ML CO-CHANNEL INTERFERENCE ESTIMATION FROM SINR MEASUREMENTS FOR MULTICELL OFDM DOWNLINK: BOUNDS AND PERFORMANCE ANALYSIS

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ABSTRACT

We consider the downlink of a multicell OFDM system where the frequency reuse factor is 1. Base-stations cooperate and pre-code the symbols intended for the mobile stations (terminals) located in different cells and operating on the same frequency, in order to precompensate the co-channel interferences (CCIs). To design the precoders, the CCI coefficients have to be estimated. We assume that the mobile stations are able to estimate their SINR that they feedback to their base-station. By properly designing a perturbation signal, it is possible for the base-station to estimate the complex valued CCI coefficients from the SINRs fed back. We investigate the maximum likelihood (ML) estimation of co-channel coefficients from SINR measurements. We obtain the associated Cramér-Rao bound and the mean square error of a practical estimator. Finally a two-cells system is considered and we investigate the impact of such a co-channel estimation on the sum rate of the system when precoding is operated based on these estimates.

Index Terms— interference estimation, cooperation, precoding, bound

1. INTRODUCTION

We consider wireless cellular systems with multiple cells and a reuse factor of 1. We assume that a number of base-stations (BSs) cooperate and are connected via a backhaul connexion. Cooperation can take place at different levels. A first option is that the cooperating BSs coordinate the frequency and power allocation between cells in order to minimize the impact of the CCI. The optimization can be conducted to maximize various utility functions [1]. A second possible option is to jointly pre-code the signals to be sent by the BSs in order to precompensate the CCI [2]. It is this second option which is considered in the present paper. It is interesting to note the similarity with digital subscriber line (DSL) systems. For some system configurations, the crosstalk between lines becomes dominant with respect to the background noise. Coordination can take place at the central office. The performance of the global system can be improved by either spectrum coordination, named dynamic spectrum management (DSM) level 2, or by precoding which is known as DSM-level 3. Actually multicell cellular systems and DSL systems with crosstalk are both instances of the so-called interference channel. The major difference however is that there is no scheduling issue to be considered in the case of DSL systems.

To be able to compensate the CCI by means of linear precoding, the co-channel coefficients have to be available for the BSs. We assume that the MSs are equipped with an SINR estimation capability and that there is an SINR feedback channel. When transmission of signal 1 in cell 1 is interfered by transmission of a signal 2 in cooperating cell 2, a well-designed perturbation built from signal 2 and added onto signal 1 makes it possible to estimate the co-channel coefficients from BS 2 on MS 1. More precisely, by properly combining SINR measurements obtained at MS 1 over 3 phases with properly designed perturbations, it is possible to estimate the complex CCI coefficients from BS 2 to MS 1. Actually this method has been proposed to estimate crosstalk coefficients in DSL networks [3]. Compared to that, the goals of the current paper are as follows. The first goal is to derive the maximum likelihood estimator of co-channel coefficients from the noise-interference power measurements. The second goal is to compute the Cramér-Rao bound (CRB) for this estimator, and also the mean square error performance of this ML estimator. The third goal is to use the estimates of the co-channel coefficients to build a precoder which precompensates the CCI for the cooperating BS. The benefit of the precoding step will be discussed as a function of the number of estimation symbols used, and of the amplitude of the perturbation applied.

2. SYSTEM DESCRIPTION

2.1. Basic system

We consider the downlink of a multicell cellular wireless system. The modulation considered is OFDM which is also used as the multiple access technique in the downlink of each cell, which leads to orthogonal frequency division multiple access (OFDMA). We assume that the transmitters at the different BSs are perfectly frequency synchronous. Moreover we assume that the OFDM symbols in the different cells are time synchronous and the cyclic prefix length is properly chosen. This means that
when the DFT is applied by the MS receiver in one cell, this
DFT is interfered by only one OFDM(A) symbol per interfering
cell. Therefore, after DFT at the MS under consideration,
the problem translates into as many parallel channels as there
carriers or tones used by the OFDM(A) modulation. Assume
a simple setup with two cells. For the sake of simplicity and
easiness of explanations we assume that a single MS is active
in each cell and is using all the carriers of the OFDM symbols.
The formalism is however compatible with scenarios where
 carriers of a cell would be allocated to different MSs. The signal
$y_i(k)$ received at carrier $k$ of the MS active in cell $i$ ($i \in [1,2]$)
is given by

