OPTIMAL SPECTRUM SHARING AND POWER ALLOCATION FOR OFDM-BASED
TWO-WAY RELAYING

Min Dong and Shahram Shahbazpanahi

Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, Oshawa, ON, Canada
Email: {min.dong, shahram.shahbazpanahi}@uoit.ca

ABSTRACT

The problem of optimally allocating power for half-duplex two-way relaying in an OFDM system is considered. Assuming two-way relay is performed using analog network coding, and with total network power constraint, we obtain the optimal power allocation across subcarriers and among a relay and two communicating nodes to maximize the achievable sum rate in the network. We show that the resulting solution is a combination of two (usually opposite) power allocation strategies, i.e., water-filling across subcarriers and SNR-balancing between communicating end nodes. Further analysis also shows the optimal power allocation on the relay itself is a water-filling solution.

Index Terms— Two-way relaying, OFDM, Power allocation, Sum rate, Water-filling, SNR-balancing

1. INTRODUCTION

Applying relaying techniques in wireless networking can potentially improve the overall network performance, such as capacity and transmission range. In contrast to one-way relaying of data from a source to a destination, two-way relaying is a technique where relays establish communication links between two nodes for simultaneous information exchange. The main idea of two-way relaying is to let relay re-transmit a processed version of the signal it receives from both communicating nodes, and each node can recover the transmitted data from the other node after cancelling the self-interference generated by its own transmission. Since the process is similar to network coding, but is done at the symbol-level, it is also called two-way relay with analog network coding [1, 2]. Previously developed two-way relaying schemes ignore the channel frequency selectivity and therefore cannot combat inter-symbol-interference (ISI). Orthogonal Frequency Division Multiplexing (OFDM) is a proven transmission technology that mitigates the problems of ISI due to channel frequency selectivity. It divides the bandwidth into multiple subcarriers and simultaneously transmit data steams over subcarriers, and thus can utilize the time-frequency resource efficiently. OFDM-based two-way relaying combines advantages of both techniques to improve the network performance and efficiency. In such a system, how to appropriately allocate power among nodes and across subcarriers is essential to optimize the overall performance.

Two-way channels without relay were first studied by Shannon [3] and two-way relaying was later introduced in [4] from information theoretic point of view. With the advances on the theories of and techniques for wireless communications, two-way relaying has recently regained its attention. Achievable rate region under either optimal strategy or practical schemes with a single relay has been studied [5–7], as well as the performance over multiple-antenna systems [8–10]. Multiple relays using distributed beamforming is studied for achievable rate region [11], and for transmit power minimization or SNR-balancing [12]. In terms of power optimization, although the problem has been studied for one-way relay under various setup [13], most of existing results for two-way relaying do not consider power allocation optimization, and are focused on single-carrier systems. The optimal power assignment for multiple-carrier systems such as OFDM has not been studied prior to our work.

In this work, we consider the problem of optimally allocating power for half-duplex two-way relaying in an OFDM system. With total power constraint in the network, we obtain the optimal power allocation across subcarriers and among relay and communicating nodes to maximize the achievable sum rate of two communicating nodes. We show that the power assignment is a combination of two often opposite power allocation strategies, i.e., water-filling across subcarriers and SNR-balancing between communicating end nodes. Further analysis also shows that the power allocation across subcarriers at the relay itself is a water-filling solution. Moreover, we show that the required information to determine the power allocation is the effective channel seen on each subcarrier, which is available at the relay, and can be broadcasted to the two end nodes. Such feature is desirable in terms of required feedback or information exchange in the network for optimal power allocation.

2. SYSTEM MODEL

We consider a two-way relay network consisting of two communicating end nodes (node 1 and 2) and one relay node (Fig.1). All nodes use OFDM for data transmission to eliminate ISI caused by the frequency selectivity of the channel. We assume that the same subcarrier will be used for the relay to receive and re-transmit the signal received from node 1 and 2; therefore for each node, its channels to and from the relay are reciprocal.

