LINK ADAPTATION IN MIMO-OFDM WITH NON-UNIFORM CONSTELLATION SELECTION OVER SPATIAL STREAMS THROUGH SUPERVISED LEARNING

Robert C. Daniels and Robert W. Heath, Jr.

Wireless Networking and Communications Group, The University of Texas at Austin
1 University Station C0806, Austin, TX 78712-0240
Email: {rcdaniels,rheath}@mail.utexas.edu

ABSTRACT
Supervised learning has been used to develop practical link adaptation algorithms for MIMO-OFDM under an equal rate per stream assumption. In this paper we develop supervised learning algorithms that select from non-uniform rates per stream. We show that the straightforward application of existing supervised learning link adaptation algorithms exhibits complexity that scales with the number of spatial streams. Therefore, we propose a decoupled stream link adaptation algorithm which reduces the complexity below the original supervised learning algorithm with uniform spatial streams. We further show that the performance loss of decoupled link adaptation is reduced in systems with non-uniform constellations per spatial stream. IEEE 802.11n and uncoded MIMO-OFDM simulations are used to validate the proposed algorithms.

Index Terms— MIMO-OFDM, link adaptation, supervised learning, non-uniform spatial streams, IEEE 802.11n

1. INTRODUCTION
Link adaptation in most MIMO-OFDM wireless systems is the process of selecting the best single constellation order and single error correction code over all subcarriers and spatial streams to maximize rate while satisfying reliability constraints. Link adaptation for MIMO-OFDM is challenging in practice due to the complicated relationship between the performance of forward error correction and the channel state [1]. Recent results have shown that supervised learning algorithms, which capture this complicated relationship from training data based on past performance, accurately complete link adaptation in these complex systems [2]. Past research has assumed stream uniformity, i.e., the same rate was employed for each spatial multiplexing stream. Channels with disparate stream quality, however, may benefit from a multitude of potential rates on each stream. Disparate stream quality occurs, for example, in spatially correlated channels, in frequency-flat channels without precoding, or when ordered stream precoding is implemented. In this paper, we derive rate adaptation algorithms where each stream selects a different rate through supervised learning.

Relaxing the stream uniformity constraint, however, leads to two primary concerns in link adaptation. First, it expands the search space of potential transmission parameters causing increased memory and processing complexity costs. Second, the streams are no longer equivalent since each stream may represent a different number of coded bits and each stream no longer dedicates the same energy-per-bit (if precoding is considered to be part of the effective channel). Prior work on supervised learning and link adaptation in MIMO-OFDM exploited this stream equivalence to reduce the design complexity [2]. Hence, new complexity reduction techniques will be required to make supervised learning algorithms manageable in MIMO-OFDM with non-uniform spatial streams.

To address the aforementioned link adaptation concerns with non-uniform constellations over spatial streams, we propose and analyze the performance of a link adaptation algorithm that decouples the spatial streams. Through decoupling, the complexity is lower than the original uniform system model. To better understand the degree of performance sacrifice with stream decoupling, we create a bound on the maximum throughput difference between decoupled and joint stream link adaptation. We discover that, given an adequate selection of constellation orders and encoder configurations, non-uniform constellation selection over each spatial stream shrinks the performance gap between joint and decoupled algorithms. Numerical experiments of link adaptation through supervised learning in an uncoded non-uniform MIMO-OFDM system confirm the utility of decoupled link adaptation algorithms. Additional simulations of an IEEE 802.11n system with non-uniform spatial streams display the effects of frequency selectivity with variable coding rates and suggest that future standards should include a larger availability of non-uniform constellations/coding rates per stream, since it is critical towards maximizing the data rate while reducing the complexity of link adaptation procedures.

