ROBUST DIRECTION-OF-ARRIVAL ESTIMATION FOR FM SOURCES IN THE PRESENCE OF IMPULSIVE NOISE

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ABSTRACT

Time-frequency methods can utilize the non-stationarity of signals to enhance the resolution capability and accuracy in direction-of-arrival estimation. In this paper, we consider the problem of non-stationary sources impinging on an array of sensors in an impulsive noise environment. We apply a robust time-frequency method in combination with morphological image processing to estimate the instantaneous frequency of the sources. Then, for the extracted time-frequency points, we employ robust methods to calculate the averaged spatial time-frequency matrix, which is then used for direction-of-arrival estimation.

Index Terms— Time-frequency analysis, direction of arrival estimation, robustness

1. INTRODUCTION

Non-stationary signals such as frequency-modulated (FM) signals arise in situations of moving targets, e.g. in radar and sonar. In such applications, the sources’ energy is confined to well-defined regions in the time-frequency (TF) domain. This fact together with the concept of spatial time-frequency distributions (STFD) can be used to enhance the performance of direction-of-arrival (DOA) estimation [1], [2]. Most methods are simply based on the assumption that the underlying noise is Gaussian. However, the assumption of Gaussian noise is not always fulfilled. In practical radio environments or due to lightning, sparking or the antenna failure, the noise is no longer Gaussian but heavy tailed. A time-frequency distribution (TFD) is severely degraded in an impulsive noise environment and robust methods are sought. In [3], the author has formulated a robust TFD based on the $L_1$-norm, while in [4], the author has suggested a robust TFD, which is based on vector median filtering to combat impulsive noise. In [5] the author has defined a myriimized filtering approach to obtain a robust TFD.

In this paper, we consider the problem of DOA estimation for FM signals under heavy tailed noise. We propose to combine robust TF methods, as in [3], [4] and [6], and the concept of averaging the STFD over the instantaneous frequency (IF) of the sources, to obtain enhanced DOA estimates. For the extraction of the required IF signature we employ morphological image processing techniques as suggested in [2]. We compare our approach with other robust methods for DOA estimation without TF processing, which are based on robust spatial covariance matrix estimation using a spatial sign operator [7] and robust adaptive trimming [8].

The paper is organized as follows. In Section 2, the signal model and conventional robust DOA estimation are briefly presented. Section 3 deals with robust TFD calculation. In Section 4, the used IF extraction technique and DOA estimation via an averaged STFD are described. Simulations and conclusions are presented in Sections 5 and 6.

2. SIGNAL MODEL

We assume $K$ narrowband non-stationary sources impinging on an uniform linear array (ULA) with $M$ sensors. The received signal vector $x(t)$ at the sensor array is given by

$$
x(t) = \sum_{k=1}^{K} a(\theta_k)e^{j\psi_k(t)} + n(t), \ t = 1, \ldots, N \quad (1)
$$

where $a(\theta_k)$ is the array response to the $k$th source signal impinging from direction $\theta_k$, $\psi_k(t)$ is the phase of the $k$th source signal with instantaneous frequency $IF_k = d\psi_k(t)/(2\pi dt)$, $n(t)$ denotes the noise due to sensor imperfections and surrounding environment and is considered to be impulsive, $N$ is the number of snapshots. The above model in matrix form is

$$
x(t) = A s(t) + n(t) \quad (2)
$$

where $A = [a(\theta_1), \ldots, a(\theta_K)]$ is commonly denoted as the array response matrix and $s(t) = [e^{j\psi_1(t)}, \ldots, e^{j\psi_K(t)}]^T$ is the source vector.

There are different methods for DOA estimation which are mainly based on the spatial covariance matrix estimate,
given for \( N \) snapshots by

\[
\hat{R} = \frac{1}{N} \sum_{t=1}^{N} x(t)x^H(t). \tag{3}
\]

The estimate in (3) is not robust and cannot be used in impulsive noise environments. We therefore require a robust estimate of spatial covariance matrix. Visuri et al. [7] have proposed a method based on the spatial sign function

\[
S\{x(t)\} = \begin{cases} \frac{x(t)}{||x(t)||} & \text{if } x(t) \neq 0 \\ 0 & \text{otherwise.} \end{cases}
\]

where \( ||x|| = (x^H x)^{1/2} \). The estimated spatial covariance matrix based on the spatial sign function is called the sign covariance matrix (SCM) and is given by

\[
\hat{R}_{\text{scm}} = \frac{1}{N} \sum_{t=1}^{N} S\{x(t)\}S\{x(t)\}^H
\]

(4)

Lim et al. [8] recently proposed another robust method for the estimation of a trimmed spatial covariance matrix, which attempts to suppress samples contaminated by impulsive noise. It essentially uses a Shapiro-Wilk W-test for normality in order to iteratively detect the contaminated samples. Let \( \hat{X} = [\hat{x}_1, \ldots, \hat{x}_M] \) be the trimmed observations and \( \hat{Z} \) denotes a matrix which contains zeros for all trimmed observations and one elsewhere. Then, the estimated covariance matrix can be written as follows

\[
\hat{R}_{\text{trim}} = 1./ZZ^H \odot \hat{X}\hat{X}^H
\]

(5)

where ./ and \( \odot \) denote elementwise division and multiplication, respectively. Assuming the noise process in (1) to be zero-mean and uncorrelated with the signal, we can decompose \( \hat{R} \) into noise and signal subspace. Then we are able to apply the popular MUSIC method [9] and estimate the DOAs as \( K \) largest peaks of the MUSIC pseudo spectrum.