![Fig. 1. Two-way relay scenario in an OFDM system](image)

Let $P_{1i}$ and $P_{2i}$ be the transmit power allocated to the $i$th subcarrier at node 1 and node 2, respectively. The relay received signal over the $i$th subcarrier is given by

$$x_i = \sqrt{P_{1i}}f_{1i}s_{1i} + \sqrt{P_{2i}}f_{2i}s_{2i} + \nu_i$$  \hspace{1cm} (1)

where $\nu_i$ is the relay noise on the $i$th subcarrier, $s_{1i}$ and $s_{2i}$ are the information symbols transmitted over the $i$th subcarrier by node 1 and node 2, respectively.
1 and 2, respectively, and $f_{1i}$ and $f_{2i}$ are the channel coefficients corresponding to the ith subcarrier from the relay to node 1 and 2, respectively.

Each relay then multiplies the signal it receives on the ith subcarrier by a weight coefficient denoted as $w_i$. The signals received at node 1 and 2 are respectively given by [12]

$$\begin{align}
\hat{y}_{1i} &= f_{1i}w_i(\sqrt{P_{T1}}f_{1i} + \sqrt{P_{T2}}f_{2i} + n_{1i}) + \nu_{1i}, \\
\hat{y}_{2i} &= f_{2i}w_i(\sqrt{P_{T1}}f_{1i} + \sqrt{P_{T2}}f_{2i} + n_{2i}) + \nu_{2i}
\end{align}$$

where $n_{1i}$ and $n_{2i}$ are the noise of the ith subcarrier at node 1 and 2, respectively. After self-interference cancellation, the residual signals are given by

$$\begin{align}
\tilde{y}_{1i} &= \hat{y}_{1i} - \sqrt{P_{T1}}w_i f_{1i}^2 = \hat{y}_{1i} - w_i f_{1i} + n_{1i}, \\
\tilde{y}_{2i} &= \hat{y}_{2i} - \sqrt{P_{T2}}w_i f_{2i}^2 = \hat{y}_{2i} - w_i f_{2i} + n_{2i},
\end{align}$$

which will be processed at node 1 and 2, respectively, to extract $s_{2i}$ and $s_{1i}$. The SNR on the ith subcarrier at node 1 and 2 is given by

$$\begin{align}
\text{SNR}_{1i} &= \frac{P_{2i}|w_i|^2|h_{1i}|^2}{\sigma^2(1 + |w_i|^2|f_{1i}|^2)}, \\
\text{SNR}_{2i} &= \frac{P_{1i}|w_i|^2|h_{2i}|^2}{\sigma^2(1 + |w_i|^2|f_{2i}|^2)}
\end{align}$$

where $h_{i} \triangleq f_{i1}f_{2i}$. Let $P_T = [P_{T1}, P_{T2}, \ldots, P_{TN_c}]^T$ be a vector of the total power allocated on each subcarrier for the relay and node 1 and 2, where

$$P_{Ti} \triangleq P_{Ti1} + P_{Ti2} + P_{Ti3},$$

are, respectively, the total transmit power and the relay transmit power on the ith subcarrier.

The overall total transmit power in the network is then given by

$$P_T = \sum_{i=1}^{N_c} P_{i1} + \sum_{i=1}^{N_c} P_{i2} + \sum_{i=1}^{N_c} P_{i3}.$$  

Assuming we have a total power budget in the network, our goal is to find optimal power allocation to maximize the achievable sum-rate of the two communicating end nodes over all subcarriers, as detailed in the next section.

### 3. SUM-RATE MAXIMIZATION

Using the following definitions, $w = [w_1, w_2, \ldots, w_{N_c}]^T$, $P_1 = [P_{11}, P_{12}, \ldots, P_{1N_c}]^T$, and $P_2 = [P_{21}, P_{22}, \ldots, P_{2N_c}]^T$, the optimization problem is then given by

$$\begin{align}
\max_{w, P_1, P_2} & \quad \frac{1}{2} \sum_{i=1}^{N_c} \log(1 + \text{SNR}_{1i}) + \frac{1}{2} \sum_{i=1}^{N_c} \log(1 + \text{SNR}_{2i}) \\
\text{subject to} & \quad P_T \leq P_T^{\text{max}}, \quad P_1, P_2 \succeq 0
\end{align}$$

where $P_T^{\text{max}}$ is the total power budget, and the scaling factor 1/2 reflects the half-duplex operation. Note that the power allocation involves both optimal distribution across subcarriers and among relay and end nodes. Next, we show that the optimization problem in (7) can be split into two maximization subproblems.

