CLOSED-FORM BLIND CHANNEL ESTIMATION IN ORTHOGONALLY CODED MIMO-OFDM SYSTEMS

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ABSTRACT

Two closed-form blind channel estimators for orthogonally coded multiple-input multiple-output orthogonal frequency-division multiplexing (MIMO-OFDM) systems are proposed. The key idea of our approaches is to estimate the channel parameters in the time domain instead of doing this in the frequency domain. Our first approach is based on the maximum likelihood (ML) technique with the relaxed finite alphabet constraint, while the second approach uses the generalized Capon estimator. The proposed techniques amount to solving eigenvector problems and, therefore, their complexity is much lower than that of the approach of [7]. They can be used with any numbers of transmit and receive antennas, and exploit coherent processing across the subcarriers to estimate the channel in the time domain. Also, the proposed techniques are applicable to more than one data block, and their associated complexity is nearly independent of the number of subcarriers and the data block size.

Index Terms— Blind channel estimation, MIMO-OFDM systems, maximum likelihood, Capon estimator

1. INTRODUCTION

Space-time coded MIMO systems can be used in conjunction with the OFDM scheme to combine the advantages of multi-antenna and multi-carrier transmissions [1]. However, the performance of MIMO-OFDM systems critically depends on the quality of the channel state information (CSI) available at the receiver. Although training-based approaches are commonly used, a promising recent trend is to estimate the channel using spectrally efficient blind techniques [2]-[7]. Most of the blind estimation methods can only deal with flat fading channels; see, for example, [2]-[5]. Hence, their implementation in MIMO-OFDM systems is subcarrier-wise. This does not enable coherent processing across the subcarriers and may lead to prohibitively high computational costs when the number of subcarriers is large.

Recently, several promising approaches to blind symbol detection and/or channel estimation in orthogonally coded MIMO-OFDM systems with frequency selective channels have been developed [6], [7]. However, the applicability of the approach of [6] may suffer from a high computational complexity when the number of subcarriers and/or the data block size are large. Moreover, the latter approach is limited by the particular case of BPSK or QPSK symbol constellations, and it can only be applied to a single data block. The approach of [7] is based on convex optimization and amounts to solving a semidefinite programming (SDP) problem. This may be a challenging task for on-line applications as the computational cost of the existing SDP solvers is rather high. Moreover, even in the case of constant modulus symbols, the approach of [7] requires channel norm estimation that has to be repeated for each subcarrier independently. This may cause a substantial performance degradation at low signal-to-noise ratios (SNRs).

In this paper, we propose two closed-form approaches to the problem of blind channel estimation in orthogonally coded MIMO-OFDM systems. Our first approach is based on the ML technique with the relaxed finite alphabet constraint, while the second approach uses the generalized Capon estimator. The proposed techniques amount to solving eigenvector problems and, therefore, their complexity is much lower than that of the approach of [7]. They can be used with any numbers of transmit and receive antennas, and exploit coherent processing across the subcarriers to estimate the channel in the time domain. Also, the proposed techniques are applicable to more than one data block, and their associated complexity is nearly independent of the number of subcarriers and the data block size.

2. BACKGROUND

The input-output time-domain relationship for a point-to-point MIMO system with $N$ transmit and $M$ receive antennas and frequency-selective finite impulse response (FIR) multipath channel with $L + 1$ efficient taps can be written as [8]

$$Z(n) = \sum_{l=0}^{L} Y(n-l) G_l + E(n)$$  (1)

where $Z(n)$ is the $T \times M$ matrix of the received data whose $(p,m)$th entry $[Z(n)]_{p,m}$ is the sample received during the $p$th burst via the $m$th receive antenna at the $n$th time interval, $T$ is the number of bursts, $Y$ is the $T \times N$ matrix of the transmitted data, $G_l$ is the $l$th $N \times M$ complex channel matrix that corresponds to the $l$th tap, and $E$ is the $T \times M$ noise matrix. Assume that the noise is spatially and temporally white with variance of $\sigma^2$ per complex dimension and that the channel length is known at the receiver. After the serial-to-parallel conversion, at the transmitter side we have $K$ parallel data streams of length $N_0$ where $K$ is the number of complex information symbols prior to encoding and $N_0$ is the number of orthogonal subcarriers. These symbol streams are then space-frequency encoded using the same orthogonal code by mapping them onto a sequence of $T \times N$ matrices $\{X(i)\}$ where $i$ is the subcarrier index [7]. Due to the inverse fast Fourier transform (IFFT) at the transmitter and fast Fourier transform (FFT) at the receiver, the frequency-selective fading channel is converted to $N_0$ parallel flat fading channels [8]. Then, (1) turns to the following frequency-domain input-output relation