$$
\begin{bmatrix}
y_1(k) \\
y_2(k)
\end{bmatrix} =
\begin{bmatrix}
\Lambda_{11}(k) & \Lambda_{12}(k) \\
\Lambda_{21}(k) & \Lambda_{22}(k)
\end{bmatrix}
\begin{bmatrix}
I_1(k) \\
I_2(k)
\end{bmatrix} +
\begin{bmatrix}
n_1(k) \\
n_2(k)
\end{bmatrix}
$$

(1)

where $I_j(k)$ denotes the complex symbol sent by BS $j$ ($j \in [1,2]$) on carrier $k \in [1, N_c]$ where $N_c$ is the number of carriers.
$\Lambda_{ij}(k)$ denotes the channel gain between BS $j$ and MS $i$ on
frequency $k$. In particular $\Lambda_{12}(k)$ and $\Lambda_{21}(k)$ are the co-channel
coefficients to be estimated. The $n_i(k)$ are circular zero mean
complex random variables with variance $\sigma^2_{n_i}$, representing the
background noise and possibly all the interference coming from
non-cooperating BSs on carrier $k$.

2.2. System with perturbation

We assume that each MS is equipped with an SINR estimation
capability. Let us focus for the sake of clarity on MS 1. Pilots
are sent from BS 1 to MS 1. We also assume that the $\Lambda_{11}(k)$
are perfectly known to the MS which achieved estimation of
these coefficients in a previous phase. The value which is mea-
sured by MS 1 is actually given by $|\Lambda_{11}(k)I_1(k)|^2/|y_1(k) -
\Lambda_{11}(k)I_1(k)|^2$ where the $I_1(k)$ are perfectly known by MS 1
as they are pilots. Based on this SINR estimation capability,
the co-channel coefficient estimation requires 3 phases. During
these 3 phases, BS 1 sends pilots symbols while payload sym-
ols are sent by BS 2.

In phase 1, the signals sent and received correspond to model
(1). The value computed during that phase is denoted by $\widehat{\text{SINR}}_1$
and given by

$$
\widehat{\text{SINR}}_1 = \frac{|\Lambda_{11}(k)I_1(k)|^2}{|y_1(k) - \Lambda_{11}(k)I_1(k)|^2}
$$

(2)

whose structure is actually

$$
\frac{|\Lambda_{11}(k)I_1(k)|^2}{|\Lambda_{12}(k)I_2(k) + n_1(k)|^2}
$$

(3)

During phase 2, thanks to the backhaul, a perturbation built
from $I_2(k)$ is added onto $I_1(k)$ with a certain scaling factor $\epsilon$,
which leads to the following observation model (similar equa-
tions could be written for the other cell):

$$
y'_1(k) = \Lambda_{11}(k)[I_1(k) + \epsilon I_2(k)] + \Lambda_{12}(k)I_2(k) + n'_1(k)
$$

where $n'_1(k)$ has the same properties as $n_1(k)$. It is important
to note that $n'_1(k)$ and $n_1(k)$ are independent as phase 1 and
phase 2 correspond to different time instants, hence noise sam-
ples. The SINR estimate denoted by $\widehat{\text{SINR}}_2$ computed during this
phase is given by

$$
\widehat{\text{SINR}}_2 = \frac{|\Lambda_{11}(k)I_1(k)|^2}{|y'_1(k) - \Lambda_{11}(k)I_1(k)|^2}
$$

