JOINT CHANNEL AND DOPPLER ESTIMATION FOR MULTICARRIER UNDERWATER COMMUNICATIONS

A.Y. Kibangou
GIPSA-LAB/CNRS/UJF
Systems Control Department
Grenoble, France

C. Siclet, L. Ros
GIPSA-LAB/Grenoble-INP/UJF
Image and Signal Department
Grenoble, France

ABSTRACT
Underwater wireless communications are subject to multipath propagation and Doppler effects. In the receiver side, it is necessary to estimate both Doppler and channel parameters in order to compensate for their effects in the transmitted signal. In this paper, in the OFDM (Orthogonal Frequency Division Multiplexing) context, we derive a new estimation scheme based on the resolution of two harmonic retrieval problems. Unlike standard methods, data resampling and estimation of residual carrier frequency offset are avoided. The efficiency of the proposed scheme is evaluated by means of simulations.

Index Terms— Underwater acoustic communications, OFDM, Doppler distortion, Hankel matrix, Vandermonde decomposition, Total least squares.

1. INTRODUCTION

The OFDM technique, a multi-carrier modulation scheme in which broadband data is effectively transmitted in parallel as K narrowband channels on K orthogonal subcarriers, has been claimed to be one of the most promising communication technologies for achieving high data rate and large system capacity[1]. It allows designing low complexity receivers to deal with highly dispersive channels [2]. These features motivate the use of OFDM in underwater environments.

Underwater acoustic channels are wideband in nature due to the small ratio of carrier frequency to the signal bandwidth, which introduces frequency-dependent Doppler shifts [3]. They also exhibit several propagation paths.

In order to adequately recover the transmitted information, algorithms at the receiver must include estimation and compensation of the Doppler scaling factor, channel estimation, and information symbols estimation. Several approaches have been suggested in the literature for estimating the Doppler scaling factor. They are based on the use of preamble and postamble of a packet consisting of multiple OFDM blocks [3, 4] or by exploiting correlation induced by the cyclic prefix [1]. Then, the received signal is resampled by using a sampling period related to the estimated Doppler scaling factor. It is also necessary to estimate and compensate for the residual carrier frequency offset (CFO) since the Doppler can vary between consecutive OFDM blocks inside a given packet.

In this paper, the received data are processed block-by-block. We make use of high resolution harmonic estimation methods to estimate both Doppler scaling factor and channel parameters (path gains and delays). The advantage of the proposed scheme is to avoid data resampling and residual CFO estimation and compensation. Notation: In the sequel, we denote by V(x, N) the L × N Vandermonde matrix with x ∈ C^L as generator. Its first column is constituted by 1s.

2. PROBLEM SETTING

The proposed receiver is based on block-by-block processing. We assume that consecutive OFDM symbols are separated by a sufficient guard interval in order to avoid intersymbol interference. Moreover, the transmitter and the receiver clocks are synchronized.

Each OFDM block is divided into two sub-blocks with respective duration T_0 and T_1. Each sub-block is followed by a guard interval of duration T_g. The first sub-block is a learning sub-block whereas the second is an informative one. As a consequence the transmitted signal s(·) results on the superposition of s_0(·) and s_1(·) respectively associated with the learning and the informative sub-blocks.

Let us first consider the informative sub-block. The bandwidth is B = K_1 Δ f, where K_1 is the number of subcarriers used for information transmission and Δ f = 1/T_1 is the frequency spacing. The kth subcarrier is at the frequency

f_k = f_c + k Δ f , \quad k \in K_1 = \{-K_1/2, \cdots, K_1/2 - 1\} \label{eq:subcarrier}

where f_c is the carrier frequency. Let c_k denote the information symbol to be transmitted on the kth subcarrier. The transmitted bandpass signal associated with the informative sub-block is given by

s_1(t) = \text{Re} \left\{ \sum_{k \in K_1} c_k e^{j 2 \pi f_k t} g_1(t - T_0 - T_g) \right\}, \label{eq:s1}

with g_1(t) = 1 if t \in [0, T_1] and g_1(t) = 0 otherwise.

