ABSTRACT
The problem of channel shortening equalization (CSE) for multi-input multi-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems is addressed. Relying on constrained minimization of the mean-output-energy (MOE) at the CSE output, we propose a blind channel shortening technique, which does not require a priori knowledge or estimation of the MIMO channel impulse responses to be shortened. Numerical results showing the attainable performance gain of the proposed method are reported.

Index Terms— Blind channel shortening, orthogonal frequency-division multiplexing (OFDM), multiple-input multiple-output (MIMO) systems.

1. INTRODUCTION
The combination of orthogonal-frequency division multiplexing (OFDM) with multiple-input multiple-output (MIMO) technology has gained a lot of interest, since it leads to very computationally-efficient equalization strategies [1]. Such techniques rely on the fact that, to counteract interblock and intercarrier interference, a cyclic prefix (CP), whose length $L_{cp}$ is equal to or greater than the maximum order $L_0$ of the MIMO channel impulse responses, is inserted at the beginning of each OFDM block. Over highly frequency-selective channels, however, fulfillment of condition $M_p \geq L_0$ might be impractical, since it can lead to significant reduction of the achievable system throughput. This drawback can be overcome by means of channel shortening [2], a preprocessing technique which shortens all the underlying MIMO channel impulse responses (CIRs) into a time window of length $L_{eff} \leq M_p$, thus allowing one to employ subsequent conventional OFDM equalization. In a MIMO scenario, channel shortening has been tackled in [3], where a finite-impulse response (FIR) channel shortening equalization (CSE) technique, designed according to the minimum-mean-square-error (MMSE) criterion, has been proposed. However such a technique is not blind, in the sense that a priori knowledge of all the CIRs to be shortened is required. When channel state information is not available at the receiver side, in order to avoid the use of long training sequences, required to reliably estimate highly time-dispersive channels, one can resort to blind channel shortening approaches. In the multicarrier (MC) single-input single-output (SISO) context, Martin et al. [4] have proposed a blind, adaptive CSE algorithm, which, even though is low-complexity and globally convergent, requires a large number of data records to converge. Recently, such an algorithm has been generalized to the MIMO case in [6]. On the other hand, although the method [5] converges much faster than [4], its global convergence is not ensured and its implementation is computationally intensive. With reference to MC single-input multiple-output (SIMO) transceivers, in [7] a blind time-domain equalization (TEQ) technique, based on a minimum mean-output-energy (MOE) criterion, has been investigated. The TEQ developed in [7], whose synthesis can be carried out adaptively, also exhibits narrowband interference suppression capabilities. However, the design proposed in [7] is targeted at a single-antenna transmitter and, hence, does not exploit spatial diversity arising from the availability of multiple transmit antennas. In this paper, we consider the extension to MIMO-OFDM systems of the blind channel shortening technique presented in [7]. In particular, relying on constrained minimization of the MOE at the CSE output, we propose a FIR TEQ, which simultaneously shortens the multiple channels, without requiring any a priori knowledge of the MIMO channels.

Notations: Upper- and lower-case bold letters denote matrices and vectors; the superscripts $T$, $H$, $-1$ and $\dagger$ denote the transpose, the Hermitian (conjugate transpose), the inverse and the Moore-Penrose inverse of a matrix; $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Z}$ are the fields of complex, real and integer numbers; the field of $m \times n$ complex [real] matrices is denoted as $\mathbb{C}^{m \times n}$ [$\mathbb{R}^{m \times n}$], with $\mathbb{C}^n$ [$\mathbb{R}^n$] used as a shorthand for $\mathbb{C}^{n \times 1}$ [$\mathbb{R}^{n \times 1}$]; $\mathbf{0}_n$, $\mathbf{0}_{n \times m}$ and $\mathbf{I}_n$ denote the $n$-column zero vector, the $n \times m$ zero matrix and the $n \times n$ identity matrix; $\mathcal{N}(\mathbf{A})$, $\mathcal{R}(\mathbf{A})$, and $\mathcal{R}^\perp(\mathbf{A})$ denote the null space, the range (column space), and the orthogonal complement of the column space of $\mathbf{A} \in \mathbb{C}^{n \times n}$ [$\mathbb{R}^{n \times n}$] in $\mathbb{C}^n$ [$\mathbb{R}^n$], respectively; for any $\mathbf{a} \in \mathbb{C}^n$, $\|\mathbf{a}\|$ denotes the Euclidean norm; $\mathbb{E}[\cdot]$ denotes ensemble averaging.