(4)

and its structure is actually

$$
\frac{|\Lambda_{11}(k)I_1(k)|^2}{|\Lambda_{11}(k)\epsilon + \Lambda_{12}(k)I_2(k) + n'_1(k)|^2}
$$

(5)

During phase 3, the process is similar to that of phase 2, but the
perturbation is designed by using a coefficient $j\epsilon$. The observation
model is given by

$$
y''_1(k) = \Lambda_{11}(k)[I_1(k) + j\epsilon I_2(k)] + \Lambda_{12}(k)I_2(k) + n''_1(k)
$$

where $n''_1(k)$ has the same properties as $n_1(k)$. The $n''_1(k)$
are independent from the $n'_1(k)$ and $n_1(k)$. The SINR estimate
denoted by $\widehat{\text{SINR}}_3$ computed during this phase is given by

$$
\widehat{\text{SINR}}_3 = \frac{|\Lambda_{11}(k)I_1(k)|^2}{|y''_1(k) - \Lambda_{11}(k)I_1(k)|^2}
$$

(6)

and its structure is actually

$$
\frac{|\Lambda_{11}(k)I_1(k)|^2}{|\Lambda_{11}(k)j\epsilon + \Lambda_{12}(k)I_2(k) + n''_1(k)|^2}
$$

(7)

3. ML ESTIMATION OF INTERFERENCE GAINS

We investigate here the structure of the ML estimator of the in-
terference coefficients $\Lambda_{12}(k)$ from the metrics computed
during the three phases described above. Therefore we need to
identify the probability density function (pdf) of the metrics at
disposal of the BS. From the metric measured during phase 1,
we can compute $1/\text{SINR}_1$ whose structure is actually

$$
\frac{1}{\text{SINR}_1} = \left| \frac{\Lambda_{12}(k)}{\Lambda_{11}(k)} I_2(k) + \frac{n_1(k)}{\Lambda_{11}(k)} \right|^2
$$

(8)

where we have assumed that the pilots sent by BS 1 are such that
$|I_1(k)|^2 = 1$. We assume that the symbols $I_2(k)$ are zero mean
gaussian distributed with variance $\sigma^2_{I_2}$. From these, it turns out
that $1/\text{SINR}_1$ is actually the sum of squares of gaussian. These
gaussians are zero mean and have variances $\sigma^2_{g,1}$ given by

$$
\sigma^2_{g,1} = \frac{1}{2} \left[ \left| \frac{\Lambda_{12}(k)}{\Lambda_{11}(k)} \right|^2 \sigma^2_{I_2} + \frac{\sigma^2}{|\Lambda_{11}(k)|^2} \right].
$$

(9)

Hence $1/\text{SINR}_1$ is $\chi^2$ distributed with 2 degrees of freedom.
This also applies to measures $1/\text{SINR}_2$ and $1/\text{SINR}_3$ with variances $\sigma^2_{g,2}$ and $\sigma^2_{g,3}$ given by

$$
\sigma^2_{g,2} = \frac{1}{2} \left[ \left| \frac{\Lambda_{12}(k) + \Lambda_{11}(k)\epsilon}{\Lambda_{11}(k)} \right|^2 \sigma^2_{I_2} + \sigma^2_{n,\Lambda}(k) \right]
$$

$$
\sigma^2_{g,3} = \frac{1}{2} \left[ \left| \frac{\Lambda_{12}(k) + \Lambda_{11}(k)j\epsilon}{\Lambda_{11}(k)} \right|^2 \sigma^2_{I_2} + \sigma^2_{n,\Lambda}(k) \right]
$$

