ENHANCED TRELLIS BASED VECTOR QUANTIZATION FOR COORDINATED BEAMFORMING

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ABSTRACT

Trellis-based quantization is a suitable method for reporting channel state information in multi base-station or base-station-relay coordinated transmission schemes [1, 2]. Due to different path loss and shadowing from each transmitting station, the receiver experiences different long-term average signal powers from each transmitter. We first propose to use a tail-biting trellis to improve the performance of the vector quantization proposed in [2]. To reduce decoding complexity, we present a trellis labeling technique where efficient encoding and decoding can be performed by traversing the trellis twice. Lastly, we introduce a mechanism for dynamically allocating quantization bits amongst different parts of the augmented channel state vector. A trellis simplification procedure that keeps the number of trellis states at each stage to four is shown to provide a satisfactory performance.

1. INTRODUCTION

Multi-site coordinated transmission has gained popularity within academia and wireless standardization circles. In particular, coordinated transmission schemes enhance the performance for users located in edge areas between two service regions (cell edge users). It is well known that coordinated transmission schemes achieve a significant rate boost for the cell edge users over non-coordinated schemes (e.g., [3, 4, 5, 6]). Potential scenarios include multiple base station (BS) coordinated transmission and base-station-relay coordinated transmission.

To reap the benefits of coordinated transmission, many schemes treat the multiple-transmission system as more or less a single multiple-input-multiple-output (MIMO) system. This treatment usually implies the availability of channel station information at all of the coordinating transmitters (CSIT). Modern frequency division duplexed (FDD) systems usually facilitate CSIT by establishing a limited feedback channel from the receiver back to the transmitters. The transmission rate on the feedback channel is low and thus CSIT must be quantized. Though important in practice, we do not study the effects of feedback delay or error in this paper.

One of the problems associated with CSIT feedback in a coordinated transmission system is the increased dimensionality one must quantize. To avoid loss of quantization precision, extra feedback bits are needed to accurately report the expanded CSIT space. As the number of coordinating transmission points increase, so does the total number of quantization bits to maintain accuracy. This leads to the codebook size scaling exponentially with the number of quantization bits, making the selection of the best codeword a complex task.

Quantizing a large dimensional space is studied in the multi-carrier context in [7, 8] where the problem space of fewer than 1 feedback bit per subcarrier is studied. Another approach to quantize CSIT is to use trellis [1]. The benefit of using a trellis-based quantization approach is the low complexity, which grows only linearly to the number of transmit antennas. Inspired by [9], in [2], the authors enhanced the first scheme where each trellis stage processes multiple channel dimensions.

In this paper, we propose two further enhancements to [2]. The first enhancement involves the use of a tail-biting trellis. It is well known that terminating a trellis by returning to the initial state results in a rate loss. In addition, truncating the trellis at the last stage (which was adopted in [1, 2]) suffers a performance degradation. Finally, a tail-biting trellis, whose paths start and end at the same state [10], provides full trellis protection without rate loss at the price of higher decoding complexity. In this paper, we show that using an Ungerboeck-based trellis with special labeling, encoding and decoding a tail-biting trellis can be done in two trellis passes.

The second enhancement exploits the fact that a receiving mobile typically experiences different average powers from each of the coordinating transmission stations due to unequal path loss and shadowing, with a ratio varying between 0 to 15 dB. Given a fixed feedback budget, it is more beneficial to allocate extra feedback bits to quantize channel response with higher power than the channel response with lower power. We propose an adaptive scheme that dynamically allocates feedback bits to different parts of the augmented CSI vector based on long term signal strength ratio.

2. SYSTEM SETUP

Fig. 1 shows the feedback system we study. It has a similar structure as in [1, 2]. Consider a coordinated transmission system with $T$ coordinating transmitters. Let transmitter $t$ has $M_t$ transmit antennas, and the receiver has $M_R$ receive antennas. The baseband channel between transmitter $t$ and the receiver is denoted by $\mathbf{H}_t$, which is a $M_R \times M_t$ matrix with complex elements. Denote the combined channel by $\mathbf{H} = [\mathbf{H}_1, \ldots, \mathbf{H}_T]$.

