Abstract—This paper considers spectrally efficient anti-jamming system design based on coded three-dimensional (3D) frequency hopping. Unlike conventional frequency hopping systems where the hopping pattern is determined by a preselected pseudo-random sequence, in the proposed scheme, part of the information is passed through a block encoder, and used to determine the selected frequency bands for signal transmission. The receiver is designed to retrieve the hopping pattern without any a priori knowledge. The proposed system can effectively mitigate random jamming interference while maintaining high spectral efficiency. Simulation results are provided to illustrate the system’s jamming mitigation performance.

Index Terms: Anti-jamming, frequency hopping, coding

I. INTRODUCTION

As a popular spread spectrum technique, frequency hopping (FH) has been widely used for military and civil applications [1]. In conventional FH, the transmitter hops in a pseudo-random manner among the available frequencies according to a pre-specified pattern, and the receiver then operates in strict synchronization with the transmitter and remains tuned to the same center frequency.

The pseudo-random frequency hopping during radio transmission minimizes the possibility of hostile jamming and unauthorized interception. However, the spectral efficiency of the conventional FH is very low. In literature, considerable efforts have been devoted to increasing the spectral efficiency of FH systems by applying high-dimensional modulation schemes, see [2]–[4], for example. These systems are common in that they hop in a wide band much larger than the actually required bandwidth for message transmission. In many communication systems, however, strict bandwidth constraint is usually imposed. Therefore, new techniques that are more efficient and reliable have to be developed.

In this paper, we propose a 3D frequency hopping scheme for spectrally efficient jamming mitigation. Unlike conventional FH systems where the hopping pattern is determined by a preselected pseudo-random sequence, in the proposed scheme, a coded information sequence is used to determine frequency bands for signal transmission. In this way, an extra dimension is added to the signal space, making the modulation a 3D scheme. Block coding is applied in a novel way such that the receiver can retrieve the hopping pattern correctly. The proposed system can effectively mitigate random jamming interference while maintaining high spectral efficiency. Today, along with advances in wireless communications, such as the emerging cognitive radios, intentional or unintentional jamming is prevalent in many applications. The proposed system provides a possible solution for efficient and secure wireless communications under jamming environment.

II. TRANSMITTER DESIGN OF CODE-CONTROLLED FH

The transmitter structure of the proposed system is illustrated in Fig. 1. The input binary stream \{a_i\} with bit interval \(T_b\) is demultiplexed (DEMUX) into two branches. In the lower branch, \(K\) bits \(\{a_1, a_2, \ldots, a_K\}\) are fed into a rate \(\frac{K}{N}\) block encoder \(ENC_1\), which produces an \(N\)-bit codeword \(b = [b_1, b_2, \ldots, b_N]\). The block encoder \(ENC_1\) is chosen in a way such that the total number of 1’s in the codeword, \(L = \sum_{i=1}^{N} b_i\), is a fixed number for any \(K\)-bit input as long as \(a = [a_1, a_2, \ldots, a_K] \neq [0, 0, \ldots, 0]\). Two popular codes possessing such property are the maximum-length codes and the Hadamard codes [5]. If an \((N, K)\) Hadamard code is employed, for example, then \(N = 2^{(K-1)}\) and \(L = N/2\). This code can correct up to \(t = N/4\) errors.

The upper branch is the concatenation of a rate-\(R\) encoder \(ENC_2\), an interleaver \(\pi\), and an \(M\)-ary symbol mapper with \(m = \log_2 M\) bits per symbol. The number of bits entering the upper branch is \(mLR\), which is assumed to be an integer. That is, a total number of \(L\) symbols are generated from the upper branch, denoted as \(d = [d_1, d_2, \ldots, d_L]\). The symbol vector \(d\) is first fed to a serial-to-parallel (S/P) converter and then transmitted simultaneously through \(L\) selected subcarriers of a \(N\)-carrier system. These \(L\) active subcarriers are determined by the \(N\)-bit codeword \(b' = [b_1, b_2, \ldots, b_N]\) generated in the lower branch. Let \({\omega_1, \omega_2, \ldots, \omega_N}\) be all the carrier frequencies in the system. For \(i = 1, \ldots, L\), if \(b_i = 1\), then carrier centered at \(\omega_i\) is selected as an active subcarrier; Otherwise, if \(b_i = 0\), then carrier centered at \(\omega_i\) becomes idle.