(10)
where we have used the notation $\sigma^2_{\alpha \lambda}(k) = \sigma^2_{n,\lambda}/|\Lambda_{11}(k)|^2$ for the inverse of the signal to noise ratio of MS $1$ for carrier $k$. We assume that such metrics are collected over $N_s$ successive symbols in each phase and that the coefficient to be estimated does not change during the whole measuring phase. Let us define $\Lambda_{12}(k)/\Lambda_{11}(k) = \alpha_r(k) + j\alpha_i(k)$ where $\alpha_r(k)$ and $\alpha_i(k)$ are respectively the real part and the imaginary part of $\Lambda_{12}(k)/\Lambda_{11}(k)$. It is interesting to rewrite the $\sigma^2_{g,\lambda}$ values as follows:

$$\sigma^2_{g,2} = \frac{1}{2} \left[ (\alpha_r^2(k) + \alpha_i^2(k) + \epsilon^2 + 2\epsilon\alpha_r(k))\sigma^2_{s,\lambda} + \sigma^2_{n,\lambda}(k) \right]$$

$$\sigma^2_{g,3} = \frac{1}{2} \left[ (\alpha_r^2(k) + \alpha_i^2(k) + \epsilon^2 + 2\epsilon\alpha_i(k))\sigma^2_{s,\lambda} + \sigma^2_{n,\lambda}(k) \right]$$

$$\sigma^2_{g,4} = \frac{1}{2} \left[ (\alpha_r^2(k) + \alpha_i^2(k))\sigma^2_{s,\lambda} + \sigma^2_{n,\lambda}(k) \right]$$

(11)

and notice that the actual SINR for link $1$ corresponds to $\text{SINR}_1 = 1/2\sigma^2_{g,2}$. During phase $l (l \in [1,3]),$ we assume that the MS measures the inverse of the $\text{SINR}_1$, which is denoted by $x_{l,n}$ for OFDM symbol $n \in [1, N_s]$. Because measures for different $n$ indices are independent $\chi^2$ variables with 2 degrees of freedom, the pdf of the vector $x_l$ of measures $x_{l,n}$ for phase $l$ is given by

$$T_{x_l}(x_l) = \frac{1}{2^{N_s}(\sigma^2_{g,l})^N_s} \exp^{-\sum_{n} x_{l,n}/\sigma^2_{g,l}}.$$  

(12)

As the three phases occur at different time intervals, and because the noise samples are independent over these three intervals, the joint pdf of $[x_1^T, x_2^T, x_3^T]^T$ is just the product of the marginals. This joint pdf parameterized with $\Lambda_{12}(k)/\Lambda_{11}(k)$ is the likelihood function. In order to obtain the ML estimate, derivatives of the log-likelihood function are computed, with respect to the real part and the imaginary part of $\Lambda_{12}(k)/\Lambda_{11}(k)$. Forcing these derivatives to 0 leads to the following set of equations:

$$(\hat{\alpha}_r^2(k) + \hat{\alpha}_i^2(k))\sigma^2_{s,\lambda} + \sigma^2_{n,\lambda}(k) = \frac{\sum_{n} x_{1,n} N_s}{N_s}$$

$$(\hat{\alpha}_r^2(k) + \hat{\alpha}_i^2(k) + \epsilon^2 + 2\epsilon\alpha_r(k))\sigma^2_{s,\lambda} + \sigma^2_{n,\lambda}(k) = \frac{\sum_{n} x_{2,n} N_s}{N_s}$$

$$(\hat{\alpha}_r^2(k) + \hat{\alpha}_i^2(k) + \epsilon^2 + 2\epsilon\alpha_i(k))\sigma^2_{s,\lambda} + \sigma^2_{n,\lambda}(k) = \frac{\sum_{n} x_{3,n} N_s}{N_s}$$

where $\hat{\alpha}_r$ and $\hat{\alpha}_i$ denote the estimates of $\alpha_r$ and $\alpha_i$. From these equations, it is easily seen that the estimates are computed as follows:

$$\hat{\alpha}_r = \frac{\sum_{n} x_{1,n} N_s - \sum_{n} x_{2,n} N_s}{2\epsilon\sigma^2_{s,\lambda} N_s} - \frac{\epsilon}{2}$$

$$\hat{\alpha}_i = \frac{\sum_{n} x_{3,n} N_s - \sum_{n} x_{1,n} N_s}{2\epsilon\sigma^2_{s,\lambda} N_s} - \frac{\epsilon}{2}.$$  

(13)

(14)

This is the structure of the ML estimators for the real part and the imaginary part of the normalized co-channel gain coefficients. It is interesting to note that actually the individual $x_{l,n}$ need not be fed back, but rather the values of $\sum_{n} x_{l,n}$ which is of high practical interest. Interestingly this solution has been proposed as an ad-hoc solution in [3] without showing that it derives from an ML approach.

