TRANSMIT STRATEGIES FOR THE MIMO TWO-WAY AMPLIFY-FORWARD CHANNEL WITH MULTIPLE RELAYS AND MMSE RECEIVER

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ABSTRACT

Two-way multi-antenna relaying with multiple relays offers great potential to increase the reliability and performance of bidirectional data transmission in relay networks. The optimal transmit strategies at the relay and at the nodes are known for the decode and forward strategy at one relay. However, for multiple linear amplify and forward relays the optimal strategy is not known. We compare three different recent proposals, DFT relaying, dual channel matching, and ANOMAX with optimal transmit strategies at the nodes under average power constraints at the relays and with linear MMSE receivers. It turns out that for a few relays DFT relaying performs reasonably whereas for more relays channel aware relaying gains significantly in performance. For large number of relays in rich multi-path fading it is important to adapt the relay strategy than the node strategies to the current channel state.

Index Terms— MIMO, Bidirectional Relay Channel, Linear Amplify and Forward, Linear MMSE, Optimization

1. INTRODUCTION

Two-way relay channels occur naturally in wireless communications scenarios, e.g. in cellular networks with additional fixed relays to increase coverage with simultaneous uplink and downlink transmission. The general discrete memoryless two-way relay channel was introduced in [1] and the multiple antenna relay channel in [2].

In [3], the optimal transmit covariance matrix for the MIMO bidirectional relay channel is characterized for decode and forward relaying. Multi-antenna bidirectional relaying schemes including maximum ratio combining (MRC) and MMSE beamforming are compared in [4]. In [5] a suboptimal transmit strategy for two-way relaying with amplify and forward relays called ANOMAX is presented and improved in terms of high SNR performance in [6]. In [7], the bit error performance is used to optimize the relay strategy. For single antennas at the user nodes, the optimal amplify and forward matrix is characterized in [8]. The case in which multiple relays assist the bidirectional communication is studied in [9] and the diversity multiplexing tradeoff is characterized using dual matching matrices at the relays. A recent overview of amplify and forward schemes is presented in [10].

The one-way case is recently studied in a large body of work considering amplify and forward or decode and forward processing.

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For instance, downlink transmission is improved via MIMO encoding and decoding at the relay station in [11]. [12] minimizes the MSE by linear relay matrix optimization.

In this paper, we study the two-way MIMO amplify and forward channel with multiple relays and linear MMSE receivers. Using a linear MMSE receiver, the end-to-end performance is described by the sum MSE at the terminals1. We compare three recently proposed relaying strategies, namely a fixed DFT matrix, dual channel matching (DCM), and ANOMAX. First, we derive expressions for the required average transmit power at the relay. This can be done in closed form for DFT and dual channel matching and the average transmit power depends only on the trace of the transmit covariance matrices at the nodes. Therefore, a straightforward SVD and power allocation can be performed. The numerical results indicate an interesting behavior as a function of the number of relays: For a low number of relays the relay processing is less important while for more relays channel adaptive relaying as DCM or ANOMAX gains more than optimized transmit covariance matrices at the nodes. For a low number of relays fixed DFT relaying has reasonable performance. Relay and node strategies developed for the single relay case must be carefully extended to the multiple relay case.

Vectors are denoted in bold letters $\mathbf{x}$. Matrices are written in bold capital letters $\mathbf{H}$. Transpose is $\mathbf{X}^T$, the conjugate transpose is $\mathbf{X}^H$. The conjugate is denoted $\mathbf{X}^\dagger$. The matrix inverse is denoted by $\mathbf{X}^{-1}$. $\text{tr}(\mathbf{A})$ denotes the trace of the matrix $\mathbf{A}$, i.e. $\text{tr}(\mathbf{A}) = \sum_{k=1}^{n} A_{k,k}$. The expectation operator is $\mathbb{E}$.

2. SYSTEM MODEL AND MSE MINIMIZATION

In this paper, we focus on nodes all equipped with multiple antennas and multiple multi-antenna relays, which has not been analysed sufficiently within the research community. Consider a two-phase MIMO amplify and forward system with multiple relays $k, 1 \leq k \leq K$. Node $A$ and $B$ have $n_A$ and $n_B$ transmit antennas, respectively. Relay $k$ has $n_k$ antennas. Denote the channels from node $A$ to the $K$ relays as $\mathbf{H}_{A1}, ..., \mathbf{H}_{AK}$ of size $[n_k \times n_A]$ and from node $B$ to the $K$ relays as $\mathbf{H}_{B1}, ..., \mathbf{H}_{BK}$ of size $[n_k \times n_B]$. The relays perform amplify and forward with matrices $\mathbf{A}_1, ..., \mathbf{A}_K$ of size $[n_k \times n_k]$. The transmit covariance matrices at node $A$ and $B$ are $\mathbf{Q}_A [n_A \times n_A]$ and $\mathbf{Q}_B [n_B \times n_B]$, respectively.