Beamforming is a simple single-dimensional precoding technique. It is characterized by an $M$-dimensional vector $\mathbf{w}$ with complex elements which corresponds to the eigenvector for the maximum eigenvalue of $\mathbf{H}^\dagger \mathbf{H}$. In this paper, we use the trellis-based approach to quantize $\mathbf{w}$ which begins by feeding $\mathbf{w}$ into a trellis decoder. The output of the trellis decoder is then channel coded and transmitted through the feedback channel. We assume $BM_t$ bits are
available in total to quantize \( w \), or \( B \) feedback bits per antenna. Similar to [2], we assume \( L \) antennas are processed at each trellis stage. We restrict \( M \) to be divisible by \( L \) and that the product \( BL \) must be an integer. The feedback receiver consists of a feedback decoder and a trellis encoder, which reverses the operations of the feedback transmitter. In this paper, we assume the feedback channel is error free and has zero-delay.

3. KNOWN BEAMFORMING VECTOR QUANTIZATION SCHEMES

In this section, we summarize and compare known methods to quantize a given \( M \)-dimensional beamforming vector \( w \) using \( BM \) bits.

1. Beamforming Vector Codebook: The conventional codebook approach designs a codebook \( C_M = \{v_1, \ldots, v_{2^{BM}}\} \) with \( 2^{BM} \) complex \( M \)-dimensional vectors. To quantize a query vector \( w \), the vector with the smallest chordal distance from \( w \) is selected. This method has the best quantization performance with a complexity of \( O(M2^{BM}) \).

2. Section-by-Section Quantization: In this sub-optimal approach, \( L \) dimensions are quantized one at a time using a codebook \( C_L = \{v_1, \ldots, v_{2^{BL}}\} \) of size \( 2^{BL} \). This method incurs a quantization complexity of \( O(M2^{BL}) \).

3. Trellis Quantization: The codebook approach suffers from an exponential complexity in \( M \), while the section-by-section approach suffers from poor performance. The trellis-based method [1, 2] incurs a quantization complexity of \( O(M2^{BL}) \) with performance better than Section-by-Section Quantization and slightly worse than random vector quantization (RVQ).

4. TAIL-BITING TRELLIS QUANTIZATION

It is well known that tail-bitng trellises offer proper trellis termination without sacrificing coding rate, at a cost of increased complexity [11, 12]. A tail-biting trellis is a trellis where valid paths start and end at the same state. Encoding a tail-biting trellis can be achieved by two runs through the trellis [10], where the first run determines the proper starting state and the second run performs the encoding at the proper starting state.

Decoding a tail-biting trellis is more challenging. In general, ML decoding requires exhaustive search by testing all of the possible starting states. Existing algorithms either iteratively converge to the optimal solution or use suboptimal heuristics ([11, 12]). To reduce the complexity of the quantizer, we design a special Ungerboeck-based trellis where encoding and decoding can both be performed in two trellis passes. Denote \( \{s(n)\}_{n=0}^{M/L}, \{x(n)\}_{n=1}^{M/L}, \) and \( \{y(n)\}_{n=1}^{M/L} \) to be the sequence of states, inputs, and outputs respectively at the \( n \)-th stage. \( s(0) \) is the initial state, \( s(M/L) \) is the final state, and \( M/L \geq 2 \). Denote \( s_1 \rightarrow s_2 \) if there is a one-stage state transition from state \( s_1 \) to state \( s_2 \) (state numbering starts from 1). This special trellis has the following properties:

Property 1 For \( BL \) bits available to quantize \( L \) antennas at each trellis stage, where \( BL \) is an integer, each trellis stage has \( 2^{BL+1} \) states and \( 2^{BL} \) branches emanating from each state.

Property 2 If \( s_1 \) is odd and \( s_2 \) satisfies \( s_2 \leq 2^{BL} \), then \( s_1 \rightarrow s_2 \). If \( s_1 \) is even and \( s_2 \) satisfies \( s_2 > 2^{BL} \), then \( s_1 \rightarrow s_2 \). No other state transitions are allowed.

