A NOVEL ACTIVE CONSTELLATION EXTENSION ALGORITHM WITH LOW PEAK POWER FOR PILOT-AIDED OFDM SYSTEMS

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ABSTRACT

The active constellation extension (ACE) technique is attractive for reducing peak power in OFDM systems. It is simple and easy to implement and provides suboptimal gain in peak power reduction. However, its peak power reduction capacity is sensitive to low target clipping ratios. In this paper, we propose a novel ACE algorithm using pilot symbols. In contrast to the existing ACE techniques, our proposed algorithm can reach any desired target clipping level with low complexity, even when the clipping level is set below the initially unknown optimum value. Simulation results are utilized to compare our proposed algorithm with the existing ACE techniques. It is shown that our proposed algorithm provides better peak-to-average power ratio (PAR) reduction performance for low clipping ratios.

Index Terms— Orthogonal frequency division multiplexing (OFDM), peak-to-average ratio (PAR), active constellation extension (ACE), pilot-aided

1. INTRODUCTION

One of the major obstacles for implementing OFDM techniques in applications is its inherently high peak-to-average power ratio (PAR), resulting in nonlinear distortion associated with amplitude limited devices such as high power RF amplifiers (PA) and digital-to-analog converters (DAC). Therefore, PAR reduction of the baseband OFDM signal prior to the PA can increase the efficiency of PA and reduce such nonlinear distortions.

Among many PAR reduction methods, the active constellation extension (ACE) technique is attractive for reducing PAR for current and future OFDM standard systems, because ACE can reduce peak power by modifying the modulation constellation without any loss of data rate and extra side information. The PAR reduction problem for ACE can be cast as a constrained convex optimization problem; however, the complexity of the optimization solvers such as the second order cone program (SOCP) or the quadratically constrained quadratic program (QCQP) are too high for practical implementation [1].

Several low complexity algorithms based on iterative clipping and filtering with ACE constraint are proposed in [1,2]. For a fast convergence rate, a smart gradient-project method with good computational efficiency was proposed in [1], and the adaptive optimal step size minimizing the clipping noise was introduced in [3]. Because the basic idea of these kind of low complexity iterative algorithms comes from clipping, these low complexity algorithms represent a suboptimal solution when the given clipping ratio is low. However, if the initial target clipping level is below a certain level, the desired peak reduction can not be achieved by any existing low complexity iterative gradient algorithms [4]. In [2], ACE is improved by minimizing the correlation between the input signal and the peak cancellation signal at a low clipping ratio. However, the peak reduction capacity is still sensitive to the the choice of a target clipping ratio. To solve the problem with low clipping ratio, the adaptive clipping control method in the random tone reservation technique is introduced in [4].

In this paper, we propose a simple and efficient ACE algorithm using pilot symbols. Pilot-assisted approaches via selected mapping (SLM) have been proposed for both PAR reduction and channel estimation in [5,6]. To the best of our knowledge, the pilot-aided ACE has not yet been introduced. In contrast to the existing ACE techniques, our proposed algorithm can achieve the desired gain of peak power reduction even when the target clipping ratio is set below the unknown optimum clipping point. Simulation results show our proposed algorithm provides more PAR reduction performance compared to the existing ACE techniques for low clipping levels.

2. OFDM SYSTEMS WITH ACTIVE CONSTELLATION EXTENSION (ACE)

An OFDM signal is the sum of $N$ independent signals modulated in the frequency domain onto subchannels of equal bandwidth. As a continuous-time equivalent signal, the oversampled OFDM signal is expressed as

$$x_n = \frac{1}{\sqrt{JN}} \sum_{k=0}^{JN-1} X_k e^{j2\pi k n/N}, \quad n = 0, 1, \ldots, JN - 1$$

where $N$ is the number of subcarriers; $X_k$ are the complex data symbols using phase-shift keying (PSK) or quadrature amplitude modulation (QAM) at the $k$th subcarrier; and $J$ is the oversampling factor where $J \geq 4$, which is large enough to accurately approximate the peaks [7]. In matrix notation, (1) can be expressed as $x = Q^*X$, where $Q^*$ is the inverse discrete Fourier transform (IDFT) matrix of size $JN \times JN$, $(\cdot)^*$ denotes the Hermitian conjugate, the complex time-domain signal vector $x = [x_0, x_1, \ldots, x_{JN-1}]^T$, and the complex symbol vector $X = [X_0, X_1, \ldots, X_{N/2-1} \, |_{x(J-1)N}, X_N/2, X_{N/2+1}, \ldots, X_{N-1}]^T$. As a random variable, we consider the PAR defined by normalizing the peak power by the symbol-wise average power as follows

$$\xi \triangleq \frac{\|x_n\|^2_F}{\frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2}.$$ 