The independent data vectors are denoted by $\mathbf{d}_A$ and $\mathbf{d}_B$. The power constraints at the nodes are $\text{tr}(\mathbf{Q}_A) \leq P_A$ and $\text{tr}(\mathbf{Q}_B) \leq P_B$.

1If a MMSE receiver with successive interference cancellation (SIC) is applied, a suitable performance measure would be the mutual information.
The relay uses transmit power given by \( \text{tr} (A_k A_k^H) \). The additive white Gaussian noise at the receivers (relays in phase 1 and nodes in phase 2) is independently and identically distributed as zero-mean complex Gaussian identity covariance. The channel matrices are Rayleigh-distributed with zero-mean and unit variance, i.e., \([H_{Ak}]_{nn} \sim CN(0,1/n_T)\). Then \( \mathbb{E}[H_{Ak} H_{Ak}^H] = I \).

### 2.1. Two-phase Amplify and Forward

The receive vector at relay \( k \) (phase 1) is given by

\[
y_k = H_{Ak} Q_{Ak}^{1/2} d_A + H_{Bk} Q_{Bk}^{1/2} d_B + n_k
\]

with AWGN \( n_k \sim \mathcal{CN}(0,\sigma^2 I) \). The transmit vector at relay \( k \) (phase 2) is given by

\[
\tilde{x}_k = A_k y_k = A_k H_{Ak} Q_{Ak}^{1/2} d_A + A_k H_{Bk} Q_{Bk}^{1/2} d_B + A_k n_k
\]

and the corresponding transmit power is

\[
p_{R_k} = \mathbb{E} (\tilde{x}_k \tilde{x}_k^H) = \text{tr} (A_k H_{Ak} Q_{Ak} A_k^H Q_{Ak}^H) + \text{tr} (A_k H_{Bk} Q_{Bk} H_{Bk}^H A_k^H) + \sigma^2 \text{tr} (A_k A_k^H).
\]

The normalized transmit vector at relay \( k \) is \( \tilde{x}_k = \frac{x_k}{\|x_k\|} \sqrt{P_k} \). Where \( \gamma \) is used to satisfy average or short-term power constraints. We assume that the channel is reciprocal and thus the channel from relay \( k \) to node \( A \) is given by the transpose of channel of the uplink, i.e., \( H_{Ak}^T \). The received vector at node \( A \) (phase 2) is given by

\[
y_A = \sum_{k=1}^K H_{Ak}^T x_k + n_A
\]

with AWGN \( n_A \sim \mathcal{CN}(0,\sigma^2 I) \). Node \( A \) subtracts the self interference (analog network coding) and obtains

\[
\tilde{y}_A = \sum_{k=1}^K \sqrt{\frac{P_k}{\gamma_k}} H_{Ak}^T A_k H_{Ak} Q_{Ak}^{1/2} d_B + \sum_{k=1}^K \sqrt{\frac{P_k}{\gamma_k}} H_{Ak}^T A_k n_k + n_A.
\]

Define the matrices \( \tilde{H}_{BA} = \sum_{k=1}^K \sqrt{\frac{P_k}{\gamma_k}} H_{Ak}^T A_k H_{Ak} \) and \( \tilde{H}_{AB} = \sum_{k=1}^K \sqrt{\frac{P_k}{\gamma_k}} H_{Bk}^T A_k H_{Ak} \) as the effective channels seen by node \( A \) and \( B \). The noise covariance matrix at node \( A \) is given by

\[
\tilde{Z}_A = \sigma^2 I + \sigma^2 \sum_{k=1}^K \frac{P_k}{\gamma_k} H_{Ak}^T A_k A_k^H H_{Ak}^H.
\]

Observe that the coloured noise matrix \( \tilde{Z}_A \) depends indirectly on the transmit strategies \( Q_A \) and \( Q_B \) via \( \gamma_k \).

#### 2.2. MMSE receiver, relay strategies and problem statement

At the receiver (node \( A \)), we apply the linear MMSE receiver and estimate the data vector \( d_B \) by

\[
\hat{d}_B = Q_{Bk}^{1/2} \tilde{H}_{BA} \left[ \hat{Z}_A + \tilde{H}_{BA} Q_B \tilde{H}_{BA}^H \right]^{-1} \tilde{y}_A
\]

Then, the normalized sum MSE of data stream \( d_B \) at node \( A \) is given by

\[
\text{MSE}_B = n_T \text{tr} \left( \tilde{H}_{BA} Q_B \tilde{H}_{BA}^H \left[ Z_A + \tilde{H}_{BA} Q_B \tilde{H}_{BA}^H \right]^{-1} \right)
\]

and the normalized sum MSE of data stream \( d_A \) at node \( B \) is given analogously.