$$Y(i) = X(i) H_i + V(i).$$  (2)

where $Y(i)$, $X(i)$, $H_i$, and $V(i)$ are the frequency-domain counterparts of $Z(n)$, $Y(n)$, $G_l$ and $E(n)$, respectively [7]. For any
complex-valued matrix $B$, introduce the following operators [7]

$$
\begin{bmatrix}
\text{Re}(B)_{p,m} & -\text{Im}(B)_{p,m} \\
\text{Im}(B)_{p,m} & \text{Re}(B)_{p,m}
\end{bmatrix}
$$

where $\text{vec} \{\cdot\}$ is the vectorization operator that stacks all columns of a matrix on each other, $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ are the real and imaginary parts, and $(\cdot)^T$ is the transpose. Using (3), (4) and the so-called oversampled FFT matrix $F$ (built from the first $L+1$ columns of $N_0$-point normalized FFT matrix) with associated $i$th row $f_i$, we have the following relationships between the channel parameters in the frequency and time domains:

$$
\hat{h}_i = \sqrt{N_0} (I_1 \otimes I_{MN}) g'_i \triangleq \mathcal{F}_i g'_i,
$$

$$
\hat{h}' = \sqrt{N_0} (F \otimes I_{MN}) g'_i \triangleq \mathcal{F} g'_i
$$

Also, defining the covariance matrices $R_i \triangleq E \{y_i y_i^T\}$ and $\hat{R} \triangleq E \{y' y'^T\}$ and using the fact that the symbol streams and noise are mutually uncorrelated at each subcarrier, we have

$$
\begin{align*}
R_i &= A(h_i) A_{s_i}^T (h_i) + (\sigma^2/2) I_{2MT} \\
\hat{R} &= \hat{A}(h'_i) A_{s_i}^T (h'_i) + (\sigma^2/2) I_{2MTN_0}
\end{align*}
$$

where $A_{s_i} \triangleq E \{s_i s_i^T\}$ and $A_{s'_i} \triangleq E \{s'_i s'^T\}$.

In practice, $\hat{R}_i$ and $\hat{R}$ are estimated as

$$
\begin{align*}
\hat{R}_i &= \frac{1}{J} \sum_{j=1}^{J} y_i[j] y_i[j]^T, \\
\hat{R} &= \frac{1}{J} \sum_{j=1}^{J} y'[j] y'[j]^T
\end{align*}
$$

3. BLIND CHANNEL ESTIMATION

3.1. The Approach Based on Relaxed ML

Assuming that the MIMO channel remains invariant during J OSTBC-OFDM data blocks and that the noise vectors are i.i.d. zero-mean white Gaussian, the joint ML estimator of the channel frequency response vector (5) and symbol vectors $s_i[p]$ ($p = 1, \ldots, J$) at the $i$th subcarrier can be expressed as

$$
\{\hat{h}_{i,ML}, \hat{s}_{i,ML}\} = \arg \min_{\hat{s}_i \in \Omega, \hat{h}_i} \sum_{j=1}^{J} \|y_i[j] - A(h_i) s_i[p]\|^2
$$

where $\Omega \triangleq \{s_i[1], s_i[2], \ldots, s_i[J]\}$ stacks all the available information symbol vectors corresponding to the $i$th subcarrier and $\Omega$ is the set of all possible values of $\Omega$. It is very difficult to solve (15) as its computational complexity grows exponentially in $J$. To reduce the complexity, let us relax the finite alphabet constraint $\Omega_i \in \Omega$ and assume $\Omega_i \in \mathbb{R}^{2K \times J}$. As $A(h_i)$ has full column rank [2], each individual term in the summation is then minimized with

$$
\hat{s}_i[p] = \left( A^T (\hat{h}_i) A(\hat{h}_i) \right)^{-1} A^T (\hat{h}_i) y_i[p].
$$