2. MIMO-OFDM SYSTEM MODEL
We consider frame-based MIMO-OFDM transmission where source binary data is fragmented. Each frame or fragment of bits is channel encoded for forward error correction and formatted into OFDM symbols. \( N_r/N_s \) is the number of available RF chains at the transmitter/receiver and \( N_s \leq N_d \equiv \min\{N_r, N_f\} \) is the number of spatial multiplexing streams (simultaneous data streams) used during transmission. For MIMO-OFDM symbol \( m \in \{0, 1, \ldots, N_0 - 1\} \) and subcarrier \( n \in \{0, 1, \ldots, N - 1\} \), \( X[m, n] \in \mathbb{C}^{N_s} \) is the transmit symbol vector. The data symbol for the \( a \)th spatial stream, \( X_s[a, n] \in \mathbb{C} \) for \( a \in \{0, 1, \ldots, N_s - 1\} \), represents \( K_s = \log_2 M_s \) encoded bits through, for example, quadrature amplitude modulation (QAM) constellations. We represent \( H[n] \in \mathbb{C}^{N_r \times N_t} \) as the frequency flat effective wireless channel matrix for subcarrier \( n \), \( E_s \) as the average transmit signal energy, and assume additive white Gaussian noise effects with variance \( \sigma^2 \) at the receiver. The wireless channel definition \( H[n] \) for subcarrier \( n \) in this model assumes that the channel impulse response does not change for all \( N_0 \) OFDM symbols within a frame. For a more specific description of the system model when the streams are uniform,
3. MIMO-OFDM LINK ADAPTATION THROUGH SUPERVISED LEARNING

Link adaptation in our MIMO-OFDM system model is the process of selecting the number of spatial streams, $N_s$, the constellation order for spatial stream $a$, $M_a$ for all, and the channel encoder configuration. Each realization of $N_s$, $\{M_1, M_2, \ldots, M_{N_s}\}$, and a single encoder configuration is called a modulation and coding scheme (MCS). In terms of classification for supervised learning, link adaptation is the process of selecting a class $i$ in $\{0, 1, \ldots, |\mathcal{I}| - 1\}$ to maximize rate, $R_i$, under a reliability constraint. For the remainder of this paper, per convention, we will assume a frame error rate (FER) constraint of $T$ where $\text{FER}_i$, the probability of incorrectly decoding source bits with MCS, for a specific channel, is denoted $\text{FER}_i(\{\mathbf{H}[n]\}_{n=0}^{N_s-1}, E_s, \sigma^2)$. Thus, classification selects

$$
\arg \max_i \{R_i : \text{FER}_i(\cdot) \leq T \}
$$

(1)
to maximize the performance for a given channel realization. If no MCS satisfies (1) the most reliable MCS will be selected.

Supervised learning completes classification by discovering relationships in training data. The training data is composed of the following.

- $W \times N_s$ classes, each corresponding to the best MCS with $a$ spatial streams $\forall a \in \{1, 2, \ldots, N_s\}$ for each of $W$ distinct channel realizations.
- $p \times N_s$-dimensional feature set, $\mathbf{z}_{a,i} \forall a, \forall \mathbf{w} \in \{0, 1, \ldots, W - 1\}$ and, extracted from each distinct channel realization.

$W$ should be large enough to provide a diverse set of channels and establish the relationship between each channel state, as captured in the feature set, and each class $i$. The feature set should be chosen to accurately characterize the reliability constraint (e.g. FER, $\forall i$) with $p$ as small as possible to reduce complexity. Finally, the classifier algorithm (e.g. Bayesian classifier, $k$-nearest neighbor ($k$-NN), etc.) should be able to identify the mapping between feature sets and the correct class (MCS) according to (1).

3.1. Joint Stream Link Adaptation

Joint stream link adaptation is the straightforward translation of supervised learning to link adaptation in MIMO-OFDM systems with non-uniform constellations. Joint algorithms allow for high accuracy, but also suffer from high complexity because no commonality/correlation between each spatial stream is exploited. The algorithm is as follows.

1. Define $k := 0$.
2. Extract the $(k + 1) \times p$-dimensional feature set ($p$ dimensions for each stream) from channel state measurements and store in query $\mathbf{q}$.
3. Use classifier $c_k$, trained with $\{\mathbf{z}_{0,0}, \ldots, \mathbf{z}_{k,W-1}\}$ and the associated $(k+1)$-stream classes, to predict the mapping $c_k : \mathbf{q} \mapsto i(k)$ where $i(k)$ is the highest rate $(k + 1)$-stream MCS meeting the reliability constraint for the current channel state.
4. Increment $k$ and repeat steps 2-4 until $k = N_d$.
5. Select highest rate MCS in $\{i(0), \ldots, i(N_d - 1)\}$.