2. SYSTEM MODEL
We consider a MIMO-OFDM system, with $M$ subcarriers, CP length $L_{cp}$, $N_T$ transmit antennas, and $N_R$ receive antennas. The information-bearing data symbols, with zero mean and variance $\sigma_r^2$, are subject to (s.t.) conventional OFDM precoding, encompassing Inverse Discrete Fourier Transform (IDFT) and CP insertion 1. After digital-to-analog (D/A) filtering and up-conversion, the continuous-time signal transmitted from the $\alpha$th antenna propagates through the physical channel modeled as a linear time-invariant filter. Assuming perfect timing and frequency synchronizion, let $N_S \geq 1$ be the temporal oversampling factor, the data block $\mathbf{y}_r[n] \in \mathbb{C}^{N_S}$ received at the $r$th antenna, with $r \in \{0, 1, \ldots, N_R-1\}$, is given [3] by

$$
y_r[n] = \sum_{\alpha=0}^{N_T-1} \sum_{\ell=0}^{L_{cp}} \mathbf{h}_{r,\alpha}[\ell] u_{\alpha}[n-\ell] + v_r[n],
$$

Even though we do not consider space-time-frequency block coding (STFBC) herein, the proposed blind channel shortening method can work with any standard STFBC rule.
where $h_{r,\alpha}[n] \in \mathbb{C}^{NS}$ gathers the samples of the baseband equivalent channel impulse response between the $\alpha$th transmit antenna, with $\alpha \in \{0, 1, \ldots, NT - 1\}$, and the $r$th receive antenna, $u_r[n]$ ($n \in \mathbb{Z}$) represents the useful signal transmitted from the $r$th antenna, $v_r[n] \in \mathbb{C}^{N_R}$ is the noise vector at the $r$th antenna, and $L_h$ is the maximum (with respect to $r$ and $\alpha$) channel order. Eq. (2.1) can equivalently be expressed as

$$y_r[n] = \sum_{\ell=0}^{L_h} H_r[\ell] u[n-\ell] + v_r[n], \quad (2.2)$$

where $H_r[\ell] \triangleq (h_{r,0}[\ell], h_{r,1}[\ell], \ldots, h_{r,N_T-1}[\ell]) \in \mathbb{C}^{N_R \times N_T}$ and $u[\ell] \triangleq (u_0[\ell], u_1[\ell], \ldots, u_{N_T-1}[\ell])^T \in \mathbb{C}^{N_T}$. By grouping the data block received from the $N_R$ antennas into the larger vector $y[n] \triangleq (y_0^T[n], y_1^T[n], \ldots, y_{N_R-1}^T[n])^T \in \mathbb{C}^{N_R^2}$, with $N_G \triangleq NS N_R$, one has

$$y[n] = \sum_{\ell=0}^{L_h} H[n][\ell] u[n-\ell] + v[n], \quad (2.3)$$

where $H[n][\ell] \triangleq (H_0[\ell], H_1[\ell], \ldots, H_{N_R-1}[\ell])^T \in \mathbb{C}^{N_R \times N_T}$ and $v[n] \triangleq (v_0[n], v_1[n], \ldots, v_{N_R-1}[n])^T \in \mathbb{C}^{N_R}$. In the sequel, we assume that the CP length is insufficient, i.e., $M_Q < L_h$; in this case, the effect of frequency-selective fading cannot be completely eliminated by CP removal, and interblock (IBI) as well as intercarrier interference (ICI) will be introduced. To overcome this drawback, TEQs [3] are employed to shorten all the underlying CIRs into a certain time window of dimension $L_{eff} \leq M_Q$, thus allowing the use of conventional OFDM demodulation, i.e., CP removal and one-tap frequency-domain equalization. Let $L_\alpha$ be the equalizer order, the input-output relationship of a TEQ can be compactly expressed as $z[n] = \mathcal{F} \tilde{x}[n]$, where $\mathcal{F} \in \mathbb{C}^{N_R \times [N_G(L_e+1)]}$ is a channel shortening matrix to be optimized, and the vector $z[n] \triangleq (y^T[n], y^T[n-1], \ldots , y^T[n-L_e])^T \in \mathbb{C}^{N_G(L_e+1)}$ is given by