4. CRAMÉR RAO BOUNDS

For a vector parameter $\theta = [\theta_1 \theta_2 \ldots \theta_p]^T$, we have that [4]

$$\text{var}(\hat{\theta}_i) \geq \left[ \text{I}^{-1}(\theta) \right]_{ii}$$  

(15)

where $\text{var}(\hat{\theta}_i)$ denotes the variance of any unbiased estimator of $\theta_i$, $\text{I}(\theta)$ is the $p \times p$ Fisher information matrix defined by

$$[\text{I}(\theta)]_{ij} = -E \left[ \frac{\partial^2 \ln p(x; \theta)}{\partial \theta_i \partial \theta_j} \right]$$  

(16)

where $x$ is the observed vector of random variables from which the estimator is built, $p(x; \theta)$ is the pdf of $x$ and $\theta$ denotes the expectation. Here the vector of parameters is made of $\alpha_r$ and $\alpha_i$ (we omit $k$ for the sake of concision) and the pdf under consideration is that of vector $[x_1^T, x_2^T, x_3^T]^T$. The CRBs on $\alpha_r$ and $\alpha_i$ can be shown to be, respectively:

$$\frac{1}{N_s\sigma^2_{g,2}} \alpha_r^2 (\hat{\sigma}^2_{g,2} + \hat{\sigma}^2_{g,3} + \alpha_r^2\hat{\sigma}^2_{g,4}) + (\alpha_r + \epsilon)\sigma^4_{g,1}\hat{\sigma}^4_{g,2}$$

$$\frac{1}{N_s\sigma^2_{g,4}} \alpha_i^2 (\hat{\sigma}^2_{g,2} + \hat{\sigma}^2_{g,3} + \alpha_i^2\hat{\sigma}^2_{g,4}) + (\alpha_i + \epsilon)\sigma^4_{g,1}\hat{\sigma}^4_{g,2}.$$

5. ESTIMATOR PERFORMANCE

5.1. Theoretical results

Considering the estimators derived in (13) and (14), it is easy to show that both estimators are unbiased, namely $E[\hat{\alpha}_r] = \alpha_r$ and $E[\hat{\alpha}_i] = \alpha_i$. About the MSEs, it can be shown that the MSEs $\text{MSE}_r$ for $\hat{\alpha}_r$ and $\text{MSE}_i$ for $\hat{\alpha}_i$ are given by

$$\text{MSE}_r = \frac{1 + [1 + (\epsilon^2 + 2\epsilon\alpha_r(k))]\sigma^2_{s,\lambda}\text{SINR}_1}{4N_s\epsilon^2\sigma^2_{s,\lambda}\text{SINR}_1^2}$$

$$\text{MSE}_i = \frac{1 + [1 + (\epsilon^2 + 2\epsilon\alpha_i(k))]\sigma^2_{s,\lambda}\text{SINR}_1}{4N_s\epsilon^2\sigma^2_{s,\lambda}\text{SINR}_1^2}.$$  

(19)

(20)

5.2. Computational results

For the purpose of illustration, we have considered the case of a single carrier, and the values (we omit the carrier index) $\Lambda_{11} = 100$ and $\Lambda_{12} = 1 + j$. The SNR defined as $\text{SNR} = |\Lambda_{11}|^2/\sigma_n^2$ takes 4 values: 0, 10, 20, and 30 dB. The value of $\epsilon$ goes from 0 to 1. Figure 1 shows the CRB for $\alpha_r$ and MSE $\text{MSE}_r$. Both are reported in dBs ($10 \log_{10}(\cdot)$). The results show that at low SNRs, the performance tends to improve for increasing $\epsilon$ while at higher SNRs, there is an optimum value of $\epsilon$.