In other words, input binary stream \{a_i\} is always divided into blocks of length \(L_b = mLR + K\). During each time interval \(T_s = (mLR + K)T_b\), the transmitted signal on the \(i\)th carrier, \(s_i(t)\), can be expressed as

\[
s_i(t) = s_i e^{j(\omega_i t + \phi_i)},
\]

where \(s_i\) is the signal symbol transmitted over carrier \(i\) and \(\phi_i\) is a random phase uniformly distributed over \((0, 2\pi]\), which is generally caused in the modulation process and can be tracked at the receiver. \(s_i\) can be represented as

\[
s_i = \begin{cases} 0, & b_i = 0; \\
d_{\ell(i)}, & b_i = 1. \end{cases}
\]

Here the subscript \(\ell(i) = \sum_{k=1}^{i} b_k\) indicates the index of the upper branch symbol modulated on the \(i\)th carrier. By defining \(d_0 = 0\), Eq. (2) can be simplified as

\[
s_i = b_i d_{\ell(i)}.
\]

As will be seen in the next section, \(b_i\) can be simply detected using an energy detector. Since the \(K\)-bit vector \([a_1, a_2, \ldots, a_K]\) is embedded in the hopping frequency selection process through \([b_i]\), these information is transmitted at no cost of power or spectrum. From the information theory point of view, message code controlled frequency hopping actually adds another dimension to the signal space, and hence formulates a 3D modulation scheme.

Remark: The proposed system can be directly extended to...
a multi-user system in different ways. For downlink transmission, different users' data can be multiplexed into one stream \( \{a_i\} \) for transmission. For the uplink, multiple access can be achieved by combining the proposed scheme with TDMA or wideband FDMA.

### III. RECEIVER DESIGN

We assume that the system is subjected to partial-band jamming, for which the total jamming power \( J \) watts is uniformly distributed over a fraction \( \rho \) of the system bandwidth \( W \) Hz. If the bandwidth of each carrier/channel is \( B \) Hz, the jamming power in one jammed channel is given by

\[
\sigma_j^2 = \left( \frac{J}{\rho W} \right) B \text{ watts.} \tag{4}
\]

The received signal \( r_i(t) \) for the \( i \)th channel can be written as

\[
r_i(t) = s_i(t) + \theta_i j_i(t) + n_i(t), \tag{5}
\]

where \( n_i(t) \) and \( j_i(t) \) represent system noise and jamming interference, respectively. They are assumed to be statistically independent zero-mean Gaussian processes, with \( E\{|n_i(t)|^2\} = \sigma_n^2 \) and \( E\{|j_i(t)|^2\} = \sigma_j^2 \). Define

\[
\theta_i = \begin{cases} 1, & j_i(t) \text{ is present in } r_i(t), \\ 0, & j_i(t) \text{ is absent from } r_i(t). \end{cases} \tag{6}
\]

Then \( \theta_i \) is a binary random variable with \( P(\theta_i = 1) = \rho \) and \( P(\theta_i = 0) = 1 - \rho \).

#### A. Receiver Structure

The block diagram of the receiver is shown in Fig. 2. The received signal for the \( i \)th channel is given by

\[
r_i = \frac{1}{T_s} \int_0^{T_s} r_i(t) e^{-j(\omega_s t + \phi_i)} dt = b_i d_{e(i)} + \theta_i j_i + n_i, \tag{7}
\]

where \( j_i \) and \( n_i \) are zero-mean Gaussian variables with \( E\{|j_i|^2\} = \sigma_j^2 \) and \( E\{|n_i|^2\} = \sigma_n^2 \).

In decoding, we first retrieve \( \hat{b}_i \) to determine the pattern of carrier selection, and then recover \( d_{e(i)} \) for the channel with \( b_i = 1 \). The estimates of \( \hat{b} = [b_1, b_2, \ldots, b_N] \), denoted as \( \hat{b} = [\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_N] \), are determined according to the following criterion.

\[
\hat{b}_i = \begin{cases} 1, & |r_i|^2 \geq \gamma; \\ 0, & |r_i|^2 < \gamma, \end{cases} \tag{8}
\]

where \( \gamma \) is the decision threshold, whose optimum value can be determined by minimizing the error probability \( p_i = P(\hat{b}_i \neq b_i) \), as will be discussed in Section III-B.

The rough estimation \( \hat{b} = [\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_N] \) is parallel to serial converted, and passed to the block decoder \( DEC_1 \) which corresponds to the \( ENC_1 \) at the transmitter. The outputs \( \hat{a} = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_K] \) are recovered signal bits for the lower branch. Following the \( DEC_1 \), we employ an encoder same as the \( ENC_1 \), which encodes \( \hat{a} = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_K] \) to produce a more accurate estimate of \( b = [b_1, b_2, \ldots, b_N] \), denoted as \( \check{b} = [\check{b}_1, \check{b}_2, \ldots, \check{b}_N] \).