Property 3 Suppose there is an input \( x \) that results in state transition \( s_1 \rightarrow s \) and output \( y \). If \( s_2 \) is such that \( s_2 \rightarrow s \), input \( x \) results in \( s_2 \rightarrow s \) with the same output \( y \).

Property 4 Suppose at stage \( n \), \( x(n) \) causes \( s(n-1) \rightarrow s(n) \). If \( x(n) \) is odd, then so is \( s(n) \). Likewise, if \( x(n) \) is even, then so is \( s(n) \).

Figure 2 shows a possible trellis where \( BL = 2 \). Next, we will describe the two-pass tail-biting encoding and decoding algorithms for trellises satisfying these properties.

Proposition 1 To encode a tail-biting path on the trellis described, two passes through the trellis are sufficient. Moreover, the starting state in the second pass is the same as the ending state of the first pass.

Proof 1 From Property 1 and 2, odd (even) states can only transition to the same last (first) \( 2^{BL} \) states. Thus, together with Property 3, two different odd (even) states transition to the same next state for a given input. From Property 4, \( x(1) \) determines if \( s(1) \) is odd or even and \( x(2) \) drives all odd (even) \( s(1) \) to the same \( s(2) \). It follows that any input sequence with length at least two will reach the same state regardless of its starting state. By finding out \( s(M/L) \) in the first encoding run and setting \( s(0) = s(M/L) \) in the second encoding run, we guarantee the tail-biting behavior.

Proposition 2 To decode a tail-biting path on the trellis described, two Viterbi passes through the trellis are sufficient.

Proof 2 As discussed in the proof to Proposition 1, given an input sequence, any odd \( s(0) \) results in the same \( \{y(n)\}_{n=1}^{M/L} \). Likewise, any even \( s(0) \) results in the same \( \{y(n)\}_{n=1}^{M/L} \) (but different from the sequence with odd \( s(0) \)). To decode, first run a conventional
Viterbi algorithm with an arbitrary odd initial state \( s(0) \). Record the path with the optimal metric that terminates in one of the odd states \( (s(M/L) \text{ is odd}) \). Repeat for an arbitrary even \( s(0) \) and record the path with optimal metric terminating in one of the even states \( (s(M/L) \text{ is even}) \). The path with the more optimal metric amongst the two paths is outputted.

From Propositions 1 and 2, it is clear that the quantization complexity doubles when using a tail-biting trellis. Hence the asymptotic complexity is preserved and still scales linearly with \( M \).

### 5. Dynamic Bit Allocation

In the trellis-based beamforming vector quantization methods discussed so far, the power of each \( L \)-dimensional segment is the same. If \( L = 1 \), this results in an equal gain vector. In coordinated transmission applications, it is normal for some transmission sites to be closer to the receiver than other sites, resulting in a difference in average received power. In the well-known maximal ratio transmission (MRT) scheme, the optimal precoder has higher power where the channel has higher power. Assuming both the coordinating transmitters and the receiver have knowledge of the long term average channel has higher power. Assuming both the coordinating transmitters and the receiver have knowledge of the long term average channel power, the resultant beamforming vector can be scaled according to the profile. Specifically, for an \( M \)-dimensional channel vector \( h = (h_1, ..., h_M) \), we create a power profile \( P_1, ..., P_M \) where \( P_i = E(|h_i|^2) \). If it is possible to feedback the power profile in a separate channel, then the feedback receiver can modify the precoder estimate \( \hat{w} \) by

\[
\tilde{w} = \text{diag}(\sqrt{P_1}, ..., \sqrt{P_M})\hat{w}.
\]

However, feeding back the exact profile requires a lot of feedback resources. We propose to take a different approach by adaptively assign the \( BM \) available quantization bits to different parts of the vector. Intuitively, more bits are allocated to quantize the channel elements with higher power than the channel elements with lower power. Specifically, for every power profile we create the corresponding bit profile \( B_1, ..., B_{M/L} \), where the bit allocation satisfies

\[
\sum_{i=1}^{M/L} B_i = \frac{M}{L} B.
\]

The bit profile can be determined by a number of methods, including maximizing the expected SNR at the receiver or maximizing the expected reliable transmission rate. We can always re-order channel elements to quantize parts with higher power first. Hence defining the association power ratio and bit profile is equivalent to defining regions for the power ratio for each bit profile. We leave the detailed analysis and design for future studies.