The objectives \( \text{MSE}_A \) and \( \text{MSE}_B \) can be optimized by choosing the relay AF matrices \( A_1, \ldots, A_K \) and the two transmit covariance matrices \( Q_A \) and \( Q_B \). These should be optimized jointly. However, if the transmit covariance matrices \( Q_A \) and \( Q_B \) change, this usually changes the AF matrices \( A_k \) and their transmit power \( P_{R_k} \) in (3). For the one-way channel, a complete solution can be found in [13]. In [14], an approximation of the relay power was applied for large number of relays. Up to now, no complete characterization of the optimal AF matrices and covariance matrices at the nodes for the two-way relay channel is available.

Therefore, we relax the constraints at the relay and allow for average power constraints there, i.e., \( \mathbb{E}[P_{R_k}] \leq P_k \) for all \( k, 1 \leq k \leq K \) and consider three different relay strategies:

1. The relay AF matrices are chosen fixed as DFT matrices of size \( n_k \times n_k \), \( A_k^D = D_{n_k} \). For this scheme no channel information is needed at the relay and the processing can be efficiently implemented.

2. The relay AF matrices are chosen according to the recently proposed ‘dual channel matching’ matrices [9] as follows

\[
A_k^M = \mathbf{P}_{Bk} H_{Ak}^H \mathbf{P}_{Ak} + \mathbf{P}_{Ak} H_{Bk}^H.
\]

Note that dual channel matching can only be used if \( n_{R_k} = n_T \). Otherwise, a dimension reduction must be performed.

3. The relay AF matrices are chosen by ANOMAX [5] as \( \text{vec}(A_k^M) = u_k \) where \( u_k \) is the dominant left singular vector of the matrix \( K_k = [H_{Bk} \otimes H_{Ak}; H_{Ak} \otimes H_{Bk}] \).

For given relay strategy \( A_k^D, A_k^M \) or \( A_k^M \), the remaining problem statement is as follows: The optimal transmit covariance matrices \( Q_A \) and \( Q_B \) solve the sum MSE minimization problem given by

\[
\min_{Q_A \succeq 0, \ Q_A \leq P_A} \text{MSE}_A + \min_{Q_B \succeq 0, \ Q_B \leq P_B} \text{MSE}_B.
\]
3. OPTIMAL NODE TRANSMIT STRATEGIES

Our first result is on the average relay transmit power if the DFT matrix is used at the relays $A_k^D$.

**Lemma 1** Using DFT amplify-and-forward relaying $A_k^D$, the average transmit power at the relay $k$ is given by

$$P_k^D = E[p_k] = tr \left( E[H_{Ak}^H H_{Ak}] Q_A \right) + \sigma^2 tr(\mathbf{I}) + \sigma^2 tr(\mathbf{I}) + \sigma^2 \lambda_{B,k} \right)^+. \quad (10)$$

For independent and spatially uncorrelated fading the average transmit power is

$$P_k^D = E[p_k] = tr(\mathbf{Q}_A + \mathbf{Q}_B) + \sigma^2 n_k. \quad (11)$$

Observe that this result also holds for the identity AF matrices $A_k^I = \mathbf{I}$. The nice observation in (11) is that the average relay power does only depend on the transmit power of the nodes $tr(\mathbf{Q}_A)$ and $tr(\mathbf{Q}_B)$, respectively. Therefore, this interdependence described above is avoided and the optimal transmit covariance matrices can be easily found (see Corollary 3 below).

The next result shows that a similar identity holds for the dual channel matrices $A_k^M$, too.

**Lemma 2** Using the dual channel matching matrices in (8), the average transmit power at the relay $k$ is characterized by

$$P_k^M = E[p_R_k] = c_k tr(\mathbf{Q}_A + \mathbf{Q}_B) + 3\sigma^2 n_k. \quad (12)$$

The constant $c_k$ depends on the number of antennas at the nodes and at the relay.

Observe that the average power depends on the transmit covariance matrices via the trace again. This implies that for fixed trace constraint on $\mathbf{Q}_A$ and $\mathbf{Q}_B$, the average power does not vary. Therefore, the optimization in (9) deconstructs into single-user optimization problems.

Denote the effective channel including the coloured noise at node $A$ by $C_{BA} = \tilde{Z}_A^{-1/2} H_{BA} V_B$ and its singular value decomposition by $C_{BA} = U_{BA} \Lambda_{BA} V_{BA}^H$.