Pilot symbols are widely used for channel estimation as well as synchronization in OFDM systems [8,9]. In [10], the optimal placement of pilots are equally spaced among the subcarriers and with
equal power. Let \( \mathcal{N} \) be the set of all data tones: \( \mathcal{N} = \{ k \text{ s.t. } 0 \leq k \leq N-1 \} = \{ \mathcal{N}_p \cup \mathcal{N}_c \} \) where \( \mathcal{N}_p \) is the set of pilot tones. Thus, the complex symbols \( X_k \) are

\[
X_k = \begin{cases} 
  P_k, & k \in \mathcal{N}_p \\
  S_k, & k \in \mathcal{N}_c
\end{cases},
\]

where \( P_k \) and \( S_k \) are pilot and data symbols transmitted at the \( k \)th subcarrier respectively.

Let \( \mathcal{I}_a \) be the index set of active subchannels, where the modified constellation \( X_k + C_k \) is within the feasible set of the ACE constraint. For PAR reduction, the PAR problem in ACE can be formulated as [11]

\[
\min_{X_k + C_k} \| x + QC \|_\infty
\]

subject to: \( X_k + C_k \) be feasible for \( k \in \mathcal{I}_a \),

\[
C_k = 0, \quad \text{for} \quad k \notin \mathcal{I}_a
\]

where \( C \) is the extension vector whose components, \( C_k \), are nonzero only if \( k \in \mathcal{I}_a \). In general, the optimal solution for this formulation of PAR reduction is a QCQP with large complexity. This large computational complexity is not appropriate for practical implementation. Thus, the low complexity ACE algorithms are introduced [1,2].

In the following section, we will review the existing clipping based ACE techniques.

### 3. REVIEW OF THE EXISTING CLIPPING BASED ACE

The existing clipping based ACE technique can be considered as repeated clipping and filtering (RCF) with ACE constraints to avoid in-band degradation due to clipping. The iterative signal update for peak power reduction is expressed as

\[
x^{(i+1)} = x^{(i)} + \mu \hat{e}^{(i)},
\]

where \( \mu \) is a positive real step size that determines the convergence speed and prevents the peak growth, \( i \) is the iteration index, the initial signal is \( x^{(0)} \), and \( \hat{e}^{(i)} \) is the anti-peak signal at the \( i \)th iteration as follows: \( \hat{e}^{(i)} = Q^{(i)} \tilde{C}^{(i)} = T^{(i)} e^{(i)} \), where \( \tilde{C}^{(i)} \) is the extension vector after the ACE constraint is applied, which is defined by

\[
\tilde{C}^{(i)} = \begin{cases} 
  C_k, & k \in \mathcal{I}_a \\
  0, & k \notin \mathcal{I}_a
\end{cases},
\]

\( T^{(i)} \) is the transfer matrix of size \( JN \times JN \), \( T^{(i)} = Q^{(i)} \tilde{Q}^{(i)} \), where \( Q^{(i)} \) is determined by the ACE constraint in (6), and \( e^{(i)} \) is the peak signal above the pre-determined clipping level \( A \) and \( e^{(i)} = [e_1^{(i)}, e_2^{(i)}, \ldots, e_{JN-1}^{(i)}]^T \), where \( e_n^{(i)} \) is the clipping sample which can be obtained as follows:

\[
e_n^{(i)} = \begin{cases} 
  (A - |x_n^{(i)}|) e^{\theta_n}, & \text{if } |x_n^{(i)}| > A \\
  0, & \text{if } |x_n^{(i)}| \leq A
\end{cases},
\]

where \( \theta_n = \arg(x_n^{(i)}) \). The clipping level \( A \) is obtained from the clipping ratio as follows: \( \gamma = \frac{\sqrt{2}}{E[|x_n^{(i)}|^2]} \). Note that (5) should be performed until either the peak signal is below the clipping level, or the peak power is minimized for a given number of iterations.

Note that the performance for peak power reduction depends on the following important factors: (i) the fast convergence rate and (ii) the generation of the anti-peak signal. For fast convergence, two computationally efficient approaches have been proposed. In [1], the optimal step size \( \mu \) is obtained by using the smart gradient-project method, and the other way proposed to find the optimal step size \( \mu \) uses the adaptive scaling algorithm minimizing the excess power of the peak signal [3].

Another important issue is how much the anti-peak signal \( \hat{e}^{(i)} \) approximates the clipping signal \( e^{(i)} \). In general, as we lower the clipping level, better PAR reduction is generally expected. However, for low clipping ratios, the existing ACE cannot reduce peak power as much as we expect, because of the increasing interpeak interference (IPI) in [4]. Moreover, the reduced power by clipping degrades the ACE capacity in the peak power reduction in [2]. To improve the ACE techniques, compensation for lost power is proposed by using the weighting factor in [2]. But, the gain in the peak power reduction still depends on the clipping ratio. In this paper, we solve this problem with the low clipping ratio by fully employing the inherent property of iterative clipping and filtering. The detailed algorithm is discussed in the next section.