If $\{c_0, c_1, \ldots, c_{N_d-1}\}$ are all ideal, the algorithm performs perfect link adaptation according to the reliability constraint.

3.2. Decoupled Stream Link Adaptation

Joint stream link adaptation requires $N_d$ different classifiers. To save silicon space, processing power, and battery energy we propose a link adaptation algorithm that decouples the spatial streams as illustrated in Fig. 1. The key assumption of stream decoupling is that $k + 1$ single stream MCSs may be mapped to an equivalent $(k + 1)$-stream MCS. For the moment, we will accept this as true and will defer the discussion of such mappings to the next subsection. The decoupled stream link adaptation algorithm is as follows.

1. Define $k := 0$. Define $a := 0$.
2. Compute the effective $(k + 1)$-stream channel.
3. Extract the $p$-dimensional feature set for stream $a + 1$ and store it in the current query $\mathbf{q}$. Use the classifier $c_a$ trained with $\{\mathbf{z}_{a,0}, \mathbf{z}_{a,1}, \ldots, \mathbf{z}_{a,W-1}\}$ and the associated 1-stream classes, to complete mapping $c_a : \mathbf{q} \mapsto i(0, a)$ where $i(0, a)$ is the highest rate 1-stream MCS meeting the reliability constraint for stream $a$ in the current channel state.
4. Increment $a$ and repeat step 3 until $a = k + 1$.
5. Map single stream MCS classes $\{i(0,0), \ldots, i(0,k)\}$ to $(k + 1)$-stream equivalent MCS $i(k)$.
6. Increment $k$ and repeat steps 2-5 until $k = N_d$.
7. Select highest rate MCS in $\{i(0), \ldots, i(N_d - 1)\}$.

We observe reduced complexity since only one classifier, $c_0$, is needed. The training data required is also reduced, which often significantly reduces the memory required in classifiers. A complexity comparison of joint and decoupled link adaptation for MIMO-OFDM is summarized in Table 1.

3.3. Decoupled Stream Mappings

Decoupled stream link adaptation requires a single stream MCS mapping from the channel for each of $N_s$ spatial streams separately. The algorithm assumes that there exists a unique mapping to an $N_s$-stream MCS. The standard MIMO-OFDM system model, however, only allows for a single channel encoder realization over all subcarriers and spatial streams per frame. Hence, because encoder configurations (primarily coding rate) are adapted to provide
Table 1. Memory complexity comparison for joint and decoupled stream link adaptation in non-uniform MIMO-OFDM systems. \( M \) is the required memory for a classifier and feature set in a single stream. The processing complexity for decoupled link adaptation is also much less because the decoupled operation does not have to complete new searches to consider permutations of the constellation order on each stream.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Memory Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint</td>
<td>( M \times \sum_{k=1}^{N_s} k )</td>
</tr>
<tr>
<td>Decoupled</td>
<td>( M )</td>
</tr>
</tbody>
</table>

We further assume that decoupled stream link adaptation never selects a single stream MCS for any spatial stream that is more than one index below the equivalent spatial stream MCS that results from joint stream MCS selection. We justify this assumption by observing that equal shifts in SNR on a dB scale above and below an MCS boundary (10% FER) do not produce equal shifts in FER. Since the MCSs are designed to be equally distributed in SNR on a dB scale and because the rate of FER increase above the 10% FER SNR boundary is much larger than the rate of FER decrease below the SNR boundary for typical MCS realizations, this assumption is often valid in practice [5].

Given the aforementioned system model and its assumptions, we acknowledge that the worst case scenario occurs when the average SNR for all but one of the spatial streams is just below the FER constraint boundary. In that scenario, joint stream MCS selection will balance the FER across the spatial streams by bumping up all but one of the single stream MCSs. Thus, we state that the maximum performance gap is

\[
\Delta T_{\text{max}} = (N_s - 1) \times \max_{i \in \{1, 2, \ldots, |I| - 1\}} \{R_i - R_{i-1}\}. \quad (3)
\]

Clearly, by decreasing the rate gap between adjacent single stream MCSs with the same channel encoder configuration, we decrease the maximum potential performance gap.