Inserting (16) into (15), the relaxed ML (RML) estimator of $h_i$ can be written as

$$
\hat{h}_{i,RML} = \arg \max_{\hat{h}_i} \text{tr} \left( A(\hat{h}_i) A^T (\hat{h}_i) A(\hat{h}_i) \right)^{-1} A^T (\hat{h}_i) \hat{R}_i
$$

where $\text{tr}(\cdot)$ stands for the matrix trace. Using the orthogonality property (10), we obtain from (17) that

$$
\hat{h}_{i,RML} = \arg \max_{\hat{h}_i} \text{tr} \left( A^T (\hat{h}_i) \hat{R} A(\hat{h}_i) \right) / \|\hat{h}_i\|^2.
$$

Note that the relaxation of the finite alphabet constraint in (15) leads to the norm ambiguity in (18). To prevent this, we impose a norm constraint on the optimization variable in (18) to obtain

$$
\hat{h}_{i,RML} = \arg \max_{\|\hat{h}_i\| = 1} \text{tr} \left( A^T (\hat{h}_i) \hat{R} A(\hat{h}_i) \right).
$$

Combining (19) for all subcarriers and using (9), we obtain the following estimate of $h'$:

$$
\hat{h}' = \arg \max_{\hat{h}} \sum_{i=0}^{N_0-1} \hat{h}^T_i \Phi^T (I_{2K} \otimes \hat{R}_i) \Phi_{\hat{h}_i}
$$
where the norm constraints $\|\mathbf{h}_i\| = \|h_i\|$, $i = 0, \ldots, N_0 - 1$ should be taken into account and $\mathbf{h}_0 \triangleq [\mathbf{h}_0^T \ldots \mathbf{h}_{N_0-1}^T]^T$ is the $2MN_0 \times 1$ vector of frequency-domain optimization variables.

It is now convenient to reformulate this problem in the time domain, where the number of variables is independent of the number of subcarriers and, hence, remains small even for large $N_0$. Using (5), we rewrite (20) in time domain as

$$
\mathbf{g}' = \arg \max_\mathbf{g} \sum_{i=0}^{N_0-1} \mathbf{g}^T \mathbf{F}_i^T \Phi^T (\mathbf{I}_{2K} \otimes \mathbf{R}_i) \Phi \mathbf{F}_i \mathbf{g}
$$

(21)

where $\mathbf{h}_i = \mathbf{F}_i \mathbf{g}$, $\mathbf{g} \triangleq [\mathbf{g}_0^T \ldots \mathbf{g}_{N_0-1}^T]^T$, and the norm constraints take the form $\|\mathbf{F}_i \mathbf{g}\| = \|h_i\|$, $i = 0, \ldots, N_0 - 1$.

The problem formulation in (21) benefits from coherent processing among all subcarriers. Using (5), (6) and (11), the estimator (21) can be approximated as

$$
\mathbf{g}' = \arg \max_{\mathbf{g}} \mathbf{g}^T \mathbf{F}^T \mathbf{P}^T (\mathbf{I}_{2K N_0} \otimes \mathbf{R}) \mathbf{P} \mathbf{F} \mathbf{g}
$$

(22)

where, instead of the original norm constraints, we use their relaxed version $\|\mathbf{g}'\| = \|\mathbf{g}\|$. The approximation (relaxation) used here is that $N_0$ individual norm constraints are replaced by one aggregate constraint. The solution to (22) belongs to the subspace spanned by principal eigenvector (or eigenvectors, in the case of eigenvalue multiplicity) of the matrix $\mathbf{F}^T \mathbf{P}^T (\mathbf{I}_{2K N_0} \otimes \mathbf{R}) \mathbf{P} \mathbf{F}$.