$$z[n] = \mathcal{H}_e \eta[n] + w[n], \quad (2.5)$$

with $\mathcal{H}_e$ denoting the $N_C(L_e+1) \times N_T(L_e+L_h+1)$ block Toeplitz complex channel matrix reported at the top of the page, $\eta[n] \triangleq (u^T[n], u^T[n-1], \ldots , u^T[n-L_e-L_h])^T \in \mathbb{C}^{N_T(L_e+L_h+1)}$, and $w[n] \triangleq (v^T[n], v^T[n-1], \ldots , v^T[n-L_e])^T \in \mathbb{C}^{N_T(L_e+1)}$. Observe that, by virtue of (2.5), the TEQ output can also be written as $\tilde{x}[n] = G \eta[n] + d[n]$, where the composite matrix $G \triangleq \mathcal{F} \mathcal{H}_e$ collects the taps of the combined channel-TEQ impulse responses and $d[n] \triangleq \mathcal{F} w[n]$ is the noise contribution. Matrix $\mathcal{F}$ must be chosen in order to obtain a channel-TEQ response matrix $G$ having the target form

$$G_{target} = \{O_{NR \times (N_T \Delta)}, G(\Delta), O_{NR \times [N_T(L_e+L_h-L_e-L_h)]}\}, \quad (2.6)$$

where $0 \leq \Delta \leq L_e + L_h - L_{eff}$ and $L_{eff}$ is a suitable shortening delay at the designer’s disposal, and $L_{eff}$, which expresses the effective order of the combined channel-TEQ impulse responses, obeys $L_{eff} \leq M_Q < L_h$ in order to achieve perfect IBI cancelation through CP insertion.\footnote{We have assumed that the TEQs introduce the same delay $\Delta$ for the symbols from all the transmit antennas. Even though a better design could be developed by allowing different delays across the inputs, the computational complexity would be correspondingly higher.}

Hereinafter, we assume that: (a) $\eta[n]$ in (2.5) is a zero-mean complex circular wide-sense stationary (WSS) random vector, with $R_{\eta \eta} \triangleq \mathbb{E}(\eta[n] \eta^H[n]) \in \mathbb{C}^{N_T(L_e+L_h+1) \times N_T(L_e+L_h+1)}$ positive-definite; (a2) $w[n]$ in (2.5) is statistically independent of $\eta[n]$ and is modeled as a zero-mean circularly symmetric complex Gaussian random vector with $\mathbb{E}(w[n] w^H[n]) = \sigma_w^2 I_{N_G(L_e+1)}$.

### 3. The Proposed Channel Shortening Approach

Pursuing a minimum mean-output-energy (MMOE) approach, the channel shortening matrix $\mathcal{F}$ is designed so as to minimize the cost function

$$\text{MOE}(\mathcal{F}) \triangleq \mathbb{E}(\|\tilde{x}[n]\|^2) = \text{trace}(\mathcal{F} R_{zz} \mathcal{F}^H), \quad (3.1)$$

under a suitable constraint aimed at preventing the trivial solution $\mathcal{F} = O_{NR \times [N_G(L_e+1)]}$, where $R_{zz} \triangleq \mathbb{E}(z[n] z^H[n]) \in \mathbb{C}^{N_G(L_e+1) \times N_G(L_e+1)}$. $R_{zz}$ denotes the autocorrelation matrix of the received data. Such a constraint must preserve a delayed version of the MIMO transmitted information-bearing vector $u[n]$; moreover, it must be chosen in a completely blind manner, i.e., without requiring knowledge or estimation of the CIRs to be shortened. In order to choose the constraint, we observe that the overall channel matrix (2.4) can be partitioned as $\mathcal{H} = (\mathcal{H}_0, \mathcal{H}_1, \ldots , \mathcal{H}_{L_e}, \mathcal{H}_{L_e+1})$, where $\mathcal{H}_d \in \mathbb{C}^{N_G(L_e+1) \times NT}$ with $d \in \{0, 1, \ldots , L_e, L_h\}$, denotes the $d$th block column of $\mathcal{H}$. Following [7], it can be readily proven that blocks $\mathcal{H}_d$, for $0 \leq d \leq \min \{L_e, L_h\}$, can be linearly parameterized as