For any given total power allocation vector $P_T$, let

$$g(P_T) = \max_{w, P_1, P_2} \frac{\frac{1}{2} \sum_{i=1}^{N_c} \log(1 + \text{SNR}_{1i}) + \log(1 + \text{SNR}_{2i})}{2}$$

subject to $P_1 + P_2 + P_T = P_T$, $P_1, P_2 \succeq 0$.

Then, the optimization problem in (7) is equivalent to the following maximization problem:

$$\begin{align}
\max_{P_T} & \quad g(P_T) \\
\text{subject to} & \quad P_T \leq P_T^{\text{max}}, \quad P_1, P_2 \succeq 0
\end{align}$$

We now solve the two maximization subproblems in (8) and (9).

3.1. Optimal Power Splitting among Nodes

To solve (8), we see that the problem can be further separated into per subcarrier optimization problems, i.e., for the ith subcarrier, we solve the following maximization

$$\begin{align}
\max_{w_i, P_{Ti1}, P_{Ti2}} & \quad \frac{1}{2} \log(1 + \text{SNR}_{1i}) + \frac{1}{2} \log(1 + \text{SNR}_{2i}) \\
\text{subject to} & \quad P_{Ti1} + P_{Ti2} + P_{Ti3} = P_{Ti}
\end{align}$$

The above problem is the same as joint power optimization of two-way relay in single carrier system. It has been shown in [14] that the solution to this problem leads to an SNR-balancing solution, where the SNR1i and SNR2i are equal. If we define $\alpha_i \overset{\triangleq}{=} |w_i|^2$, then the optimal values for $w_i$, $P_{Ti1}$, and $P_{Ti2}$ are obtained as

$$\begin{align}
\alpha^*_i(P_{Ti}) &= \arg \max_{w_i} \frac{(P_{Ti} - \sigma^2 \alpha_i)|h_{1i}|^2}{\sigma^2(1 + \alpha_i|f_{1i}|^2)} \\
&= \frac{P_{Ti}}{2\sigma^2} \sqrt{1 + \frac{P_{Ti}}{\sigma^2} + \frac{P_{Ti}}{|f_{1i}|^2}}
\end{align}$$

and the maximum SNR achieved under the SNR-balancing approach at each end node is given by

$$\begin{align}
\alpha^{\text{SNR}}_i(P_{Ti}) \overset{\triangleq}{=} \alpha^*_i(P_{Ti}), \\
\text{SNR}^{\text{SNR}}_i(P_{Ti}) &= \frac{0.5P_{Ti}^2 |f_{1i}|^2 |f_{2i}|^2}{\sigma^2(\sqrt{\sigma^2 + P_{Ti}|f_{1i}|^2} + \sqrt{\sigma^2 + P_{Ti}|f_{2i}|^2})^2}
\end{align}$$

3.2. Optimal Power Distribution across Subcarriers

With the optimal power splitting among nodes for any given power assignment $P_T$, we now can find the power distribution across subcarriers that maximize the sum rate

$$g(P_T^{\text{opt}}) \overset{\triangleq}{=} \max_{P_T} \sum_{i=1}^{N_c} \log(1 + \text{SNR}^{\text{SNR}}_i(P_{Ti}))$$

subject to $P_T \leq P_T^{\text{max}}$, $P_T \succeq 0$, where $P_T^{\text{opt}}$ is the resulting optimal power distribution across subcarriers. Note that the SNR in (14) can be rewritten as