4. Simulations

We complete numerical simulations first using the IEEE 802.11n standard with 20 MHz channels, 2 receive antennas, 2 transmit antennas, and MCSs through MCS15 (uniform streams) along with MCS3 through MCS38 (non-uniform streams) [6]. Link adaptation is performed with \( k \)-nearest neighbor (\( k \)-NN) classifiers and ordered post-processing SNR feature sets according to [2]. In these simulations, however, for joint stream MCS selection, since non-uniform spatial streams are considered, we cannot perform SNR ordering over the spatial streams, but instead only order per-spatial stream. This increases the dimensionality of the feature set in joint MCS selection by a factor of \( N_s = 2 \) when compared to the features used in [2] and decoupled stream MCS selection. Fig. 2 shows the throughput generated from complex Gaussian channels for 1, 4, and 8 delay taps with uniform power delay profiles. Each packet consists of 128 bytes, uses no precoding, is perfectly synchronized, and the receiver implements zero-forcing equalization with error-free channel estimation.

The gains for non-uniform modulation (max throughput gain \( \approx 4 \) Mbps) are not indicative of the maximum gains in practice. SVD precoding which creates a larger disparity between stream quality and higher MIMO dimensionality which creates more potential streams. Adding selectivity to the channel increases the quality of spatial streams on average. We observe less channels where non-uniform constellations are advantageous. To maximize the gains of non-uniform constellation selection, precoding is desired to maintain channel quality disparity among streams. The gap (\( \approx 10 \) – 20 Mbps) between the performance of joint and decoupled stream link adaptation is significant, which was expected for IEEE 802.11n since there is only a small number of available mappings between single stream rates and double stream rates. These simulations encourage future MIMO-OFDM standards to consider a more comprehensive list of available coding rates and non-uniform constellations to enable high-performance, reduced-complexity link adaptation.
Fig. 2. Throughput of $2 \times 2$ IEEE 802.11n systems with joint stream (JS) and decoupled stream (DS) link adaptation using $k$-NN classifiers. The throughput resulting from ideal link adaptation was not plotted because the curves were not significantly distinguishable (the mean/max throughput loss from $k$-NN link adaptation was 0.0%/0.2%/1.7%/5.3%/2.2%/6.7% for 1, 4, and 8 tap channels respectively). The uniform curves only adapt over MCS0 through MCS15 while the non-uniform curves leverage the remaining MCS33 through MCS38.

Fig. 3. $3 \times 3$ MIMO-OFDM uncoded system performance for BPSK, QPSK, 8-PSK, and 16-PSK modulation per stream. SVD precoding is implemented to create stream disparity. Decoupled link adaptation performance approaches joint link adaptation performance for non-uniform modulation.

To further demonstrate this point a second set of supervised learning link adaptation simulations were completed, this time with BPSK, QPSK, 8-PSK, and 16-PSK modulation in $3 \times 3$ MIMO-OFDM with frequency flat channels, SVD precoding, and no forward error correction. All combinations of different dimension PSK constellations are available to create non-uniform spatial streams up to $N_s = 3$. Fig. 3 shows the resulting throughput for $k$-NN classifiers and ordered post-processing SNR feature sets. We observe significant gains for non-uniform modulation ($\approx 0.6$bps/Hz) over uniform modulation. Moreover we can demonstrate that decoupled link adaptation approaches the performance of joint link adaptation when non-uniform modulation is enabled. This confirms our intuition that non-uniform modulation can serve to both increase the overall throughput and allow low-complexity, high performance link adaptation for channels with stream quality disparity.

5. CONCLUSION

Supervised learning provides accurate link adaptation algorithms in challenging frequency selective channels. The complexity of link adaptation is reduced by decoupling spatial streams. Non-uniform constellation selection not only increases the observable throughput to a small degree, but more importantly reduces the performance gap of low-complexity decoupled link adaptation algorithms.

6. REFERENCES