$$\mathcal{H}_d = \Theta_d \Xi_d, \quad 0 \leq d \leq \min \{L_e, L_h\}, \quad (3.2)$$

with $\Theta_d \in \mathbb{R}^{N_G(L_e+1) \times [N_G(L_e+1)]}$, $\Xi_d \in \mathbb{R}^{N_G(L_e+1) \times N_G(L_e+1)}$ being the matrix constraint. Note that, with $\Theta_d \in \mathbb{R}^{N_G(L_e+1) \times [N_G(L_e+1)]}$, $\Xi_d \in \mathbb{R}^{N_G(L_e+1) \times N_G(L_e+1)}$ being the matrix constraint. Note that, thanks to the introduced parameterization, the constraint in (3.4) does not require any a priori knowledge about the CIRs; hence, the proposed CSE can be synthesized in a completely blind manner, requiring only estimation of $R_{zz}$ from the received data.

By exploiting the generalization to the matrix case of the well-known generalized sidelobe canceler (GSC) decomposition (see, e.g., [9]), the matrix $\mathcal{F}_{min}(\delta)$ can be equivalently expressed as

$$\mathcal{F}_{min}(\delta) = \mathcal{F}^{(f)}_{min}(\delta) + \mathcal{F}^{(a)}_{min}(\delta) \Pi_{\delta}, \quad (3.5)$$

where $\mathcal{F}^{(f)}_{min}(\delta)$ and $\mathcal{F}^{(a)}_{min}(\delta)$ are the frequency and amplitude parts of the CSE, respectively.
where \( F_{\text{mooe}}^{(f)}(\delta) = \Gamma_3 \Theta_3^T I_N = \text{data-independent}, \ F_{\text{mooe}}^{(o)}(\delta) = -\hat{F}_{\text{mooe}}^{(o)}(\delta) \mathbf{R}_{zz} \Pi \hat{\mathbf{R}}_{zz} \Pi^{T} \mathbf{R}_{zz}^{-1} \mathbf{R}_{zz}^{-1} = \text{data-dependent}, \) and the signal blocking matrix \( \Pi \in \mathbb{R}_{N_G \times (L_e - \delta) \times N_G (L_e + 1)} \) is chosen so that its columns form an orthonormal basis for the null space of \( \Theta_3 \), i.e., \( \Pi \Theta_3 = \mathbf{0}_{N_G (L_e - \delta) \times N_G (\delta + 1)} \).

Let us prove that, resorting to the MOE criterion (3.4), in the high-signal-to-noise-ratio (SNR) regime, the cascade

\[
G_{\text{mooe}}(\delta) \triangleq \mathbb{F}_{\text{mooe}}(\delta) \mathcal{H} \in \mathbb{C}^{N_R \times N_T} (L_e + L_h + 1)
\]

(3.6)
of the proposed channel shortening equalizer and the original MIMO channel (2.4) exhibits the target form (2.6). Substituting the expressions of \( \mathbb{F}_{\text{mooe}}^{(f)}(\delta) \) and \( \mathbb{F}_{\text{mooe}}^{(o)}(\delta) \) into (3.6), one obtains

\[
G_{\text{mooe}}(\delta) = \Gamma_3 \Theta_3^T ;
\]

(3.7)

where, accounting for (a2), \( \mathbf{R}_{zz} = \mathcal{H} \mathbf{R}_{yy} \mathcal{H}^H + \sigma_w^2 I_{N_G (L_e + 1)} \).