3. ROBUST TIME-FREQUENCY DISTRIBUTION

TFDs have been extensively used over the last two decades for the analysis of non-stationary signals. However, TFDs are highly sensitive to non-Gaussian noise and their performance in an impulsive noise environment is severely degraded.

In the following, we want to calculate a robust TFD of some scalar signal \( x(t) \). We use \( \mathcal{G}(t,l) \) to denote the time-lag kernel function, which is defined as the convolution w.r.t. time \( t \) between the kernel \( \varphi(t,l) \) and the local auto-correlation function \( R_{xx}(t,l) = x(t+l)x^*(t-l) \) [3]. In this paper, we consider the Wigner-Ville distribution (WVD), for which we have \( \varphi(t,l) = \delta(t) \) where \( \delta(t) \) denotes the Kronecker delta, and the modified B-distribution (MBD), for which we have \( \varphi(t,l) = (||/\cosh^2(t)||)^\beta \) where \( \beta \in [0,1] \) is a cross-term suppression parameter.

To obtain a robust version of the TFD, the problem is formulated in [3] as an optimization of the following type,

\[
D_{xx}(t,f) = \arg \min_{\zeta} \sum_{l} h(l) |\mathcal{G}(t,l)e^{-j4\pi f l} - \zeta|^{\alpha}
\]

(6)

where \( h(l) \) is the lag-window and \( \alpha \) is the parameter chosen depending on the underlying noise distribution. In the case of Gaussian noise \( \alpha = 2 \) is chosen, and the corresponding solution to (6) is given by

\[
D_{xx}(t,f) = \frac{1}{\sum_{l} h(l)} \sum_{l} h(l)|\mathcal{G}(t,l)e^{-j4\pi f l}|
\]

(7)

Thus, from the above equation the standard TFD for each time instant \( t \) and frequency point \( f \) can interpreted as an estimate of the mean over complex observations \( \{\mathcal{G}(t,l)e^{-j4\pi f l}\} \) for \( l \in [-L/2, L/2] \). In order to robustify the TFD, the order of the cost function can be either chosen according to the fractional lower order moments (FOM) or based on a cost function for the median, i.e. \( \alpha = 1 \). The solution in this case is given by

\[
D_{xx}(t,f) = \frac{1}{a_h(t)} \sum_{l=-L/2}^{L/2-1} d(t,f,l) \mathcal{G}(t,l)e^{-j4\pi f l}
\]

(8)

where \( d(t,f,l) \) and \( a_h(t,f,l) \) are given by

\[
d(t,f,l) = \frac{h(l)}{|\mathcal{G}(t,l)e^{-j4\pi f l} - D_{xx}(t,f)|}
\]

(9)

and \( a_h(t,f,l) = 1/\sum_{l} d(t,f,l) \), respectively. Eq. (8) can be solved iteratively. Therefore, an initial TFD is calculated using (7). Then iterations for the robust TFD are performed until convergence. The other way to solve Eq. (6) is to use vector medians [4]. In this case, \( D_{xx}(t,f) \) is computed as follows

\[
D_{xx}(t,f) = \text{med}\{\Re\{h(l)\mathcal{G}(t,l)e^{-j4\pi f l}\}\}, \quad l \in [-L/2, L/2]
\]

(10)

Also, marginal medians can be used for independent estimation of real and imaginary parts [3]. We remark that the calculation of the vector median method is computationally simpler than the iterative procedures.

4. SPATIAL TIME-FREQUENCY DISTRIBUTION

For DOA estimation we use the concept of STFD matrices, which are defined in terms of the auto- and cross-TFDs between the sensor signals

\[
[D_{x|x}]_{ij} = D_{x_i x_j}(t,f), \quad i,j = 1, \ldots, M
\]

(11)
where $D_{x_i x_j}(t, f)$ denotes the cross-TFD between $x_i(t)$ and $x_j(t)$, respectively. In this paper, we use a robust WVD based on the iterative procedures, as in Eq. (8), to compute the STFD.

By averaging the STFD matrix over a subset of signal IF signatures, we are able to obtain an enhanced estimate of the spatial covariance matrix, which is then used for DOA estimation [1]. Since this requires the signal IF signature, its extraction is described next.