Let us now consider the learning sub-block. The pilot symbols d_q, known to the receiver, are transmitted using K_0 < K_1 subcarriers. The qth subcarrier of the learning sub-block is at the frequency

φ_q = f_c + q Δ f_0 , \quad q \in K_0 = \{-K_0/2, \cdots, K_0/2 - 1\} \label{eq:pilot}

with Δ f_0 = B/K_0. In passband, the transmitted signal associated with the learning sub-block is given by

s_0(t) = \text{Re} \left\{ \sum_{q \in K_0} d_q e^{j 2 \pi q φ_q t} g_0(t) \right\}, \label{eq:s0}

with g_0(t) = 1 if t \in [0, T_0] and g_0(t) = 0 otherwise.
We consider a multipath underwater channel with impulse response \( h(t, \tau) = \sum_{p=1}^{P} A_p(t) \delta(t - \tau_p(t)) \), where \( A_p(t) \) and \( \tau_p(t) \) are the real-valued gain and the delay associated with the \( p \)th path.

In the sequel, we adopt the following assumptions:

- All path are affected by a similar Doppler scaling factor \( \alpha \) such that \( \tau_p(t) = \tau_p - \alpha t \).
- The path delays \( \tau_p \), the gains \( A_p \), and the Doppler scaling factor \( \alpha \) are constant over the block duration \( T = T_0 + T_1 \).
- The maximal path delay \( \tau_p \), the guard interval \( T_g \), is sufficiently large to avoid interference between the learning and the informative sub-blocks inside a given OFDM block and between consecutive blocks. We have \( T_g > \max\{\tau_p\} \).

The analytic representation of the received signal \( y(t) \) satisfies \( y(t) = y_0(t) + y_1(t) \), with:

\[
y_0(t) = \sum_{q \in K_0} \sum_{p=1}^{P} d_q A_p g_0 ((1 + a) t - \tau_p) e^{-j2\pi \varphi_q (1+a) t},
\]

\[
y_1(t) = \sum_{k \in K_1} h_k(t)c_k,
\]

with

\[
h_k(t) = \sum_{p=1}^{P} A_p g_1 ((1 + a) t - \tau_p - T_0 - T_g) e^{-j2\pi f_k (1+a) t}.
\]

By assumption, there is no interference between \( y_0(t) \) and \( y_1(t) \). Indeed, there exists a time \( t' \) so that \( y(t) = y_0(t) \) if \( t < t' \) and \( y(t) = y_1(t) \) if \( t > t' \). As a consequence, the first part of the received signal will serve to estimate the Doppler scaling factor \( \alpha \) and the channel parameters (gains and delays). The estimated parameters and the second part of the received signal will be used for estimating the informative symbols.

### 3. DOPPLER AND CHANNEL ESTIMATION

In this section we consider the received signal corresponding to the time interval \( t < t' \). In the sequel, it is denoted \( z(t) \). It can equivalently be written as:

\[
z(t) = \sum_{q \in K_0} d_q B_q e^{j2\pi \varphi_q (1+a) t}
\]

with \( B_q(t) = \sum_{p=1}^{P} A_p g_1 ((1 + a) t - \tau_p) e^{-j2\pi \varphi_q \tau_p} \). One can note that \( B_q(t) \) can be viewed as a mixture of complex valued harmonics with piecewise constant magnitude. \( z(.) \) also has a similar behaviour.

Parameter estimation of this kind of signals, also called harmonic retrieval problem, has been studied in the literature in the case where the magnitude variations are deterministic and continuous or random and stationary [5, 6], that is not the case of the problem under consideration. However, we can define an observation window where the magnitudes are constant. For this purpose, we assume that the bounds of the path delays, \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \), and that of the Doppler scaling factor are known\(^1\). We get:

\[
\tau_{\text{min}} \leq \tau_p \leq \tau_{\text{max}}, \quad \lambda_{\text{min}} \leq 1 + a \leq \lambda_{\text{max}}.
\]

By selecting \( t_0 \) and \( t_1 \) such that

\[
\frac{\tau_{\text{max}}}{\lambda_{\text{min}}} \leq t_0 < t_1 \leq \frac{T_0 + \tau_{\text{min}}}{\lambda_{\text{max}}}
\]

we define an observation window where the magnitude of the harmonics are constant. Indeed, for \( t \in [t_0, t_1] \), we get

\[
B_q(t) = B_q = \sum_{p=1}^{P} A_p e^{-j2\pi \varphi_q \tau_p}.
\]