The set of carriers \( I \) selected for transmission is given by \( I = \{k \mid b_k = 1, 1 \leq k \leq N\} \). To retrieve the upper branch signals, \( \{r_i, i \in I\} \) is first demodulated, then fed to the deinterleaver \( \pi^{-1} \), followed by decoder \( DEC_2 \). The decoder output, multiplexed with the estimated lower branch bits, is then the recovered signal.

#### B. Optimal Threshold Selection

The error probability of \( b_i \) in the lower branch, denoted as \( p_i \), depends on jamming conditions. If the output of the block encoder \( ENC_1 \) always has an equal number of 1’s and 0’s, as in the Hadamard code, it then follows from Eqs. (8) and (6) that:

\[
p_i = \sum_{b_i=0,1} P\{b_i \neq \tilde{b}_i \} P\{b_i \}
\]

\[
= \frac{1}{2} \sum_{b_i=0,1} P\{b_i \neq \tilde{b}_i | \theta_i = 0, \tilde{b}_i \} P\{\theta_i \}
\]

\[
= \frac{1}{2} \left[ (1 - \rho) p_{i1} + \rho p_{i2} \right] + \frac{1}{2} \left[ (1 - \rho) p_{i3} + \rho p_{i4} \right] \tag{9}
\]

where \( \rho = P(\theta_i = 1) \) and \( 1 - \rho = P(\theta_i = 0) \), and the conditional probabilities \( p_{i1}, \ldots, p_{i4} \) can be calculated as follows.

The probability of \( p_{i1} \) is given by

\[
p_{i1} = P\{|r_i|^2 < \gamma | b_i = 1, \theta_i = 0\}
\]

\[
= P\{|d_{e(i)} + n_i|^2 < \gamma\}
\]

\[
= 1 - Q_1\left(\frac{|d_{e(i)}|}{\sqrt{\gamma}}, \frac{\sqrt{\gamma}}{\sigma_n}\right), \tag{10}
\]

where \( |d_{e(i)}| \) is the square-root of the modulated signal power, and function \( Q_1(a, b) \) is the Marcum’s Q function defined as \( Q_1(a, b) = \int_b^\infty x e^{-(x^2+a^2)/2} I_0(ax) dx \). Here \( I_0(x) \) is the zeroth order Bessel function of the first kind.
Similarly,
\[ p_{i2} = P(|r_i|^2 < \gamma |b_i = 1, \theta_i = 1) \]
\[ = 1 - Q_1\left( \frac{||d_{i(d1)}||}{\sqrt{\sigma_n^2 + \sigma_j^2}} \frac{\sqrt{\gamma}}{\sqrt{\sigma_n^2 + \sigma_j^2}} \right), \]  \hspace{1cm} (12) \]
and
\[ p_{i3} = P(|r_i|^2 \geq \gamma |b_i = 0, \theta_i = 0) \]
\[ = e^{\gamma/\sigma_n^2}, \]  \hspace{1cm} (13) \]
\[ p_{i4} = P(|r_i|^2 \geq \gamma |b_i = 0, \theta_i = 1) \]
\[ = e^{\gamma/(\sigma_n^2 + \sigma_j^2)}. \]  \hspace{1cm} (14) \]
The optimal threshold \( \gamma_{opt} \) can be determined through
\[ \gamma_{opt} = \arg \min_{\gamma} p_i \]  \hspace{1cm} (15) \]
The closed-form solution of \( \gamma_{opt} \) is hard to find. Fortunately, simulation results have shown that satisfying performance can still be achieved even if \( \gamma \) varies over a certain range. In practice, when a rough estimation of jamming interference and noise power is obtained, \( \gamma \) can be set to be slightly above the power level of the interference and the noise.

IV. SPECTRAL EFFICIENCY

To ensure orthogonality between the subcarriers, the minimum frequency spacing between adjacent carriers should be \( \Delta f = 1/T_s \). The total bandwidth \( W \) required for the system depends on whether adjacent carriers are overlapped or not. If the overall frequency band is divided into \( N \) nonoverlapping subchannels and signals are frequency-multiplexed, the total system bandwidth, denoted here as \( W_{FDM} \), is given by
\[ W_{FDM} = \frac{2N}{(mLR+K)T_s} \]
The spectral efficiency \( \eta_{FDM} \) is then given by
\[ \eta_{FDM} = \frac{(mLR+K)}{2N} \text{ bits/s/Hz}. \]  \hspace{1cm} (16) \]