4.1. IF Extraction

In this paper, we employ morphological image processing techniques, as proposed in [2], to extract the IF of a multi-component signal. As the input image we consider a sensor averaged robust MBD which is obtained either by using the iterative procedures or by a vector median approach. The obtained image is binarized, which is then used to perform the IF extraction by using different morphological operations. In order to smooth objects opening and closing is performed [2]. Then the medial axis is approximated by thinning operations. All junction points are removed and the separated segments are labeled. As an example we consider two linear FM sources, and the median-based MBD, as described above. Figure 1 shows the non-robust MBD of the two sources without noise, and in impulsive noise, as well as the robust MBD and the extracted IF using the described technique.

![Fig. 1. (a) MBD of the noise-free signal (b) Non-robust MBD of the signal at SNR = -5 dB (c) Median-based robust MBD (d) IF extraction with median-based MBD.](image)

From Figure 1, we can infer that the application of morphological operations on non-robust TFD may most likely lead to an incorrect IF extraction. On the other hand, the robust median-based approach is able to provide an accurate estimate of the TF signature.

4.2. DOA Estimation

Let $IF_p$ for $p \in \{1, \ldots, P\}$ denote the $P$ extracted segments where each segment corresponds to a single source. To get the DOA estimate for each segment $IF_p$, an averaged STFD matrix is calculated for each $(t_k, f_k) \in IF_p$. The averaged STFD matrix is computed by

$$D_{av} = \frac{1}{\text{#IF}_k} \sum_{k \in IF_p} D_{x, x_j}(t_k, f_k)$$

Unlike the conventional spatial covariance matrix, the averaged STFD matrix provides an effective improvement by amplification of source eigenvalues w.r.t the noise eigenvalues [10]. The final estimates for $\theta_p$ are obtained by successively applying MUSIC [9] and then combining the DOAs, which belong to the same source. The algorithm is summarized in Table 1.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>Compute the spatially averaged robust MBD based on iterative or median, Eq. (7) and (10), procedures as $D_{x, x_j}(t, f) = \sum_{i=1}^{M} D_{x, x_j}(t, f)/M$</td>
</tr>
<tr>
<td>2.</td>
<td>Perform the morphological image processing techniques to extract single-source IF segments, as in [2]</td>
</tr>
<tr>
<td>3.</td>
<td>For each segment, compute the average STFD matrix, by using the robust WVD as in Eq. (8)</td>
</tr>
<tr>
<td>4.</td>
<td>Estimate the DOA of each averaged STFD, using the MUSIC algorithm</td>
</tr>
<tr>
<td>5.</td>
<td>Combine broken segments whose DOAs are close w.r.t. estimation variance</td>
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5. SIMULATIONS

To evaluate the performance of the proposed method and to compare it with standard robust methods for DOA estimation in impulsive noise, we have conducted Monte-Carlo simulations. In the setup, $M = 8$ sensors in an ULA geometry and two linear FM signals, as shown in Figure 1, from DOAs $[-3^\circ, 2^\circ]$ have been simulated. All sources have the same power and $N = 128$ snapshots are used for estimation. The noise $n(t)$ in Eq. (2) is modeled as an $\epsilon$-contaminated mixture, given by

$$n(t) \sim (1-\epsilon)N_C(0, \sigma^2I) + \epsilon N_C(0, \kappa \sigma^2I)$$

(13)
where \( N_C(\mu, \Sigma) \) is used to denote an \( M \)-variate circular complex Gaussian distribution with mean \( \mu \) and covariance matrix \( \Sigma \), \( \epsilon \) is a positive contamination number \( \epsilon \in [0, 1] \), and \( \kappa \) is the degree of impulsiveness, usually \( \kappa \gg 1 \). In the simulations, \( \epsilon = 0.2 \) and \( \kappa = 20 \) was used. For IF extraction, the robust MBD using the median-based procedures of Eq. (10) and the iterative procedures of Eq. (8) with a window length of \( L = 21 \) was used. For the computation of STFD matrices, the iterative robust WVD with a window length of \( L = 29 \) was used. Figures 2 and 3 show the RMSE of DOA estimates versus SNR for both sources.

\[ \text{RMSE (deg)} \]

\[ \text{SNR (dB)} \]

**Fig. 2. RMSE of DOA estimate for the source from } -3^\circ.**

\[ \text{RMSE (deg)} \]

\[ \text{SNR (dB)} \]

**Fig. 3. RMSE of DOA estimate for the source from } 2^\circ.**

It can be observed in both figures that all TF methods converge to a smaller RMSE when compared with the non-TF methods, since effectively single-source DOA estimation is carried out. In terms of standard robust methods, the Visuri method provides the lowest RMSE while the method without any impulsive remedy requires a high SNR to achieve reasonable results. We have seen that the median-based techniques is able to provide a cleaner image which eases the IF extraction when compared with the iterative based methods which is computationally more expensive. In the case of known IF, the robust TF averaging provides the best RMSE.

6. CONCLUSION

We have presented robust TF processing for the problem of DOA estimation. For FM sources, the proposed method performs better than the conventional robust DOA estimation techniques. Moreover, the proposed method does not necessarily require the estimation of number of sources, since single source DOA estimation (in combination with a clustering technique) is possible using STFD matrices averaged over individual IF segments.

7. REFERENCES


