B-SPLINE BASED JOINT CHANNEL AND FREQUENCY OFFSET ESTIMATION IN DOUBLY-SELECTIVE FADING CHANNELS

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ABSTRACT
In this paper, a joint data-aided channel and frequency offset estimator is proposed for doubly-selective fading channels. The joint estimator is based on the B-spline model approximating the fading process and the dichotomous search frequency estimation technique. The estimator relies on the Bayesian approach. It is examined for different scenarios in Rayleigh fading channels. Simulation results show that the proposed estimator achieves a high accuracy performance, which is close to that with perfect knowledge of the frequency offset, over a wide range of signal to noise ratios, for different Doppler frequencies and throughout all the frequency acquisition range.

Index Terms— Joint channel and frequency offset estimation, doubly-selective fading channel, B-spline, Bayesian estimation, dichotomous search

1. INTRODUCTION
Accurate channel estimation is very important in communication systems, where reliable transmission is required. This is challenging in frequency-selective and time-variant fading channels, especially in the presence of a frequency offset.

Most of the frequency offset estimators proposed in the literature have been devoted to correlation-based estimation, such as [1, 2] for frequency-selective time-invariant channels and [3, 4, 5] for frequency-flat time-variant fading channels. However, the performance of such estimators is inferior to that of the estimator based on the generalized periodogram [6], and unlike that estimator, they are operable only for high signal to noise ratios (SNR) and/or they possess a limited frequency acquisition range [7, 8]. Periodogram-based joint channel and frequency offset estimation for frequency-flat time-variant fading channels has been considered in [9, 10], where joint estimators exploiting basis expansion model (BEM) of the channel time variations have been proposed. BEMs have been widely used for frequency-flat time-variant channel estimation [11, 12, 13]. However, these estimators yield a severe degradation in the performance in the presence of a frequency offset. Joint channel and frequency offset estimation for frequency-selective time-invariant channels has been addressed in [14]. For doubly-selective fading channels, BEM-based channel estimation has been considered in [15]. The estimation of doubly-selective fading channels in the presence of a frequency offset for multicarrier systems, based on complex exponential BEM, has been addressed in [16].

In this paper, we focus on estimating jointly the doubly-selective fading channels and frequency offset by using B-spline BEM. The joint estimation allows more efficient and higher accuracy performance, compared to techniques dealing separately with these two problems. The proposed estimator is based on representing the fading process by BEMs and employing frequency estimation based on the dichotomous search [17] of the generalized periodogram peak.

2. SIGNAL AND CHANNEL MODELS
We consider a known (pilot) signal transmitted through a doubly-selective fading channel. The received signal and channel models, respectively, can be expressed as

\[ r(nT_s) = \sum_{l=0}^{L-1} s(nT_s - \tau_l)h_l(nT_s) + z(nT_s), \] (1a)

\[ h_l(nT_s) = g_l(nT_s)e^{j2\pi f_0 n}, \quad n = 0, 1, \ldots, N - 1, \] (1b)

where \( s(nT_s) \) is the transmitted pilot symbol, the first \( L - 1 \) \( \{s(nT_s)\}_{n=0}^{N-1} \) of which are the precursors, \( z(nT_s) \) is the complex-valued additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma^2 \), \( \tau_l \) and \( g_l(nT_s) \) are the \( l \)th path delay and fading process, respectively, \( f_0 \) is the true frequency offset, \( f_0T_s \triangleq F_0 \) is the normalized frequency offset (termed here as the frequency offset), \( T_s \) is the symbol interval; \( L \) and \( N \) are the number of paths and received symbols, respectively.