Replacing \( \mathbf{R}_{zz} \) into (3.7) and recalling that \( \Pi \Theta_3 = \mathbf{0}_{N_G (L_e - \delta)} \), one has

\[
G_{\text{mooe}}(\delta) = \Gamma_3 \Theta_3^T \left[ I_{N_G (L_e + 1)} - \mathcal{H} \mathbf{R}_{yy}^{1/2} \left( \Pi \mathcal{H} \mathbf{R}_{yy}^{1/2} \right)^{T} \right] \mathcal{H}.
\]

(3.8)

At this point, it can be verified that, in the high SNR region, \( \lim_{\delta \to 0} G_{\text{mooe}}(\delta) \triangleq G_{\text{mooe}}^{[0]}(\delta) \) becomes

\[
G_{\text{mooe}}^{[0]}(\delta) = \Gamma_3 \Theta_3^T \left[ I_{N_G (L_e + 1)} - \mathcal{H} \mathbf{R}_{yy}^{1/2} \left( \Pi \mathcal{H} \mathbf{R}_{yy}^{1/2} \right)^{T} \right] \mathcal{H}.
\]

(3.9)

where we have used the properties of Moore-Penrose inverses [8] and, in the second equality, we have exploited the fact that \( (\Pi \mathcal{H} \mathbf{R}_{yy}^{1/2} \Pi)^T = \mathbf{R}_{yy}^{1/2} (\Pi \mathcal{H})^T \), due to the nonsingularity of \( \mathbf{R}_{yy} \). Since \( \Pi \mathcal{H} \) represents the orthogonal projector [8] on the null space of \( \mathcal{H} \), it is apparent from (3.9) that \( G_{\text{mooe}}^{[0]}(\delta) \) belongs to the subspace \( \mathcal{V} (\Pi \mathcal{H}) \equiv \mathcal{R}^\perp (\mathcal{H}^T \Pi)^{T} \).

Relying on (3.9), we can enunciate the following Theorem, whose proof, similar in spirit to that provided in [7], is omitted for brevity:

**Theorem 3.1** Given \( 0 \leq \delta \leq \min \{ L_e, L_h \} \), let the matrix \( S_{\delta} \in \mathbb{C}^{N_G (L_e + 1) \times N_G (L_e + L_h - \delta)} \) be defined as

\[
S_{\delta} \triangleq (\mathcal{H}_{L_e + 1}, \mathcal{H}_{L_e + 2}, \ldots, \mathcal{H}_{L_e + L_h})
\]

Assuming \( N_G > N_T \), if \( \Pi \Theta_3 \mathbf{S}_{\delta} \) is full-column rank, i.e.,

\[
(\mathbf{c}1) \quad N_G (L_e - \delta) \geq N_T (L_e + L_h - \delta),
\]

\[
(\mathbf{c}2) \quad \text{rank}(\Pi \Theta_3 \mathbf{S}_{\delta}) = N_T (L_e + L_h - \delta),
\]

then, for \( \sigma_w^2 \to 0 \), the combined channel-TEQ matrix (3.8) assumes the target form (2.6) where \( \Delta = 0 \), \( L_{ef} = \delta \), and \( \mathcal{G}(\Delta) = \Gamma_3 \Theta_3^T [\mathcal{H}_0, \mathcal{H}_1, \ldots, \mathcal{H}_3] \).

Condition (c1) imposes the following upper bound \( L_{ab} \) on the maximum MIMO channel order:

\[
L_{ab} \triangleq (L_e - \delta) \cdot \frac{N_G - N_T}{N_T};
\]

(3.10)

therefore, for a given value of the pair \( (N_T, N_R) \) characterizing the MIMO architecture, we have three parameters at our disposal to maximize \( L_{ab} \): the TEQ order \( L_e \), the channel shortening delay \( \delta \), and the oversampling factor \( N_S \). In particular, it is worthwhile to comment on the choice of \( \delta \). It is apparent from (3.10) that, assuming \( N_G > N_T \), the upper bound \( L_{ab} \) linearly grows with \( L_e \). Therefore, one can conclude that the choice \( \delta = 0 \) is preferable, since, in this case, the proposed MMOE-TEQ is asymptotically able to shorten more selective channels as \( L_e \) increases. Nevertheless, simulation results, not reported here for brevity, show that, in a more realistic noisy scenario, the MMOE-TEQ performances improve as \( \delta \) grows, showing a remarkable performance degradation for values of \( \delta \) approaching zero.

