduced method pursues fast convergence time remained; this convergence behavior is fundamentally different from the conventional proportional NLMS algorithms.

Next, an experiment is conducted to study the convergence behavior of the proposed gain allocation scheme. To this end, optimal filter \( h_{l_{opt}} \) is assumed to be known and applied to the gain allocation schemes in the PNLMS, MPNLMS and the proposed R-PNLMS and R-MPNLMS (the reference point is set to the present iteration, i.e., \( n_r = n \)) algorithms are carried out for the problem of room impulse response identification (RIR). Here, 100 different RIRs (with RT60=0.2s) are generated using the method in [6] and 100 independent experiments are conducted for each of the RIRs. The filter length is set to 0.2 seconds (e.g. 1600 filter coefficients for a sample rate of 8000 Hz). The signal-to-noise ratio (SNR) is 30 dB. To evaluate the algorithm performance, the normalized misalignment measure (in dB) is employed and given by
\[ \epsilon(n) = 10 \log_{10} \left( \frac{\| h_{l_{opt}} - \hat{h}_l(n) \|_2^2}{\| h_{l_{opt}} \|_2^2} \right). \] (11)

Fig. 2 shows the averaged convergence rate for NLMS, PNLMS, MPNLMS and the two proposed R-PNLMS and R-MPNLMS, all with the optimal gain allocation schemes. Some observations can be drawn from Fig. 2:

- The PNLMS and MPNLMS have to balance between the initial convergence rate and overall convergence rate, which is also observed in [4].
- The R-PNLMS and R-MPNLMS have a fundamentally different convergence behavior, i.e., achieving both fast initial and overall convergence rate.

4. IMPLEMENTATION COMPROMISE

Intrinsically, the gain allocation scheme in the family of the proportionate NLMS algorithms, requires the a priori information in the transfer function of the acoustic channel (i.e., magnitude of \( h_{l_{opt}} \)), which is not available. For practical implementation, an estimated version, e.g., \( \hat{h}_l(n) \) is employed as a suboptimal choice in each iteration, as in PNLMS and MPNLMS. However, this implementation can not work in the introduced concept of relative gain allocation, which requires the difference between the estimation and optima.

In this study, one possible method is explored using the decision-directed technique. Taking the reference point \( n_r \), as the present iteration, we can approximate \( |h_l(n_r) - h_{l_{opt}}| \) by \( |h_l(n_r-1) - h_l(n)| \). This approximation is valid when \( \hat{h}_l(n) \) is close to the optimal coefficient \( h_{l_{opt}} \), e.g., \( |h_l(n_r-1) - h_l(n)| \) is smaller than some pre-defined constance. Detailed implementation of the decision-directed R-PNLMS (DR-PNLMS) and R-MPNLMS (DR-MPNLMS) are summarized in Table 2. Actually, this implementation incorporates both the conventional and the proposed gain allocation schemes: at the beginning \( h_l(n) \) is far from its optimal, conventional gain allocation scheme is employed; when \( |h_l(n_r-1) - h_l(n)| \) becomes smaller than \( \phi \), \( h_{l_{opt}} \) can be replaced by \( \hat{h}_l(n) \) and thus switching to the relative gain scheme.

5. EVALUATION

Monte Carlo simulations are conducted to evaluate the performance of the proposed DR-PNLMS and DR-MPNLMS algorithms in the context of room impulse response (RIR) identification. Throughout our simulations, algorithms were tested using a zero mean white Gaussian noise (WGN) as inputs while another uncorrelated WGN sequence was added to give an SNR of 30 dB. RIRs are generated using the method implemented in [6]. We assume that the length of the adaptive filter is equivalent to that of the unknown RIR.

Fig. 3 and Fig. 4 compares the convergence behaviors for RT60 equals to 100 ms and 200 ms, respectively. In both of the cases, the proposed algorithms share same initial convergence rate as that of the conventional proportionate NLMS.
Table 2. Decision-Directed Relative Gain Scheme

<table>
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<tr>
<th>Gain Allocation Scheme:</th>
<th>$\gamma_l(n) = \max { \rho \times \max { \delta, F(\hat{h}_l(n)), \ldots, F(\hat{h}_L(n)) } }$</th>
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<tbody>
<tr>
<td>$g_l(n) = \frac{\gamma_l(n)}{\sum_{l=1}^{L} \gamma_l(n)}$</td>
<td>where, for DR-PNLMS: if $</td>
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<tr>
<td>DR-MPNLMS: if $</td>
<td>\hat{h}_l(n-1) - \hat{h}_l(n)</td>
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</table>

and is faster than the NLMS; moreover, the better overall convergence behavior is also achieved using the relative gain allocation schemes.

6. CONCLUSION

The concept of relative gain allocation scheme for the PNLMS algorithms was presented and shown to be optimal over the existing schemes. The proposed method is novel since it explores fast convergence upon the presently estimated filter coefficients. The proposed gain allocation concept is fundamentally different from the family of the proportionate NLMS algorithms, which intrinsically are a balance between initial and overall convergence rate. While the optimal performance of the introduced relative gain allocation is difficult to achieve, the proposed decision-directed methods was implemented with satisfactory compromise, as shown in the Monte Carlo evaluations. Both of the proposed RD-PNLMS and RD-MPNLMS outperform the previous versions of NLMS. Further studies could consider applying the proposed method to new applications such as acoustic echo cancelation (AEC) or network echo cancelation (NEC).

7. REFERENCES