$$\text{SNR}^{\text{opt}}_i(P_{Ti}) = \frac{P_{Ti}}{2\sigma^2} \left( \frac{|f_{1i}|^2 |f_{2i}|^2}{\sqrt{\frac{P_{Ti}^2}{\sigma^2} + |f_{1i}|^2} + \sqrt{\frac{P_{Ti}^2}{\sigma^2} + |f_{2i}|^2}} \right)^2.$$
It follows from (16) that the objective function in (15) is not concave with respect to $P_T$. Therefore, the optimization problem is not convex, and the solution may not be easy to find. We therefore resort to finding upper and lower bounds for the objective function. An upper bound for (16) is given by

$$\text{SNR}_i(P_{Ti})_{ub} = \frac{P_{Ti}}{2\sigma^2} \left( \frac{|f_{i1}| + |f_{i2}|}{|f_{i1}|} \right)^2. \quad (17)$$

In this case, $\text{SNR}_i(P_{Ti})_{ub}$ is linear with respect to $P_{Ti}$. It is clear that the optimal power allocation to maximize the upper bound of the sum rate using $\text{SNR}_i(P_{Ti})_{ub}$ is the classical water-filling solution $P^*_T$ for power distribution across subcarriers, where

$$P^*_T = \left( \frac{1}{\lambda} - \sigma^2 \left( \frac{|f_{i1}| + |f_{i2}|}{|f_{i1}|} \right)^2 \right)^+.$$

(18)

where $\lambda$ is such that $\sum_i P^*_T = P_T$.

Note that the power allocation given in (18) is the traditional water filling solution given the effective channel gain being the harmonic mean of the channel gains on the two relay paths (scaled by 0.5) over each subcarrier: $f_i^{\text{eff}} = \frac{|f_{i1}| |f_{i2}|}{|f_{i1}| + |f_{i2}|}$.

Since (18) is the optimal solution that maximizes the upper bound of the sum rate, it is a suboptimal solution for the original problem (15), and therefore provides a lower bound on the sum rate,

$$\sum_{i=1}^{N_c} \log(1 + \text{SNR}_i(P_{Ti}^*)) \leq g(P^{\text{ub}}_{P_T}) \leq \sum_{i=1}^{N_c} \log(1 + \text{SNR}_i(P_{Ti})_{ub}) \quad (19)$$

Both bounds become tighter as $P_T/\sigma^2 \gg 1$, and the solution in (18) approaches to the optimal solution at the high per-subcarrier transmit SNR regime. We will show in the next simulation section that half of the total power should be allocated to the relay. Furthermore, from (18) it is clear that

$$P_{Ti}^* = \left( \frac{1}{2\lambda} - \sigma^2 \left( \frac{|f_{i1}| + |f_{i2}|}{|f_{i1}|} \right)^2 \right)^+ \quad (21)$$

where $\lambda$ is such that $\sum_i P_{Ti}^* = 0.5P_T$. In other words, the power distribution over the subcarriers of the relay node itself is again a water-filling solution. Note that the appealing aspect of this result is that, unlike traditional water-filling which requires channel side information (CSI) at the transmitter side, here as the relay acts as a receiver for each side of the transmission and therefore CSI’s are readily available, the water-filling at relay can be easily obtained.

For the power splitting between two end nodes for the rest $0.5P_T$, we can see from (11)-(13) that at high received SNR, i.e., $P_{Ti}|f_{i1}|^2/\sigma^2 \gg 1$ and $P_{Ti}|f_{i2}|^2/\sigma^2 \gg 1$ (this typically happens when $P_T$ is sufficiently high, and/or the link condition between relay and end nodes is good),

$$\alpha_{i}^* \approx \frac{1}{|f_{i1}| |f_{i2}|} \quad (22)$$

$$P^*_i(P_{Ti})_{ub} \approx \frac{|f_{i1}|}{|f_{i1}| + |f_{i2}|} \Rightarrow P^*_i(P_{Ti}) \approx \frac{|f_{i1}|}{|f_{i1}| + |f_{i2}|} \cdot 0.5P_T \quad (23)$$

Note that the power allocation at both end nodes can be determined once they obtain the effective channel $f_i^{\text{eff}}$. In other words, with $f_i^{\text{eff}}$ available,

- Node 1 and 2 compute $P_{Ti}$ required on each subcarrier $i$ based on (18);
- Determine the fraction of $P_{Ti}$ required as in (23): $f_i^{\text{eff}}/|f_{i1}|$ (or $f_i^{\text{eff}}/|f_{i2}|$), where $f_{i1}$ (or $f_{i2}$) is assumed known at node 1 (or node 2) as the receiver-side CSI due to channel reciprocity.