There are two key benefits with using Dynamic Bit Allocation. First, the SNR at the receiver improves due to the adaptability of quantization bit allocation. We will present numerical benefits of the scheme in the Simulations section. It is well known that the gain provided by extra feedback bits diminishes as total available feedback bits increases. Hence this scheme is most beneficial for small \( BM \). Also, it is easy to see that Dynamic Bit Allocation works well when the ratio of average power amongst channel elements is larger.

The second benefit is the increased system parameter flexibility. Consider two base-station coordinating where each one has four antennas (\( M = 8 \)), allocating a total of 6 bits of feedback (\( B = 0.75 \)). Without Dynamic Bit Allocation, one cannot use a segment length of \( L = 2 \) to reduce complexity since \( BL = 1.5 \) is not an integer.

With Dynamic Bit Allocation, it is possible to use \( L = 2 \) by assigning 4 bits to the higher powered channel elements and 2 bits to the lower powered ones.

Dynamic Bit Allocation has a side effect of distorting the uniformity of the trellis by having varying number of states and transitions. The notion of tail-biting needs to be modified for this trellis. Recall that, due to the special trellis structure, any of the odd (even) starting states are equivalent. We modify the definition of a tail-biting path to a path where both starting and ending states are odd, or both are even. It is easy to see that encoding and decoding still only need two passes through the trellis.

Even though it is possible to work with a non-uniform trellis, we propose a suboptimal simplification method to maintain uniformity and to reduce complexity by using parallel transitions to keep the number of states to four for all stages. The following steps describe the simplification process:

1. For a trellis stage with \( 2^{BL+1} \) states, group them into four states. Specifically, \( G_1 = \{ s | s \text{ is odd}, s \leq 2^{BL} \} \), \( G_2 = \{ s | s \text{ is even}, s \leq 2^{BL} \} \), \( G_3 = \{ s | s \text{ is odd}, s > 2^{BL} \} \), and \( G_4 = \{ s | s \text{ is even}, s > 2^{BL} \} \). These are the four states.

2. If \( s_n \in G_1, s_0 \in G_3 \), and \( s_n \rightarrow s_0 \) with output \( y \), then \( G_1 \rightarrow G_3 \) with output \( y \). After eliminating repeats (identical state transition and outputs), each state will emanate \( 2^BL \) branches.

Fig. 3 shows an example of the trellis simplification process. The top trellis has four stages. The first two stages have \( B_1 L = 2 \) and the last two stages have \( B_2 L = 2 \). The simplified trellis has four states at each stage and the states are color-coded to represent the four state groupings.

### 6. Simulations

In this section, the performance of the proposed schemes are compared using Monte Carlo simulations. Fig. 4 compares the trellis quantization scheme in [2] against the tail-biting trellis scheme proposed in this paper. We choose \( M = 8 \) antennas to simulate the case for two cooperating base stations with 4 antennas each. In this simulation, \( L = 2 \) and \( BM = 4 \). The channel model is i.i.d. Rayleigh
7. CONCLUSION

In this paper, we proposed two enhancements to the trellis-based vector quantization scheme proposed in [1, 2]. The first enhancement incorporates tail-biting trellis in the trellis-based quantization problem. Special trellis with efficient two-pass encoding and decoding algorithms are shown. The second enhancement leverage the fact that different antenna elements often experience different average power due to differing path loss. By dynamically allocating available quantization bits to different parts of the vector, the scheme adapts to achieve higher quantization performance. A simplified trellis with four states is presented to reduce complexity and maintain uniformity across trellis stages.

8. REFERENCES


