A Novel Digital Calibration Technique for Gain and Offset Mismatch in Parallel \( \Sigma \Delta \) ADCs

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Abstract—Time interleaved sigma-delta architecture is a potential candidate for high bandwidth analog to digital converters required for reconfigurable, versatile and multistandard receivers. However, this architecture is very sensitive to the unavoidable gain and offset mismatch resulting from the manufacturing process. This paper presents a novel digital calibration method for gain and offset mismatch. This new method takes advantage of the digital signal processing on each channel to reconstruct the useful signal and requires only few logic components for implementation. The run time calibration is estimated to 10 and 15 clock cycles for offset and gain mismatches respectively.

Index Terms—sigma-delta, calibration, gain and offset mismatch, analog-to-digital conversion, time-interleaving.

I. INTRODUCTION

The current trend in telecommunication systems aims to incorporate more and more applications (multimedia, Internet, TV, GPS, WiFi, etc.) using different communication standards in one channel receiver. To reach this aim, Mitola [1] has proposed the software radio concept allowing the reconfigurability of the channel receiver to switch between different standards. The main idea of this concept is to put the digital processing as close as possible to the receiver antenna in order to ensure a software management of the radio spectrum. This solution requires a wide-band and high-resolution Analog to Digital Converter (ADC) which remains the main bottleneck.

Many ADC topologies based on sigma-delta modulators and using parallelism to increase the conversion bandwidth were proposed. The Time Interleaved Sigma-Delta (T\( \Sigma \Delta \)) converter [2] provides good performance and has the lowest hardware complexity compared to Parallel Sigma-Delta (P\( \Sigma \Delta \)) [2] and Frequency Band Decomposition (FBD) [3] architectures. However, gain and offset mismatch among channels create spurious tones that considerably reduce the Signal to Noise plus Distortion Ratio (SNDR).

After a brief introduction of the T\( \Sigma \Delta \), section III presents the effect of offset mismatch and the proposed correction method. Section IV sheds light on the effect of the gain mismatch and the relevant calibration method. Finally, section V concludes with the performance of the proposed calibration method.

II. REVIEW OF THE T\( \Sigma \Delta \) ARCHITECTURE

The block diagram of T\( \Sigma \Delta \) ADC is presented in Fig. 1 [2]. The T\( \Sigma \Delta \) ADC is composed of \( M \) parallel low-pass \( \Sigma \Delta \) modulators. The input signal \( x[n] \) is distributed among the \( M \) modulators through an analog multiplexer. Then, the signal rate, on each channel, is increased by a factor \( N \) using interpolation by zeros. Afterward, the output of each modulator is filtered by the digital filter \( H(z) \) to suppress the out of band quantization noise. Finally, the signal is demultiplexed by a digital demultiplexer to reconstruct the output signal \( y[n] \).

The theoretical Signal to Noise Ratio (SNR) of a T\( \Sigma \Delta \) ADC depends on three parameters of the modulators: its order (\( P \)), the number of quantizer levels and the interpolation factor (\( N \)). The conversion bandwidth depends on the number of channels (\( M \)). The number of taps in digital filters (\( L \)) determines the hardware complexity needed to reach the expected SNDR. It has been shown in [4] that the digital filter ensuring high attenuation of the out of band quantization noise with the lowest hardware complexity is a Comb-filter whose order is the first even number higher than \( P + 1 \).

![Fig. 1. T\( \Sigma \Delta \) ADC architecture with gain \( g \) and offset \( o \) at the output of each modulator modeling the analog imperfections.](image-url)

The major problem of the T\( \Sigma \Delta \) converter is the mismatch among the \( \Sigma \Delta \) modulators which then creates harmonic distortion in the digital output. Many factors contribute to these mismatches, namely, mismatches among the passive elements, comparator gain and offset, finite gain amplifier, temperature, etc. These imperfections introduce a gain \( g_i \) and an offset \( o_i \) at the output of each modulator (Fig. 1). Channel offset mismatch causes additive tones at integer multiples of the channel sampling rate \( \frac{s}{M} \) and channel gain mismatch results in images of the useful signal spectrum appearing at intervals of \( \frac{s}{M} \) [5]. Several solutions have been proposed in literature to correct these errors. They can be classified into three approaches:
1) The first approach aims to compensate the gain and the offset of each channel by adding a digital sigma-delta modulator on each channel [6] or by using a Pseudo Random Binary Sequence (PRBS) controlled chopper at the modulators input [7].

2) The second approach aims to equalize gain and offset of all modulators by using an extra \( \Sigma \Delta \) modulator as a reference modulator [8].