Therefore, to determine the power allocation among end nodes and relay, all we need is to let the relay broadcast $f_i^{\text{eff}}$ to both node 1 and 2. This is a desirable feature for potentially simple and reduced feedback/overhead design on information exchange.

4. SIMULATION RESULTS

We consider an OFDM system with total number of subcarriers $N_c = 128$. We adopt the standard ITU channel model PedA to model the links between the relay and two end nodes [15]. The average channel gain is assumed to be 1, and therefore $E(|f_{i1}|^2) = 1$ and $E(|f_{i2}|^2) = 1$. Fig.2 plots the amplitude of a realization of channel frequency response for $[f_{i1}] \triangleq [|f_{i1}|, \ldots, |f_{i1,N_c}|]^T$ and $[f_{i2}] \triangleq [|f_{i2}|, \ldots, |f_{i2,N_c}|]^T$, respectively, as well as the effective channel $[f_i^{\text{eff}}] \triangleq [|f_i^{\text{eff}}|, \ldots, |f_{i,N_c}^{\text{eff}}|]^T$. In the following evaluation and comparison, we normalize the sum rate and allocated power by $N_c$ so that the quantities are not a function of $N_c$ chosen. Specifically, we define average sum rate per subcarrier $R_{sc}$ as the total sum rate divided by $N_c$ and average power per subcarrier $\bar{P}_{sc}$ as $\frac{P_T}{N_c}$.

Fig.3(a) shows both the upper bound and lower bound on the average sum rate per subcarrier $R_{sc}$ at various $P_T$, as well as the differences between the optimal channel gains for the various subcarriers $\lambda_{i}$.

As seen, the bounds are tight with typically less than 0.2 bits of gap, and the different becomes negligible at high transmit SNR $P_T^*$. Also plotted in Fig.3(a) is the result when equal power allocation is used, i.e., node 1 and 2 and the relay split the total power equally and spread the power across subcarriers equally. We see that optimal power allocation provides significant sum rate improvement.

At high SNR, we see about 1 bit gap for the average sum rate per subcarrier.

In Fig.4, we plot the fraction of power $P_{Ti}$ used by the relay and node 1 and 2 for each subcarrier, as well as the power distribution across subcarriers $\{P_{Ti}\}$. As verified in the plots, the relay uses 0.5 of the total allocated power on all the subcarriers. The power assigned for subcarrier 1 and 2 depends on the channel realization, and to SNR-balancing, they depend on $f_{i1}$ and $f_{i2}$, respectively.

5. CONCLUSION

In this paper, we consider the two-way relay communication in an OFDM system. A single relay node is used to perform analog network coding for two-way relaying. Given total network power constraint, the optimal power allocation maximizing the achievable sum rate of the two end nodes turns out to be a clear two-step power
allocation strategy, i.e., water-filling across subcarriers and SNR-balancing among communicating end nodes. As it also turns out, the power allocation on the relay itself is a water-filling solution. We also show that the required information to determine the power allocation on each node in the network is the harmonic mean of the relay path gain on each subcarrier, which is available at the relay, and can be broadcasted to the two end nodes. This imposes light information exchange in the network for optimal power allocation.

**Fig. 2.** An example of OFDM channel $f_1$ and $f_2$. $N_c = 128$.

**Fig. 3.** Average sum rate per subcarrier vs. $\frac{P_{\text{avg}}}{P_{\text{max sumRate}}}$ for the given OFDM channel response example.

### 6. REFERENCES