As a consequence, the received signal is given by:

\[
z(t) = \sum_{q \in K_0} d_q B_q e^{j2\pi \varphi_q (1+a) t}
\]

Thus, inside the observation window defined by the bounds (7), we have two harmonic retrieval problems to solve. The first one makes use of the received signal \( z(.) \) in order to estimate the Doppler scaling factor \( \alpha \) and the magnitudes \( B_q \), whereas the second makes use of the estimated magnitudes \( B_q \) in order to estimate path delays and gains.

The harmonic retrieval problem have been extensively studied in the literature (see [7, 8, 9] for example). In this paper an ESPRIT like [8] high resolution method called HTLS (Hankel Total Least Squares) [10, 9] is used.

### 3.1. Doppler scaling factor estimation

Let us consider the \( N \) samples \( z_{n}, n = n_0, \cdots, N + n_0 - 1 \), of the received signal:

\[
z_n = \sum_{q \in K_0} d_q B_q e^{j2\pi \varphi_q (1+a) n T_v}
\]

\( T_v \) being the sampling period. By setting \( N = L + M - 1, L > K_0, M > K_0 \), we first build the Hankel matrix \( Z \in \mathbb{C}^{L \times M} \) having respectively as first column and last row the vectors

\[
Z_1 = \left[ z_{n_0}, z_{n_0+1}, \cdots, z_{n_0+L-1} \right]^T
\]

and

\[
Z_L = \left[ z_{n_0+L-1}, z_{n_0+L}, \cdots, z_{n_0+N-1} \right].
\]

It admits the following Vandermonde decomposition:

\[
Z = S_1 \text{diag}(\alpha) T_1^T.
\]

with \( \alpha = (\alpha_{-K_0/2}, \cdots, \alpha_{K_0/2-1})^T, \alpha_q = d_q B_q e^{j2\pi \varphi_q (1+a) n T_v}, S_1 = V(\phi, L), T_1^T = V(\phi, M), \phi = (\phi_{-K_0/2}, \cdots, \phi_{K_0/2-1})^T, \) and \( \phi_q = e^{j2\pi \varphi_q (1+a) T_v} \).

One can note that the Vandermonde matrices \( S_1 \) and \( T_1 \) possess a shift-invariance property expressed as follows:

\[
S_{1+q} \text{diag}(\phi) = S_1, \quad T_{1+q} \text{diag}(\phi) = T_1
\]

\(^1\)The knowledge of the minimal path delay is difficult. Therefore, we get the absolute minimum \( \tau_{\text{min}} = 0 \).

\(^2\)\( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \) are related to the maximal and minimal velocities of the underwater vehicles, which can be \( a \) priori known.
where the line on the bottom (resp. on the top) of a matrix stands for deleting the first (resp. the last) row.

Let us now consider the singular values decomposition of the Hankel matrix \( Z \):
\[
Z = U_1 \Sigma V_1^H, \tag{13}
\]
where \( U_1 \in \mathbb{C}^{L \times K_0}, \ V_1 \in \mathbb{C}^{M \times K_0} \), and \( \Sigma \in \mathbb{C}^{K_0 \times K_0} \) contain respectively the left and right singular vectors and the singular values of \( Z \). We can also note that \( U_1 \) and \( S_1 \) generate the same subspace. Hence, it exists a nonsingular matrix \( Q_1 \in \mathbb{C}^{K_0 \times K_0} \) such that
\[
U_1 = S_1 Q_1. \tag{14}
\]
We can then deduce:
\[
\bar{U}_1 = \bar{S}_1 Q_1, \quad \hat{U}_1 = \hat{S}_1 Q_1 \tag{15}
\]
By combining (12) and (14), we get:
\[
\hat{U}_1 = U_1 Q_1^{-1} \text{diag}(\phi) Q_1. \tag{16}
\]
Hence, the poles \( \phi_q \) are the eigenvalues of \( \Phi = Q_1^{-1} \text{diag}(\phi) Q_1 \).