3) The third approach uses randomization of the channel selection to spread out the spurious tones energy over the whole bandwidth. A randomization technique was proposed in [2] using an extra channel.

The main drawback of these calibration methods is that they require extra material resources (\( \Sigma \Delta \) modulator, PRBS generator, etc.). Furthermore, some solutions require feed-back signals at the input of the \( \Sigma \Delta \) modulator which increases the implementation constraints.

In order to overcome the drawbacks of these solutions, this paper presents an innovative digital calibration method based on a mix of the two first approaches. This new method consists in:

- Estimating the offset value on each channel and then subtracting it from the useful signal. This approach is chosen because even if all the offsets resulting from manufacturing process are equal, the offset value will be added to the useful signal at the output of the \( T \Sigma \Delta \) ADC and it will be difficult to distinguish it from the offset value in the useful signal leading to a significant decrease of the SNDR.
- Equalizing the gain of \( \Sigma \Delta \) modulators to the gain of one of them used as a reference modulator. This suppresses spurious tones without any additional modulator.

The proposed solution does not need neither additional modulator nor reference signal generator. It just requires an accumulator on each channel in addition to the existing digital resources in the \( T \Sigma \Delta \) architecture. Furthermore, it presents high accuracy for offset and gain estimation with very short convergence time.

To illustrate the influence of these errors and show the efficiency of the proposed correction method, a basic example is considered throughout this paper without any loss of generality. In this example, a 4 channel \( T \Sigma \Delta \) is considered using 4\(^{th}\) order \( \Sigma \Delta \) modulator with an interpolation factor of 80 and a 6\(^{th}\) order Comb-filter. The input signal is a sinusoidal signal with a normalized amplitude of 0.6 located at the normalized frequency 0.02. The SNDR without any channel mismatches is estimated to 102 dB.

III. OFFSET MISMATCH AND DIGITAL BACKGROUND CALIBRATION

Due to the time demultiplexer at the output of the \( T \Sigma \Delta \), the presence of the offset \( o_i \) on each channel adds a periodic signal to the useful signal of period \( M \). This periodic signal causes additive tones at integer multiples of the channel sampling rate \( \frac{f_s}{M} \) on the spectrum at the output of the \( T \Sigma \Delta \) ADC leading to a considerable drop in the SNDR.

Fig. 2 a) shows the PSD at the output while considering the following offsets , normalized to the reference voltage, for all channels : \([\text{-}0.202, 0.717, 0.765, 0.1832] \times 10^{-4}\). In this case, the SNDR is reduced by 60 dB even with these very weak offset values. It is therefore essential to remove the offset on each channel.

![Fig. 2](image1)

Compensation for the offset requires first the estimation of its value and then the subtraction of the estimated value from the output signal of each modulator. To perform the estimation phase, the Comb-filter on each channel, dedicated to the reconstruction of the digital useful signal, is used. Indeed, the Comb-filter has a very small bandwidth and a high out of band attenuation for a high interpolation factor \( (N = 80) \) (see Fig. 3). Therefore, a good estimation of the offset value is obtained at the output of the Comb-filter, due to its very small bandwidth, when the input of the modulator is connected to the ground.

![Fig. 3](image2)

Fig. 3. PSD at the output of the \( \Sigma \Delta \) modulator and frequency response of the Comb-filter.

Fig. 4 presents the estimation error defined as the difference between the true value of the offset and the estimated value at the output of the Comb-filter. It can be noticed that an accuracy of \( 10^{-7} \) for the estimated value is reached after 10 clock cycles \( ((M \times N)/f_s) \).

Fig. 2 b) shows the PSD of the output signal after the correction of the offset on each channel. The compensation of the offset allows to decrease the amplitude of spurious tones.
Spurious tones form the correction up to 2 dB of difference with the ideal SNDR. SNDR is improved by 60 dB compared to the case without correction up to 2 dB of difference with the ideal SNDR.

IV. GAIN MISMATCH AND DIGITAL BACKGROUND CALIBRATION

The multiplication of the output signal of each channel by a gain $g_i$ is equivalent, due to the digital demultiplexer at the output, to multiplying the useful signal at the output of the $\Sigma \Delta$ by a periodic signal of period $M$ formed by the different gains $g_i$. This multiplication will result, in the frequency domain, in images of the useful signal spectrum appearing at intervals of $\frac{4f_i}{M}$.[5]

Fig. 5 a) shows the PSD at the output of the $\Sigma \Delta$ where the following gains $[1.0113, 1.0146, 1.0029, 0.9884]$ are introduced on all channels. It can be noticed that a $1\%$ of gain mismatch leads to a drop of 62 dB of the SNDR. It is therefore mandatory to calibrate gain mismatch to maintain the performance.