### 4. PILOT-AIDED ACTIVE CONSTELLATION EXTENSION

Although there are many ways to use pilot symbols to estimate channel state information (CSI) in OFDM, our approach in this paper involves improving the ACE technique using pilot symbols. The main idea is to estimate the correlation coefficients between the original pilot symbols and their successive clipping noise to adjust the clipping effect, and to keep the correlation property inherited from clipping while keeping the reduced minimum distance. We assume the correlation coefficient at each iteration is constant for any \( k \in \mathcal{N} \).

First, we estimate the correlation factor between the original constellation of an OFDM symbol and the clipping noise constellation, \( C_k \), at each \( i \)-th iteration, then obtain the uncorrelated clipping noise, \( D_k^{(i)} \), by compensating the bias of clipping noise as follows,

\[
D_k^{(i)} = C_k^{(i)} - \kappa^{(i)} X_k^{(0)}
\]

where \( \kappa^{(i)} \) is the correlation factor at the \( i \)-th iteration. By making use of the Bussgang theorem, \( C_k^{(i)} \) can be expressed by [12]

\[
C_k^{(i)} = (\alpha - 1) X_k^{(0)} + D_k^{(i)},
\]

where the first term represents the correlation part of the original signal components, \( X_k^{(0)}, D_k^{(i)} \) is a zero-mean noise process uncorrelated with the signal \( X_k^{(0)} \), i.e., \( E[D_k^{(i)} X_k^{(0)*}] = 0 \) and \( E[D_k^{(i)} X_k^{(0)*}] = 0 \), and the attenuation factor \( \alpha \) is given by

\[
\alpha = \frac{E[|x_n^{(i)}|^2]}{E[|x_n^{(i)}|^2]} = 1 - e^{-\gamma^2} + \frac{\sqrt{2} \gamma}{2} \text{erfc}(\gamma).
\]

Note that \( \kappa^{(0)} = \alpha - 1 \). However, the input signal for the subsequent clipping does not guarantee a Gaussian process. Therefore, the correlation factor can be determined by considering the unbiased clipping noise power, i.e., minimizing the mean-squared error of (8) as follows:

\[
\kappa^{(i)} = \arg \min_{\kappa^{(i)}} \| D_k^{(i)} \|_2
\]

Here, the optimal correlation factor using pilot symbols can be estimated as

\[
\kappa^{(i)} = \frac{\Re[\langle C_k^{(i)}, P_k^{(0)} \rangle]}{\langle P_k^{(0)}, P_k^{(0)} \rangle},
\]

where \( \Re \) specifies the real part, \( \langle, \rangle \) is the complex vector inner-product, \( C_k^{(i)} = \{ C_k^{(i)} | k \in \mathcal{N}_p \} \), and \( P_k^{(0)} = \{ X_k^{(0)} | k \in \mathcal{N}_p \} \).
be obtained by center of the clipping noise. The desired extension constellation can be approximated clipping noise \( \tilde{\mathcal{C}} \), including the pilot symbols as well as the signal in the non-feasible area, to zero. The uncorrelated extension constellation, \( \tilde{\mathcal{D}} \), is defined by

\[
\tilde{D}_k^{(i)} = \begin{cases} 
D_k^{(i)}, & k \in \mathcal{I}_a \\
0, & k \notin \mathcal{I}_a \quad \text{and} \quad k \notin \mathcal{I}_d 
\end{cases},
\]

Note that the shift of the bias of clipping noise in (8) can maximize the possible number of active subcarriers, which can improve the peak power reduction capacity of ACE. However, uncorrelated clipping noise does not approximate the clipping noise signal for low clipping ratios. The reason is the direction of the correlation term in (8) is not only opposite to that of the desired direction of clipping noise, but the correlation term in (8) increases as the clipping ratio decreases.

Therefore, to avoid the existing ACE problem for low clipping ratios, we restore \( \tilde{D}_k^{(i)} \) with an optimal correlation factor \( \kappa_{opt}^{(i)} \) to the center of the clipping noise. The desired extension constellation can be obtained by

\[
\tilde{C}_k^{(i)} = \tilde{D}_k^{(i)} + \kappa_{opt}^{(i)} \chi_k^{(0)}
\]

Fig. 1 shows an illustrative example of the modified vector when our proposed pilot-aided ACE algorithm is applied. The constellation points correspond to the extension vector of the first quadratic constellation in QPSK for the clipping ratio of 2dB at the first iteration. The circles are for pilot symbols, and the dots are for data symbols. The shaded area indicates the feasible region corresponding to the first quadratic constellation in QPSK. In Fig. 1a, the clipping noise constellation is moving outside the feasible region, which determines the non-active subcarriers. This figure explains why the existing ACE has a problem of reducing peak power. The uncorrelated clipping noise, \( D_k \) is shown in Fig. 1b by compensating the bias of the clipping noise with the correlation factor in (12). In Fig 1c, ACE constraints are applied on \( D_k \) by setting all pilot points to zero. To well approximate the anti-peak signal, the modified vector after ACE is shifted to the center of the original clipping noise in Fig 1d.