The paths are assumed to be independent and the fading of each path follows Jakes’ model [18]. The covariance matrix of such fading process is given by

\[ R_u = P_L \otimes R_\sigma, \quad \text{with} \quad R_\sigma = J_0(2\pi f_0 T_s (u - v)), \quad u, v = 1, 2, \ldots, N, \] (2b)

where \( P_L = \text{diag}\{\sigma^2g_l^2\}_{l=0}^{L-1} \) is the power delay profile, \( \sigma^2 \) is the variance of the \( l \)th path, \( J_0(\cdot) \) denotes the Kronecker product, \( J_0(\cdot) \) is the zero-order Bessel function of the first kind, \( f_0T_s \) is the true Doppler frequency and \( f_0T_s \) is the normalized Doppler frequency (termed here as the Doppler frequency).

The received signal and channel models, respectively, in matrix form can be written as

\[ r = Sh + z, \] (3a)

\[ h = A_{f_0}g, \quad A_{f_0} = I_L \otimes \overline{X}_{f_0}, \] (3b)

where \( r \) and \( z \) are \( N \times 1 \) received signal and noise vectors with elements \( r(nT_s) \) and \( z(nT_s) \), respectively, \( S = [\text{diag}(s_0), \ldots, \text{diag}(s_{L-1})] \) is an \( N \times NL \) pilot matrix, \( s_l \) is a \( 1 \times N \) vector with elements \( s(nT_s - \tau_l) \), \( h = [h_0^T, \ldots, h_L^T, \ldots, h_{L-1}^T]^T \) is an \( NL \times 1 \) channel response vector. \(^T\) denotes the matrix transpose, \( h_l \) is an \( N \times 1 \) vector with elements \( h_l(nT_s) \), \( g = [g_0^T, \ldots, g_L^T, \ldots, g_{L-1}^T]^T \) is an \( NL \times 1 \) fading process vector, \( g_l \) is an \( N \times 1 \) vector with elements \( g_l(nT_s) \), \( I_L \) is an \( L \times L \) identity matrix and \( \overline{X}_{f_0} = \text{diag}\{e^{j2\pi f_0 n}\} \) is the frequency offset matrix.
3. JOINT CHANNEL AND FREQUENCY OFFSET ESTIMATION

Accurate estimation of the fading process \( g_i(nT_s) \) requires complicated techniques such as the Wiener filtering [19]. A simpler solution can be obtained based on representing \( g_i(nT_s) \) using a basis expansion model (BEM) with \( M \) basis functions as

\[
\tilde{g}_l(nT_s) = \sum_{m=1}^{M} a_l(m) B(nT_s, m), \quad l = 0, 1, \ldots, L - 1,
\]

where \( B(nT_s, m) \) are basis functions and \( a_l(m) \) are unknown expansion coefficients. In matrix form, it can be written as

\[
\mathbf{\tilde{g}} = \mathbf{B} \mathbf{a}, \quad \mathbf{B} = \mathbf{I}_L \otimes \mathbf{B},
\]

where \( \mathbf{B} \) is an \( N \times M \) basis function matrix with elements \( B(nT_s, m) \), \( \mathbf{a} = [a_0, a_1^T, \ldots, a_{L-1}^T] \) is an \( ML \times 1 \) expansion coefficient vector and \( \mathbf{a} \) is an \( M \times 1 \) vector with elements \( a_l(m) \).

Thus, the estimation problem of the \( L \times N \) time-variant fading process \( g_i(nT_s) \) is transformed into estimation of the \( L \times M \) time-invariant expansion coefficients \( a_l(m) \). Usually \( M < N \), which makes the BEM-based approach attractive. The model mismatching error due to the approximation of the fading process can be neglected when choosing \( M \) high enough, so that \( g \) can be assumed equal to \( \tilde{g} \); this is a practical assumption as detailed in [11, 12, 20]. Thus, the signal model can be now regarded as

\[
\mathbf{r} = \mathbf{X}_P \mathbf{\Phi} \mathbf{a} + \mathbf{z}; \quad \mathbf{\Phi} = \mathbf{SB}, \quad \mathbf{h} = \mathbf{A}_p \mathbf{B}.
\]