### 3.2. The Approach Based on Capon Estimator

The idea of this method is to use the Capon receiver, that is, to solve the following optimization problem:

$$
\min_{\mathbf{w}_{k,i}} \mathbf{w}_{k,i}^T \mathbf{R} \mathbf{w}_{k,i} \quad \text{subject to} \quad \mathbf{w}_{k,i}^T \mathbf{a}_k(\mathbf{h}_i) = 1.
$$

(23)

The solution to (23) yields a filter that passes the symbols corresponding to the $k$th column $\mathbf{a}_k(\mathbf{h}_i)$ of the matrix $\mathbf{A}(\mathbf{h}_i)$ while suppressing all the symbols corresponding to the other columns of this matrix and noise components.

The solution to (23) is given by

$$
\mathbf{w}_{k,i}(\mathbf{h}_i) = \frac{1}{\mathbf{a}_k^T(\mathbf{h}_i) \mathbf{R}_i^{-1} \mathbf{a}_k(\mathbf{h}_i)} \mathbf{R}_i^{-1} \mathbf{a}_k(\mathbf{h}_i).
$$

(24)

According to (24), a separate weight vector has to be obtained for each entry of $\mathbf{s}_k$. For any $\mathbf{h}_i$ and any $k$th entry of $\mathbf{s}_k$, let us define the Capon spectrum as

$$
P_{k,i}(\mathbf{h}_i) = \mathbf{w}_{k,i}^T(\mathbf{h}_i) \mathbf{R} \mathbf{w}_{k,i}(\mathbf{h}_i) = \frac{1}{\mathbf{a}_k^T(\mathbf{h}_i) \mathbf{R}_i^{-1} \mathbf{a}_k(\mathbf{h}_i)}.
$$

(25)

This spectrum characterizes the output power of the $k$th Capon receiver used for the $i$th subcarrier. Clearly, if we do not bound the norms of channel vectors, then $P_{k,i}(\mathbf{h}_i) \rightarrow 0$ when $\|\mathbf{h}_i\| \rightarrow \infty$. However, assuming values of $\mathbf{h}_i$ satisfying norms constraints $\|\mathbf{h}_i\| = \|\mathbf{h}_i\|$, the Capon spectrum is expected to have its maximum at $\mathbf{h}_i = \mathbf{h}_i$.

Although any of the Capon spectra $P_{k,i}(\mathbf{h}_i)$, $k = 1, \ldots, 2K$ can be used to estimate the channel frequency response vector at the $i$th subcarrier, let us improve this estimate by combining them as [9]:

$$
Q_i(\mathbf{h}_i) = \sum_{k=1}^{2K} \frac{1}{P_{k,i}(\mathbf{h}_i)} = \sum_{k=1}^{2K} \mathbf{a}_k^T(\mathbf{h}_i) \mathbf{R}_i^{-1} \mathbf{a}_k(\mathbf{h}_i)
$$

$$
= \mathbf{h}_i^T (\sum_{k=1}^{2K} \Phi_k^T \mathbf{R}_i^{-1} \Phi_k) \mathbf{h}_i
$$

(26)

where the last two equalities in (26) are obtained from (8) and (9). Using (5), we can rewrite (26) in time domain as

$$
Q_i(\mathbf{g}) = \mathbf{g}^T \mathbf{F}^T \mathbf{P}^T (\mathbf{I}_{2K} \otimes \mathbf{R}_i^{-1}) \mathbf{P} \mathbf{F} \mathbf{g}.
$$

(27)

Let us combine the spectra (27) for all the subcarriers as

$$
Q(\mathbf{g}) = \sum_{i=0}^{N_0-1} Q_i(\mathbf{g}) = \sum_{i=0}^{N_0-1} \mathbf{g}^T \mathbf{F}^T \mathbf{P}^T (\mathbf{I}_{2K} \otimes \mathbf{R}_i^{-1}) \mathbf{P} \mathbf{F} \mathbf{g}.
$$

(28)

Relaxing the individual norm constraints by replacing them, as before, by the aggregate constraint, and using the sample covariance matrix $\mathbf{R}$ instead of the true covariance matrix $\mathbf{R}$, the proposed Capon-based estimator can be written as

$$
\mathbf{g}_{\text{Capon}}^{\prime} = \arg \min_{\|\mathbf{g}\| = \|\mathbf{g}'\|} \mathbf{g}^T \mathbf{F}^T \mathbf{P}^T (\mathbf{I}_{2K N_0} \otimes \mathbf{R}^{-1}) \mathbf{P} \mathbf{F} \mathbf{g}.
$$