At this point, the synthesis of the constraint matrix \( \Gamma_3 \) is considered. The degrees of freedom provided by \( \Gamma_3 \) can be exploited to maximize the shortening signal-to-noise-plus-interference ratio (SSINR) at the MOE-TEQ output. Accounting for (2.5) and invoking (a2), the SSINR can be defined as

\[
\text{SSINR} \triangleq \frac{E(||\hat{F}_{\text{mooe}}(\delta) \mathcal{H}_{\text{win}} \eta_{\text{win}}[n]||^2)}{E(||\hat{F}_{\text{mooe}}(\delta) \mathcal{H}_{\text{wall}} \eta_{\text{wall}}[n]||^2) + E(||\hat{F}_{\text{mooe}}(\delta) \mathcal{H} \mathbf{d}[n]||^2)},
\]

(3.11)

where

\[
\mathcal{H}_{\text{win}} \triangleq (\mathcal{H}_\Delta, \mathcal{H}_{\Delta + 1}, \ldots, \mathcal{H}_{\Delta + L_{ef}}) \subseteq \mathbb{C}^{N_G (L_e + 1) \times N_T (L_e + L_{ef} + 1)};
\]

\[
\eta_{\text{win}}[n] \triangleq (u^T[n - \Delta], u^T[n - \Delta - 1], \ldots, u^T[n - \Delta - L_{ef}])^T \subseteq \mathbb{C}^{N_T (L_e + 1)};
\]

\[
\mathcal{H}_{\text{wall}} \triangleq (\mathcal{H}_0, \mathcal{H}_1, \ldots, \mathcal{H}_{\Delta - 1}, \mathcal{H}_{\Delta + 1}, \ldots, \mathcal{H}_{L_e + L_h}) \subseteq \mathbb{C}^{N_G (L_e + 1) \times N_T (L_e + L_h - L_{ef})};
\]

\[
\eta_{\text{wall}}[n] \triangleq (u^T[n], u^T[n - \Delta + 1], u^T[n - \Delta - L_{ef} - 1], \ldots, u^T[n - L_e - L_h])^T \subseteq \mathbb{C}^{N_T (L_e + L_h - L_{ef})}.
\]

Under Theorem 1, it results that, for \( \sigma_w^2 \) approaching zero, the MOE-TEQ perfectly shortens the MIMO channel, regardless of the choice of \( \Gamma_3 \), i.e., the cascade of the proposed equalizer and the MIMO channel (3.9) assumes the desired expression (2.6), whatever the constraint matrix may be. By virtue of this
remark, we can assume that, for moderate-to-high SNR values, \( \mathbb{E}(\|e_{\text{MMSE}}(\delta)H_{\text{wall}}\|^2) \approx 0 \). Therefore, invoking assumption (a1) and accounting for \( F_{\text{mmoe}}(\delta) = \Gamma_3 F_{\text{mmoe}}(\delta) \), the optimization problem can be formulated as

\[
\Gamma_3 = \arg \max_{\Gamma_3} \frac{\text{trace}\{\Gamma_3 F_{\text{mmoe}}(\delta)H_{\text{wall}} \tilde{\Gamma}_{\text{win}}H_{\text{win}} F_{\text{mmoe}}^H(\delta) \Gamma_3^H\}}{\sigma_{\delta}^2 \text{trace}\{\Gamma_3 F_{\text{mmoe}}(\delta)F_{\text{mmoe}}^H(\delta) \Gamma_3^H\}},
\]

(3.12)
s.t. a suitable constraint, with \( \tilde{\Gamma}_{\text{win}} \triangleq \mathbb{E}(\eta_{\text{win}}[n] \eta_{\text{win}}^H[n]) \in \mathbb{C}^{N_T(L_w+1) \times N_T(L_w+1)} \). We can constrain matrix \( \Gamma_3 \) to fulfill the following relation

\[
\Gamma_3 F_{\text{mmoe}}(\delta)F_{\text{mmoe}}^H(\delta) \Gamma_3^H = \mathbf{I}_{N_R},
\]