The vector \( \mathbf{a} \) is assumed to be zero mean Gaussian with \( a \) priori PDF \( p(\mathbf{a}) = \pi^{-M} |\mathbf{R}_a|^{-1} e^{-\frac{1}{2} \mathbf{a}^H \mathbf{R}_a^{-1} \mathbf{a}} \), and \( \mathbf{R}_a \) is the covariance matrix of \( \mathbf{a} \) that can be obtained as [13]

\[
\mathbf{R}_a = \mathbf{P}_L \otimes \mathbf{R}_a, \quad \mathbf{R}_a = \left( \mathbf{B}^H \mathbf{B} \right)^{-1} \mathbf{B}^H \mathbf{B} \mathbf{R}_a \left( \mathbf{B}^H \mathbf{B} \right)^{-1}.
\]

By using the Bayesian approach (as detailed in [8, 9]), we arrive at the following frequency estimator

\[
\hat{F}_0 = \arg \max_{F \in \Psi} \{ Y_F \} = \arg \max_{F \in \Psi} \left\{ \mathbf{r}^H \mathbf{X}_P \mathbf{SB} \left( \mathbf{B}^H \mathbf{S}^H \mathbf{SB} + \sigma^2 \mathbf{R}_a \right)^{-1} \times \mathbf{B}^H \mathbf{S}^H \mathbf{X}_P^H \mathbf{r} \right\},
\]

where

\[
Y_F = \mathbf{L}_F^H \left( \mathbf{G} + \sigma^2 \mathbf{R}_a^{-1} \right)^{-1} \mathbf{L}_F, \quad \mathbf{G} = \mathbf{\Phi}^H \mathbf{G} \mathbf{\Phi} \mathbf{S}^H \mathbf{SB},
\]

\[
\mathbf{L}_F = \mathbf{\Phi}^H \mathbf{X}_P^H \mathbf{r} = \mathbf{B}^H \mathbf{S}^H \mathbf{X}_P^H \mathbf{r},
\]

(\cdot)^H denotes the Hermitian transpose, \( Y_F \) is the generalized periodogram [6, 8], and \( \Psi = [-\psi/2, \psi/2] \) is the frequency acquisition range that can be considered either wide \( (\psi = 1) \) or narrow \( \psi << 1 \).

After obtaining \( \hat{F}_0 \) and substituting it in (6), the minimum mean square error (MMSE) estimator of the vector \( \mathbf{a} \) is given by

\[
\hat{\mathbf{a}} = \left( \mathbf{G} + \sigma^2 \mathbf{R}_a^{-1} \right)^{-1} \mathbf{L}_F \hat{F}_0.
\]

Finally, the Bayesian joint channel and frequency offset (BJ) estimator is obtained by substituting (10) into (6b) as

\[
\hat{\mathbf{h}} = \mathbf{A}_p \mathbf{B} \left( \mathbf{B}^H \mathbf{S}^H \mathbf{SB} + \sigma^2 \mathbf{R}_a \right)^{-1} \mathbf{B}^H \mathbf{S}^H \mathbf{X}_P^H \mathbf{r}.
\]

Table 1. Joint channel and frequency offset estimator.

Compute \( \mathbf{G} = \left( \mathbf{B}^H \mathbf{S}^H \mathbf{SB} + \sigma^2 \mathbf{R}_a \right)^{-1} \),

Calculate \( D^{(m)}_F(l) = \sum_{n=0}^{N-1} r(nT_s)^* (nT_s - \tau) B^* (nT_s, m) e^{-j2\pi F_n} \)

Rearrange \( L_F = [D_F(0), \ldots, D_F(l), \ldots, D_F(L-1)]^T \)

Determine \( Y_F = \sum_{l=1}^{M} \sum_{m=1}^{L} \mathbf{G}_l \mathbf{P}_l (u) L_F(v) \)

Find \( F_p \) arg max \( \{ Y_F \} \)