To estimate \( \Phi \), we have to estimate \( U_1 \) and then deduce its eigenvalues. The estimation of \( \Phi \) is carried out by solving \( \hat{U}_1 = \hat{U}_1 \Phi \).

The solution in the total least squares sense is given by:
\[
\Phi = -Y_{12}^{-1} Y_{22}^{-1} \tag{17}
\]
where \( Y = \left( \begin{array}{c} Y_{11} \\ Y_{21} \\ Y_{22} \end{array} \right) \in \mathbb{C}^{2K_0 \times 2K_0} \) is the matrix containing the right singular vectors of \( (U_1 \quad \hat{U}_1) \).

Let us denote by \( \phi_q \) the estimated poles, i.e. the eigenvalues of \( \hat{\Phi} \).

If the sampling period \( T_s \) is chosen such that \( T_s \leq \frac{1}{\Delta f_{\text{max}} \\max(\varphi_q)} \), then the angle of \( \phi_q \) denoted \( \varphi_q \) belongs to \([-\pi, \pi]\). In fact, the estimated eigenvalues equal the actual poles up to a permutation. For removing, such an ambiguity, the estimated eigenvalues can be sorted with their pulsation in an ascending order. Therefore, we deduce the following estimator for the Doppler scaling factor:
\[
\hat{\alpha} = -1 + \frac{1}{K_0} \sum_{q=-K_0/2}^{K_0/2-1} \frac{\varphi_q}{2 \pi \varphi_q T_s} \tag{18}
\]

### 3.2. Path delays and gains estimation

By vectorizing the Vandermonde decomposition (11), we get:
\[
\text{vec}(Z) = (T_1 \odot S_1) \alpha, \tag{19}
\]
where \( \odot \) denotes the Kratli-Rao product. Let us denote \( \alpha \) the least squares solution of (18), where the poles in \( S_1 \) and \( T_1 \) have been replaced by their estimated values. Thus, we get:
\[
\hat{B}_q = \frac{d_p^2}{|d_q|^2} \hat{a}_q e^{-j2 \pi \varphi_q + (1+\hat{a}) n \alpha T_s}, \quad q \in K_0, \tag{20}
\]
\( \hat{a}_q \) being the corresponding entry of \( \hat{a} \).

Note that we can rewrite \( B_q \), defined in (8), as
\[
B_q = \sum_{p=1}^{P} A_pe^{-j2 \pi \varphi - K_0/2 \tau_p} e^{-j2 \pi \varphi \Delta f_0 \tau_p}, \quad q = 0, 1, \ldots, K_0 - 1. \tag{21}
\]
\( B_q \) can be viewed as samples of a mixture of \( P \) harmonics, the poles of which being \( e^{-j2 \pi \varphi \Delta f_0 \tau_p} \). Obviously, we assume that the number of significative paths \( P \) is known. Provided, \( K_0 \geq 2P \), we can then solve this harmonic retrieval problem using the HTLS method, as in the previous subsection, and then deduce the path delays \( \tau_p \).

For this purpose, by setting \( K_p = L + M - 1, L > P, M \geq P \), we build the Hankel matrix \( B \in \mathbb{C}^{L \times M} \) whose first column and last row are respectively \( \vec{B}_1 = (B_0 \quad \cdots \quad B_{L-1})^T \) and \( \vec{B}_L = (B_{L-1} \quad B_L \quad \cdots \quad B_{K_0-1}) \). As previously, we get the following Vandermonde decomposition:
\[
B = S_2\text{diag}(\beta) T_2^T. \tag{22}
\]
with \( S_2 = \mathcal{V}(\varphi, L), \ T_2^T = \mathcal{V}(\varphi, M) \), \( \psi = (\psi_1, \cdots, \psi_P)^T \), \( \psi_p = e^{-j2 \pi \varphi - K_0/2 \tau_p} \), \( \beta = (\beta_1, \cdots, \beta_P)^T \), and \( \beta_p = A_p e^{-j2 \pi \varphi - K_0/2 \tau_p} \).

As in the previous subsection, the poles \( \psi_p \) are the eigenvalues of \( \Psi \) the solution in the total least squares sense of
\[
\tilde{U}_2 = U_2 \Psi. \tag{23}
\]
\( U_2 \in \mathbb{C}^{L \times P} \) being the matrix of left singular vectors of \( B \).