### 5. SIMULATION RESULTS

In this section, we demonstrate the performance of our proposed technique by computer simulations. We use an OFDM system with 2048 subcarriers and QPSK. We also approximate the continuous-time peak signal of an OFDM signal by using the oversampling rate factor \( J = 8 \). For the sake of comparison fairness with the existing clipping based ACE techniques [1, 2], the optimal adaptive scaling in [3] is used to find the fast convergence factor.

Fig. 2 demonstrates the achievable peak power of an OFDM signal with an initial 12dB PAR, for different target clipping ratios \( \gamma \) from 0dB to 12dB. The peak power is used as a performance measure with respect to the original average power without considering the power of the anti-peak signal added by ACE. We find the minimum achievable peak power of 6.26dB can be obtained with a target clipping ratio of 5.6dB and 5.8dB for ACE with CS (clipped signal) and ACE with weighting CS, respectively. On the other hand, our proposed algorithm can achieve the desired clipping level even when the initial target clipping ratio is set below optimal points for the other ACEs. As mentioned in the previous section, the performance of the peak power reduction for the existing clipping based ACE algorithm depends on the clipping ratio. For ACE with CS, the smaller the clipping ratio is with respect to the optimal value of 5.6dB, the smaller the peak power reduction gain is. Therefore, the pre-determined target clipping ratio should be carefully chosen for ACE with CS. However, it is difficult to determine the optimal clipping ratio for every symbol, because the minimum PAR as well as the optimal clipping level changes symbol by symbol. As shown in [2], ACE with weighting CS provides an improvement of the peak power.
Fig. 3: PAR CCDF comparison of existing ACE techniques and the proposed technique for different clipping ratios: $\gamma = 0\text{dB}, 2\text{dB},$ and $4\text{dB},$ respectively. The arrows indicate the trends of PAR reduction with decreasing clipping ratio.

reduction for low clipping ratios, which is observed in Fig. 2. For example, the improved peak power gain compared to ACE with CS is $2.2\text{dB}, 1\text{dB},$ and $0.15\text{dB}$ for the clipping ratio $\gamma = 0\text{dB}, 2\text{dB},$ and $4\text{dB},$ respectively. As the clipping ratio decreases, the improvement in the peak power reduction increases; however, ACE with weighting CS is still sensitive to the unknown optimum value of clipping ratio. Contrary to the existing ACE techniques, it is obvious that our proposed algorithm can reach any desired target clipping level.

Next, our proposed algorithm is quantified in terms of the complementary cumulative density function (CCDF) of PAR (2), which is the probability that the PAR of the transmitted OFDM symbol, $\xi,$ exceeds the threshold PAR $\xi_o$. Note that it is worthwhile to use the PAR metric which considers the power change by the ACE techniques. While the existing ACE techniques increase the average power to reduce the peak power, our proposed algorithm decreases the average power. In Fig. 3, the CCDF of the PAR is plotted by using different algorithms: ACE with CS, ACE with weighting CS, and our proposed method. The right-most dashed line curve is plotted for the original OFDM signal. The dotted line curves correspond to ACE with CS, the lines with the dash-dots are for ACE with weighting CS, and the solid lines are for our proposed algorithm. For each case, plots are provided for clipping ratios of $\gamma = 0\text{dB}, 2\text{dB}$ and $4\text{dB}$. As the target clipping ratio decreases, the PAR reduction capacity decreases when ACE with CS and ACE with weighting CS are applied, i.e., the curves trend to the right. Although ACE with weighting CS can improve the PAR reduction for low clipping ratios, the performance of PAR reduction is still degraded as the clipping ratio decreases, which is consistent with the trend shown in Fig. 2. On the other hand, our proposed algorithm provides more PAR reduction with the decreasing clipping ratio, i.e., the trend is to the left.

6. CONCLUSION

In this paper, we have proposed a new ACE algorithm for PAR reduction by using pilot symbols, where the pilot symbol is used to estimate the correlation factor for the maximization of the ACE constraint and to well approximate the antipeak signal for peak reduction. It is observed that the existing clipping based ACE techniques are sensitive to the low clipping ratio. The lower the target clipping ratio is from the optimal clipping value, the smaller both the peak power and PAR reduction gain are. However, our proposed algorithm can reach any desired target clipping level, even when the clipping level is set below the initially unknown optimum value. Simulation results show that our proposed algorithm provides better PAR reduction with decreasing clipping ratio.

7. REFERENCES