Locate \( Y_1 = Y_{F_p-1}, \quad Y_2 = Y_{F_p}, \quad Y_3 = Y_{F_p+1} \)

For \( Q \) iterations do

\[
\Delta F = \Delta F/2
\]

If \( Y_3 < Y_1 \) then \( Y_3 = Y_2 = F_p = F_p - \Delta F, \)

else \( Y_1 = Y_2 = F_p = F_p + \Delta F \),

\[
D^{(m)}_{F_p}(l) = \sum_{n=0}^{N-1} r(nT_s)^* (nT_s - \tau) B^* (nT_s, m) e^{-j2\pi F_n} \]

\( L_{F_p} = [D_{F_p}(0), \ldots, D_{F_p}(l), \ldots, D_{F_p}(L-1)]^T \)

\( Y_2 = \sum_{l=1}^{M} \sum_{m=1}^{L} \mathbf{G}_l \mathbf{P}_l (u) L_{F_p}(v) \)

Finally \( \hat{F}_0 = F_p, \quad \mathbf{h} = \mathbf{A}_p \mathbf{B} \mathbf{G} \mathbf{L}_{F_p} \)

4. EFFICIENT IMPLEMENTATION

Most of the complexity in the proposed estimator is consumed by the frequency offset estimation part when calculating \( L_{F_p} \) in (9c) that is used for evaluation of the generalized periodogram \( Y_F \) in (9a). For a coarse evaluation (search), FFT of a size \( N_{FPT} \geq N \) with a frequency step \( \Delta F = 1/N_{FPT} \) can be used. For a fine search, we use the dichotomous search [17]. This approach is free of nonlinear operations and well suited for real-time implementation [7]. The proposed estimator is summarized in Table 1.

5. SIMULATION RESULTS

Different basis functions can be used in the BEM such as complex exponential [11, 16], Karhunen-Loève [21], discrete prolate spheroidal- al [12] and B-splines [13]. It is shown in [10] that the channel estimation based on the B-splines is less sensitive to the accurate knowledge of statistical characteristics of the fading and simpler for implementation than that based on the other BEMs. The B-splines of order \( \eta \) are symmetrical, bell-shaped functions that are given by [22]

\[
B_{\eta}(x) = \frac{1}{\eta!} \sum_{i=0}^{\eta+1} (-1)^i \binom{\eta + 1}{i} \left( \frac{x}{PT_s} + \eta + 1 \right)^i
\]

where \( \eta = (N-1)/(M-\eta), \) \( PT_s \) is the sampling interval separating two adjacent B-spline functions, and \( \chi = \max \{0, x \} \). In this case, \( B(nT_s, m) = B_{\eta} \left[ nT_s - (m - \frac{N-1}{2})PT_s \right] \). The accuracy and complexity of B-spline approximation depends on the spline degree \( \eta \). In many situations, the cubic B-spline (\( \eta = 3 \)) provides the best trade-off between complexity and accuracy [22]. We use the cubic B-spline in the simulation below whenever \( M \geq 4 \). However, other BEMs can also be used in the joint estimator.

We consider a binary pseudo-random pilot signal for which the signal to noise ratio (SNR) is calculated as

\[
\text{SNR} = \frac{\mathbb{E} \left\{ (\mathbf{h}^H \mathbf{S}_g)^H (\hat{\mathbf{h}}^H \mathbf{S}_g) \right\}}{\mathbb{E} \left\{ \mathbf{z}^H \mathbf{z} \right\}} = \frac{\epsilon}{\sigma^2_h},
\]

where \( \epsilon = \sum_{n=0}^{L-1} \sigma^2_{h,n} \). The mean square error (MSE) of estimation is averaged over 10,000 simulation trials, where the MSE of the frequency offset and the channel estimation, respectively, in each
simulation trial are calculated as

\[
\begin{align}
\text{f-MSE} &= \left( F_0 - \hat{F}_0 \right)^2, \\
\text{h-MSE} &= \frac{\sum_{l=0}^{L-1} \sum_{n=0}^{N-1} |h_l(nT_s) - \hat{h}_l(nT_s)|^2}{\sum_{l=0}^{L-1} \sum_{n=0}^{N-1} |h_l(nT_s)|^2}. \tag{14b}
\end{align}
\]