Otherwise, if carriers are arranged in a way similar to that of Orthogonal Frequency Division Multiplexing (OFDM), the required bandwidth, \( W_{OFDM} \), is given by
\[ W_{OFDM} = \frac{N+1}{mLR+K)T_s} \]
The corresponding spectral efficiency \( \eta_{OFDM} \) is
\[ \eta_{OFDM} = \frac{(mLR+K)}{N+1} \text{ bits/s/Hz}. \]  \hspace{1cm} (17) \]

Therefore, by using OFDM modulation technique, we could almost double the spectral efficiency. However, as will be demonstrated in the simulation examples, there is always a tradeoff between the spectral efficiency and anti-jamming capability.

Consider a conventional FH system which employs the same \( R \)-rate convolutional code and \( M \)-ary symbol mapping as that of the proposed scheme. If the system hops at the symbol rate among \( N' \) frequency slots, the required bandwidth is \( W_{CFH} = \frac{2N'}{2^M R} \), and the bandwidth efficiency \( \eta_{CFH} \) is given as below:
\[ \eta_{CFH} = \frac{mR}{2N'} \text{ bits/s/Hz}. \]  \hspace{1cm} (18) \]
If \( N' = N \), i.e., the number of channels are equal for the two schemes, we can see that the proposed scheme has much higher spectral efficiency than the conventional frequency hopping. If the constraint \( \eta_{CFH} = \eta_{FDM} \) is imposed, then \( N' \approx \frac{N}{L} \). That is, the number of available channels becomes very limited for the conventional FH, which significantly degrades its anti-jamming property. For example, in the case that Hadamard code is employed in the proposed scheme, then \( L = N/2 \), and \( N' \approx 2 \). That is, the system can only hop over two bands. The hopping randomness is minimal.

V. SIMULATION RESULTS

Simulations are conducted to evaluate the system performance. For purpose of comparison, the signal power \( S \) for each incoming bit and the total injected jamming power \( J \) are fixed for all systems. The \( (N, K) = (128, 8) \) Hadamard code is employed for lower branch encoding. \( L = N/2 = 64 \). Convolutional code with \( R = 1/2 \) is used for upper branch encoding. 8-PSK constellation is used for symbol mapping.

Under these settings, it then follows from Eq. (16) and Eq. (17) that the spectral efficiency for non-overlapping modulation is \( \eta_{FDM} \approx 0.41 \text{ bits/s/Hz} \), and \( \eta_{OFDM} \approx 0.81 \text{ bits/s/Hz} \) for overlapping modulation. The overall bit error rate (BER) performance of the proposed system using these two modulation schemes are presented in Fig. 3. The jamming ratio \( J \) varies as we evaluate the system performances. It is demonstrated that the full-band jamming, i.e., \( J = 1 \), results in worst performance. The underlying argument is that with the employment of coding and interleaving, the jammer has to spread the noise over the whole bandwidth to achieve best jamming effects. We can also see from the figure that when overlapping modulation is employed, the system is more
susceptible to jamming noise and the BER performance is much worse than the non-overlapping scheme.

For a conventional scheme that does not have any jamming mitigation and detection capabilities, all carriers are modulated with signals. Suppose the same half-rate convolutional code and the non-overlapping modulation are used. The spectral efficiency for such a system is 0.75 bits/s/Hz. Compared with the proposed scheme using non-overlapping modulation, we can see from Fig. 4 that its performance is much worse than that of the proposed scheme.

Fig. 5 compares the BER performance of the proposed and the conventional scheme, both using non-overlapping modulation. The conventional scheme employs an $R = 1/4$ convolutional code such that its spectral efficiency is close to that of the proposed system. It is shown that 2dB gain can be achieved by the proposed scheme, which substantiates the effectiveness of the scheme’s anti-jamming capability.

Fig. 6 is the BER comparison of the proposed scheme and the conventional FH. The parameters of the two systems are set such that their spectral efficiencies are almost the same. It can be seen that under the bandwidth constraint, the hopping channels for the conventional FH is limited to 2, which significantly degrades its performance in the case of jamming ratio $\rho = 1/2$. As can be seen from Fig. 6, the proposed scheme shows much stronger anti-jamming capabilities.

VI. CONCLUSIONS

In this paper, we propose an anti-jamming system to mitigate jamming interference while maintaining high spectral efficiency. Multiple frequency bands are randomly selected and modulated with signals for transmission. Block coding is used for the receiver to correctly retrieve frequency hops. The scheme can effectively mitigate jamming interference, which is demonstrated by simulations.

REFERENCES