**Corollary 3** Under individual average relay power constraints, the optimal transmit covariance matrix $\mathbf{Q}_B = U_{Q_B} \Lambda_{Q_B} U_{Q_B}^H$ is given by $U_{Q_B} = V_{BA}$. The power allocation in $\Lambda_{Q_B}$ is given by

$$\lambda_{Q_B,k} = \left( \sigma^2 \lambda_{BA,k}^{-1/2} + \sigma^2 \lambda_{BA,k}^{-1} \right)^+ \quad (13)$$

where $(\cdot)^+ = \max(\cdot, 0)$ and $\nu$ is chosen such that $\sum \lambda_{Q_B,k} \leq P_B$.

This result follows from [15] where the optimal transmit strategy of a single-user MIMO link with MMSE receiver is derived. Note that at high SNR, the optimal transmit converges to equal power allocation and at low SNR only one spatial stream per node is supported.

4. NUMERICAL RESULTS

The results on the average transmit power at the relay in (11) and (12) also serve as a baseline for fair comparison between the AF strategies.

4.1. Average sum MSE

In Figure 1 we show the average sum MSE (averaged over 10000 channel realizations) for different relay strategies (dual channel matching (DCM) versus DFT) and equal power allocation versus optimal transmit covariance matrices under average relay power constraints. Both terminals as well as the relays have two antennas each. Note that with two transmit antennas at each node, the achievable sum MSE is between zero (achieved for high SNR) and four (for low SNR).

![Fig. 1. Average sum MSE over SNR for dual channel matching versus DFT with equal power allocation or optimal transmit covariance matrices.](image)

The gain of using channel aware AF matrices (DCM compared to DFT) can be observed clearly for one $(K = 1)$ relay. The gain of transmit covariance matrix optimization at then nodes can be observed for medium SNR and one or two relays. For high SNR or more relays, the gain of choosing $\mathbf{Q}_A$ and $\mathbf{Q}_B$ is limited.

In Figure 2, the medium SNR range is shown and we compare DFT, DCM, and ANOMAX. The power normalization $\gamma$ is chosen such that the average relay transmit power is equal for all three relaying strategies. It turns out that the original version of ANOMAX performs worse than DFT AF. This was observed and an improved version of ANOMAX is proposed in [6] which is close to maximum sum rate performance. Furthermore, it can be observed that for $K = 6$ multiple relays, the DCM clearly outperforms the fixed DFT relaying.

When DFT relaying, ANOMAX, and DCM are compared, it is important to observe the tradeoff between performance (here the average sum MSE), the control overhead and complexity, and the restrictions. DFT is less complex and requires no channel information at all. DCM requires information at the relay $k$ on its own channels $H_{Ak}$ and $H_{Bk}$ and has the additional restriction that $n_{TA} = n_{TB}$. ANOMAX requires also information at the relay on its own channels but can be applied to asymmetric channels as well.

5. CONCLUSION

The optimal AF relaying strategy for MIMO two-way channels with multiple relays is still unknown. The paper presents one approach to
compare recently proposed AF strategies with optimal node precoding under a fair average relay power constraint. When a single relay is applied the fixed AF strategy using a DFT matrix performs reasonable well. However, with multiple relays the channel aware DCM outperforms DFT significantly. Further reduction in sum MSE can be achieved by node optimization. Multiple relays improve the sum MSE performance and reduce the impact of the transmit strategies at the nodes.

A. SKETCH OF PROOF OF LEMMA 2

Due to space constraints, we give the sketch of the proof. In the average power expression there are four different expectations of products of channel matrices. It can be shown using [16, Lemma 4.1] that all four expectations are weighted identity matrices.

The weights depend on the number of antennas and for the simple case \( n_T_A = n_T_B = n_K = 2 \), the following identities hold

\[
E[H_A^H H_A H_B^H H_B] = 2I,
E[H_A^H H_A H_B^T H_B] = I,
E[H_A^H H_B H_A^T H_B] = I,
E[H_A^H H_B H_A H_A] = \frac{5}{4} I.
\]

This adds up to \( c_K = 10.5 \) for this example. For the noise terms, we obtain also four expectation expressions as

\[
E[H_B^H H_B] = I,
E[H_A^H H_A] = \frac{1}{2} I,
E[H_A^T H_A] = \frac{1}{2} I,
E[H_B^H H_B] = I.
\]

The fact that all expectations are weighted identity matrices is important to find the optimal node covariance matrices \( Q_A \) and \( Q_B \). The weights are necessary to perform a fair system comparison.

B. REFERENCES