![Fig. 4. Estimation error a), zoom in the interval time $0 \leq n \leq 30$ b), zoom form the 10th sample c).](image)

Fig. 4. Estimation error a), zoom in the interval time $0 \leq n \leq 30$ b), zoom form the 10th sample c).

Fig. 5. PSD at the $\Sigma \Delta$ output before and after correction in the presence of gain mismatch.

The gain calibration method proposed in this paper supposes that the correction of the offset on each channel was already performed and then aims to equalize the gain on all channels to the gain of one of them considered as the reference channel. The block diagram of this method is depicted in Fig. 6 where the first channel is considered as the reference channel. The different steps of this method are :

- The input signal is a constant signal applied to all modulators simultaneously. The magnitude of this signal $V_{in}$ is fixed, without any loss of generality, to $\frac{V_R}{2}$ where $V_R$ is the reference voltage. Any other values in the input dynamic range of the $\Sigma \Delta$ modulator can be used. This signal may be one of the reference voltages of the DAC in the sigma-delta modulator.
- The output signal of the $i^{th}$ modulator is composed of the constant input signal $V_{in}$ multiplied by the gain $g_i$ and the quantization noise shaped by the modulator. The Comb-filter $H(z)$ extracts the useful signal value $V_{in} \times g_i$ on each channel with the same accuracy for offset estimation (section III).
- The Sign Data Least Mean Square (SD-LMS) [9] algorithm calculates the weight value $w_i$ to equalize the gain of the $i^{th}$ channel to that of the reference channel (channel 1 in Fig. 6):

$$g_i \times w_i = g_1 |_{i=2...M} \quad (1)$$

It was verified by simulation that an accuracy of $10^{-6}$ for the estimated weights is required to regain the expected SNDR.

The SD-LMS algorithm is used for its implementation simplicity and its short convergence time compared to other varieties of the LMS algorithm. The estimation of $w_i$ using this algorithm is given by :

$$SD - LMS: \hat{w}_i[n+1] = \hat{w}_i[n] + \mu (y_i[n] - y_i[n]) \times \text{sgn}[y_i[n]] \quad (2)$$

where $\mu$ is the step of the algorithm.

The step of the SD-LMS algorithm $\mu$ controls the convergence time and the accuracy for the estimated weights. Fig. 7 shows the estimated weight $w_2$ at each iteration $n$ using the SD-LMS algorithm for different values of $\mu$ and Fig. 8 shows the maximum estimation error, once the convergence is achieved, with respect to $\mu$. It can be noticed that the value $\mu = 1$ ensures the shortest convergence time with the best accuracy ($5 \times 10^{-7}$ compared to the desired accuracy $10^{-6}$). Moreover, this value simplifies the implementation of this algorithm by removing the multiplication operation.

Fig. 9 shows the estimated weights with the SD-LMS algorithm with the step $\mu = 1$. In these conditions, the convergence of the algorithm to the desired weight values is reached after 15 clock cycles.

Fig. 5 b) shows the PSD at the output after applying the calibration algorithm for 15 clock cycles. As a result, there is a decrease of the amplitude of spurious tones to the level of
quantization noise thus regaining the expected SNDR of 102 dB.

In order to consider a real example, gain and offset are considered simultaneously in the basic example with values given by the following vectors: \( O = [-0.202, 0.717, 0.765, 0.1832] \times 10^{-4} \) and \( g = [1.0113, 1.0146, 1.0029, 0.9884] \). The offset calibration was applied first and then the gain calibration algorithm is applied taking into account the residual offset mismatch remaining after the offset calibration. Fig. 10 shows the PSD at the output before and after calibration. It can be noticed that the gain calibration algorithm regains the expected performance even in the presence of weak offset mismatch.

Fig. 7. Estimation of the weight \( w_2 \) with the SD-LMS algorithm for different values of \( \mu \).

Fig. 8. Maximum estimation error of the weight \( w_2 \) with respect to \( \mu \).

Fig. 9. The estimation of the weights using SD-LMS algorithm with \( \mu = 1 \).

Fig. 10. PSD at the \( T\Sigma\Delta \) output before and after correction in the presence of gain and offset mismatch simultaneously.

V. Conclusion

This paper has proposed a new digital calibration method for gain and offset mismatches. This method uses Comb-filter on each channel without any additional material resources except an accumulator in each channel for the implementation of the LMS algorithm. The proposed method has a very short calibration time estimated at 10 and 15 clock cycles for offset and gain calibration respectively. This method could be used also with other parallel ADC architectures such as the Parallel Sigma Delta (\( \Sigma\Delta \)) using Hadamard modulation.

REFERENCES