We consider a doubly-selective fading channel that has \( L = 5 \) paths, with an exponentially decaying power delay profile. The frequency acquisition range is wide (\( \psi = 1 \)). Unless otherwise specified, the size of FFT in the coarse search is \( N_{FFT} = N = 128 \) and the number of dichotomous iterations in the fine search is \( Q = 5 \).

The proposed estimator is compared to an ideal reference Bayesian channel (RBC) estimator, where the frequency offset is assumed to be known. This estimator is given as in (11) but with \( \hat{F}_0 \) being replaced with \( F_0 \).

Fig. 1 shows the \( M \)-dependent h-MSE in the slow (\( f_D T_s = 0.005 \)), moderate (\( f_D T_s = 0.02 \)) and fast (\( f_D T_s = 0.05 \)) fading channels. There is a threshold \( M \), below which the error rapidly increases due to a high modeling mismatch error, and above which the error stays almost constant. The exploitation of the fading covariance matrix and the noise variance prevents a degradation in the performance for high \( M \). For a higher \( f_D T_s \), the estimator requires a higher \( M \) to achieve its best performance. This \( M \) can be determined such that the sampling factor \( \gamma = 1/(f_D T_s P) \) is approximately 5, which is defined by approximating properties of B-splines [20].

Fig. 2 shows the \( f_D \)-dependent h-MSE for different \( M \). The best performance is achieved for \( f_D T_s \) smaller than a threshold that increases with \( M \). It is seen from Fig. 1 and Fig. 2 that the proposed BJ estimator achieves a high accuracy performance which is close to that of the ideal RBC estimator. It even outperforms the RBC for small \( M \). Thus, the frequency offset (nonlinear) estimation involved in the BJ estimator helps in reducing the modeling mismatch error, even when \( f_D T_s = 0 \) (according to simulation results not shown here).

Fig. 3 shows the SNR-dependent h-MSE for different \( f_D T_s \). We notice a threshold SNR, below which the h-MSE of the BJ estimator diverges slightly from that of the RBC estimator. This characteristic appears due to the involvement of the nonlinear frequency estimation and the occurrence of the outliers [23].

Fig. 4 shows the \( f_D \)-dependent MSE performance for different \( N_{FFT} \) and \( Q \). For the frequency estimation alone, \( N_{FFT} = 4N = 512 \) and \( Q = 8 \) is a necessity for a high accuracy. However, the BJ estimator does not require that high accuracy in the frequency offset estimation and can achieve a good joint estimation (h-MSE) performance with a significantly lower \( N_{FFT} \) (with as small \( N_{FFT} \) as \( N_{FFT} = N \)) and using a few dichotomous iterations. For \( f_D T_s = 0.05 \), an RBC-like performance is achieved using as small \( Q \) as
A novel data-aided joint channel and frequency offset estimator has been proposed for doubly-selective fading channels. This estimator is based on approximating the fading process using the B-spline model. This model simplifies the solution and allows the estimator to achieve a high accuracy performance. The joint estimator is based on the Bayesian approach and provides a high accuracy performance when some prior statistical characteristics of the channel are known, namely the covariance matrix of the fading and the variance of the AWGN. To reduce the complexity of the frequency offset estimation, a two stage technique is exploited for searching a peak of the generalized periodogram, an FFT-based coarse search and dichotomous fine search. Simulation results for different scenarios in Rayleigh fading channels have shown that the proposed estimator maintains, over wide SNR, frequency offset and Doppler frequency ranges, a high accuracy performance, which is very close to that of the Bayesian channel estimator operating with perfect knowledge of the frequency offset.

7. REFERENCES