6. CCI PRECOMPENSATION

6.1. Precoder

Equation (1) shows the structure of the CCI. Once estimates of the coefficients $\Lambda_{ij}(k)$ are available at the BSs, thanks to the backhaul these BSs can jointly precode the data in order to
precompensation for this interference structure. From these estimates, and from the original data \([s_1(k) s_2(k)]^T\), the precoder computes precoded symbols \([s_1(k) s_2(k)]^T\) as follows:

\[
\begin{bmatrix}
  s_1(k) \\
  s_2(k)
\end{bmatrix} = \begin{bmatrix}
  \Lambda_{12}(k) & \Lambda_{11}(k) \\
  \Lambda_{21}(k) & 1
\end{bmatrix}^{-1} \begin{bmatrix}
  I_1(k) \\
  I_2(k)
\end{bmatrix}
\]

which means that only the off-diagonal terms need to be available for the precoding. The transmission corresponds to the following observation model:

\[
\begin{bmatrix}
  y_1(k) \\
  y_2(k)
\end{bmatrix} = \begin{bmatrix}
  \Lambda_{11}(k) & \Lambda_{12}(k) \\
  \Lambda_{21}(k) & \Lambda_{22}(k)
\end{bmatrix} \begin{bmatrix}
  s_1(k) \\
  s_2(k)
\end{bmatrix} + \begin{bmatrix}
  n_1(k) \\
  n_2(k)
\end{bmatrix}.
\]

6.2. Computational results and discussion

In order to check the validity of the estimation method and its impact on the performance with precoding, a setup with 2 cells has been considered. BS1, MS1, MS2, and BS2 are on the same line. Denoting by \(r\) the cell radius or half the distance between BS1 and BS2, MS1 (resp. MS2) is at a distance \(7r/8\) (resp. \(3r/2\)) from BS1 or \(9r/8\) (resp. \(r/2\)) from BS2. Hence MS1 is closer to the cell border and is more interfered. In the link budget, a propagation exponent of \(-3\) is assumed; hence all powers received are proportional to \(d^{-3}\), where \(d\) is the distance between the MS, and BSj. A number of tones \(N_s = 128\) is assumed. Between MS, and BSj, channels are generated having impulse responses of length 32. All taps are zero-mean complex Gaussian i.i.d. For each SNR value the figures reported are averaged over 50 channel realizations. One realization means the set of all impulse responses needed between all MS, and all BSj. The figure of merit considered here is the rate to each MS computed as \(\sum_{k=1}^{N_s} \log_2(1 + \text{SINR}(k))\), meaning summation over all carriers. We compare the rate obtained without compensation of CCI, with perfect compensation, and with the estimation-based compensation. During the estimation phases, all symbol constellations are QPSK normalized to be of unit variance. Figure 2 reports results for \(N_s = 3\) estimation symbols and two values of \(\epsilon = 0.1; 0.25\). Please note that the same \(\epsilon\) is used for all carriers and all SNRs. The results are in solid lines for MS 1 and in dotted lines for MS 2. The curves without marker show the rate with perfect cancellation of the CCI. The curves with \(\star\) show the rate with CCI. Clearly stream 1 (to MS 1) is more impacted and the rate penalty appears at a lower SNR than for stream 2. The curves with \(\circ\) (resp. \(\cdot\)) show the results for estimation based CCI compensation with \(\epsilon = 0.1\) (resp. \(\epsilon = 0.25\)). For stream 2 compensation pays above 23 dB or 27 dB depending on the value of \(\epsilon\) used. For stream 1 the advantage is clear above 18 or 23 dB depending on the \(\epsilon\) used.

7. REFERENCES