Let us denote \( \psi_p \) the estimated poles, i.e. the eigenvalues of \( \Psi \). By replacing the poles by their estimated values in \( S_2 \) and \( T_2 \), we can solve the following equation:
\[
\text{vec}(B) = (T_2 \odot S_2) \tilde{\beta}. \tag{24}
\]
We can then estimate the gains \( A_p \) as the magnitudes of the entries of the least squares solution \( \hat{\beta} \) of (22):
\[
\hat{A}_p = |\hat{\beta}_p|. \tag{25}
\]
We also deduce \( \theta_p = \frac{\hat{\beta}_p}{|\hat{\beta}_p|} = e^{-j2 \pi \varphi - K_0/2 \tau_p} \).

The estimation of the path delays is a more complicate task. Indeed, we cannot guarantee that \( 2 \pi \tau_p f_0 \) belongs to \([-\pi, \pi]\). As a consequence, we cannot directly deduce the value of the path delays from the angle of the entries \( \psi_p \) of \( \hat{\beta} \). However, it exists an integer \( m_0 \) such that
\[
\psi_p = -2 \pi \Delta f_0 \tau_p + 2 \pi m_0. \tag{26}
\]
For different integers \( m \), we define a set of possible path delays \( \tau_{p,m} \) such that \( f_{p,m} < T_3 \):
\[
\tau_{p,m} = \frac{m \Delta f_0}{2 \pi} + m \Delta f_0 \text{ and } \theta_{p,m} = e^{-j2 \pi \varphi - K_0/2 \tau_{p,m}}. \tag{27}
\]
Let us now consider the square distance
\[
\kappa_{p,m} = |\theta_{p,m} - \theta_p|^2 = 1 - e^{-j2 \pi \varphi - K_0/2 \tau_{p,m}}. \tag{28}
\]
After few manipulations, it can be seen that this square distance is minimal only for \( \tau_{p,m} = \tau_p \), the actual path delay:
\[
\hat{\tau}_p = \arg \min \kappa_{p,m}. \tag{29}
\]

### 4. SIMULATION RESULTS

In these simulations, the range of frequency used by the underwater vehicles is \([15 \text{kHz} \quad 28 \text{kHz}]\). The Doppler scaling factor is generated as a random value such that \( |a| \leq 10^{-3} \), meaning that the maximal relative speed is 1.5 m/s. The carrier frequency is set equal to \( f_c = 21 \text{ kHz} \), whereas the guard interval is \( T_g = 10 \text{ ms} \). We use \( K_0 = 16 \) sub-carriers for the learning sequence and \( K_1 = 512 \) for the informative one. The duration of the learning sequence is \( T_0 = 2T_g \). Hence, the duration of one OFDM block is 80.5 ms.
The information rate transmitted is then equal to 1184 b/s when the used modulation is a QPSK one. The sampling frequency is $1/T_e = 56.056$ kHz. All the results presented below are averaged values over 100 independent Monte Carlo runs. The additive noise was a complex valued white Gaussian noise.

![Fig. 1](image1)

**Fig. 1.** Channel impulse responses used for simulation: $P = 4$ (left) and $P = 6$ (right)

We first evaluate the Doppler scaling factor estimation by considering 100 OFDM symbols. The Doppler scaling factor varies between two consecutive symbols. In Figure 2, one can note the effectiveness of the Doppler estimation scheme. However, the performance is degraded when decreasing the SNR, as depicted in figure 3 where the performance is evaluated by means of the Normalized Mean Square Error (NMSE). We obtain good results for moderate to higher SNR values. In particular, the estimation of the path delays is more precise for higher SNR values. However, the improvement on the estimation of the path delays has a slight effect on the overall performance characterized by the NMSE on the received signal.

![Fig. 2](image2)

**Fig. 2.** Estimation of the Doppler scaling factor ($SNR = 10$ dB, $P = 4$)

![Fig. 3](image3)

**Fig. 3.** Normalized Mean Square Error for $P = 6$, $a = 10^{-3}$.

levels of SNR, should be investigated. In addition, estimated parameters allow reconstructing the channel matrix, which can be used for designing a zero-forcing or a MMSE equalizer.

6. REFERENCES