(29)

### 3.3. Approximation of the Norm Constraint

Clearly, the norm constraint used in the proposed estimators (22) and (29) can not be satisfied in its current form, because the norm of the true channel vector $\mathbf{g}'$ is unknown. However, we can estimate the norm of $\mathbf{g}'$ as follows. First of all, we obtain the following estimates of $\|\mathbf{h}_i\|$ for all $i = 0, \ldots, N_0 - 1$ [7]

$$
\|\mathbf{h}_i\| = \sqrt{\text{tr}(\mathbf{R}_i) - MT \hat{\sigma}^2} / \text{tr}(\mathbf{A}_i).
$$

(30)

Note that $\mathbf{A}_i$ in (30) is assumed to be known exactly as the symbol constellations are assumed to be known at the receiver.

Based on the estimates (30) and using (6), we can directly compute the estimate of $\mathbf{g}'$ as

$$
\|\mathbf{g}'\| = \sqrt{\frac{1}{N_0} \sum_{i=0}^{N_0-1} \|\mathbf{h}_i\|^2}.
$$

(31)

The estimate (31) can be used along with (22) and (29) to find the corresponding estimates of $\mathbf{g}'$. These estimates amount to solving principal eigenvector problem in (22) or minor eigenvector problem in (29) and subsequent rescaling the resulting eigenvectors to make sure that their norm is equal to $\|\mathbf{g}'\|$ of (31).

It is worth noting that in the case of constant modulus constellations, the channel norm is immaterial for signal detection. Therefore, in this case the norm constraints can be dropped in (22) and (29).

### 4. SIMULATIONS

In each simulation run, the entries of $\mathbf{G}_t$ are independently drawn from a Gaussian distribution with zero mean and variance $\sigma_0^2$, and are then kept fixed for this run. It is assumed that $L + 1 = 4$,
$N_0 = 64$, $N = M = T = 4$, $K = 3$, and $J = 1$. The $3/4$-rate OSTBC of [8, Equation (7.4.10)] with QPSK symbols is used for space-frequency coding. Diagonal loading with the factor of $5\sigma^2$ has been used to improve the robustness of the Capon-based method against the finite sample effect. The proposed RML-based and Capon-based estimators are compared with the subcarrier-wise blind channel estimation approach of [2] and the more recent technique of [7] which uses coherent processing over the subcarriers and is based on a more computationally expensive SDP approach.

Fig. 1 displays the averaged over subcarriers normalized mean squared channel estimation errors

$$
NMSE = \frac{1}{N_0 N_{\text{runs}}} \sum_{n=1}^{N_{\text{runs}}} \sum_{i=0}^{N_0-1} \left\| \hat{h}_i^{(n)} \right\|^2 - \left\| {h}_i \right\|^2
$$

for all the methods tested versus SNR, where $\hat{h}_i^{(n)}$ is the estimate of $h_i$ in the $n$th simulation run, and $N_{\text{runs}}$ is the number of simulation runs. Fig. 2 shows the symbol error rates (SERs) versus the SNR. These SERs are obtained by the ML decoder that uses the channel estimates of the methods tested. Additionally, the SER of the informed ML decoder is shown in Fig. 2 as a benchmark. The latter decoder uses the exact channel knowledge.

It can be seen from Fig. 1 that the channel estimation performances of the proposed methods are substantially better than that of the method of [7] at low SNRs (up to 0 dB), whereas at higher values of SNR the performances of the Capon-based method and the technique of [7] are nearly identical. However, the approach of [7] outperforms the RML-based technique at high SNRs. The approach of [2] performs much worse than the other techniques tested. The latter fact is the result of coherent processing over all the subcarriers and the use of quite a parsimonious channel parameterization in the proposed methods and in the technique of [7].

From Fig. 2, it follows that the same relationship between the performances can be observed in terms of SERs of the ML decoder. It is worth noting that the proposed Capon-based technique has the best SER performance among the methods tested. Indeed, its performance is very close to that of the benchmark informed ML decoder for all values of SNR.

5. REFERENCES