(3.13)

which implies that the disturbance at the MIMO channel shortening equalizer output be white. It can be proved that maximizing the SSINR (3.12) under the constraint (3.13) [referred hereinafter to as the white disturbance constraint (WDC)] is equivalent to maximizing the sum of the SSINRs corresponding to each receive antenna. By resorting to the eigenvalue decomposition of \( F_{\text{mmoe}}(\delta)F_{\text{mmoe}}^H(\delta) = U \Sigma U^H \), where \( \Sigma \in \mathbb{C}^{N_R(d+1) \times N_R(d+1)} \) is nonsingular thanks to the definite positiveness of \( R_{\text{mmoe}} \), it turns out, after straightforward calculations, that \( \Gamma_3 = V_+ \Sigma^{-1/2} U^H \), where \( V_+ \in \mathbb{C}^{N_R(d+1) \times N_R} \) collects the eigenvectors corresponding to the \( N_R \) largest eigenvalues of the matrix \( Q_{\text{mmoe}}(\delta) \triangleq \Sigma^{-1/2} U^H F_{\text{mmoe}}(\delta)H_{\text{win}} \tilde{\Gamma}_{\text{win}}H_{\text{win}} F_{\text{mmoe}}^H(\delta) \Sigma^{-1/2} \). Observe that evaluation of the optimum \( \Gamma_3 \), under WDC, requires a priori knowledge of the channel-dependent matrix \( H_{\text{win}} \), which is instead unknown at the receiver. Nevertheless, the computation can be carried out in a completely blind manner, by noticing that, for moderate-to-high SNR values, the autocorrelation matrix of the pre-filtered data \( \xi(n) \triangleq \Sigma^{-1/2} U^H F_{\text{mmoe}}(\delta) z(n) \) assumes approximately the expression \( R_{\xi\xi} = Q_{\text{mmoe}}(\delta) + \sigma_w^2 I_{N_R(d+1)} \). The latter relation shows that the eigenvectors associated to the \( N_R \) largest eigenvalues of \( Q_{\text{mmoe}}(\delta) \) can be evaluated as the eigenvectors corresponding to the \( N_R \) largest eigenvalues of \( R_{\xi\xi} \), which, on the other hand, may be directly estimated from the received data.

### 4. NUMERICAL RESULTS

In this section, we present the performance of the proposed blind TEQ design (referred to as “Proposed”), in comparison with the nonblind MMSE-based channel-shortening equalizer developed in [3] (referred to as “MMSE”), which requires a priori knowledge or estimation of the channel matrix (2.4). We have considered the MMSE TEQ of [3] employing the orthonormality constraint, since it ensures better performance. We also reported the performance of the conventional OFDM receiver without channel-shortening (referred to as “No TEQ”), which corresponds to the case of \( F = (I_{N_R} \times N_Q, O_{N_R} \times (N_Q L_e)) \). The MIMO-OFDM system, having \( N_T = 1 \) and \( N_R = 2 \) antennas, employs \( M = 32 \) subcarriers and a CP of length \( M_p = 8 \). The information bits are mapped to quaternary phase-shift keying (QPSK) symbols using Gray labeling. No oversampling is implemented at the receiver (\( N_S = 1 \)). The samples of all the baseband equivalent channel impulse responses, with channel order \( L_h = 20 \), are generated at each Monte Carlo run as i.i.d. circularly symmetric complex Gaussian random variables, with zero mean and unit variance. Correspondingly, the equalizer order is set equal to \( L_e = 50 \). The SNR is defined as \( \sigma_y^2 / \sigma_w^2 \). As performance measure, we resort to the bit-error-rate (BER) at the output of OFDM demodulator as a function of the SNR, which is defined as \( \text{BER} \triangleq \sum_{m=0}^{M-1} \text{BER}_m / M \), where \( \text{BER}_m \) is the output BER at the \( m \)th subcarrier. From Fig. 1, it is immediately apparent that, in the considered scenario, the detection process is completely unreliable without a TEQ. In particular, it can be observed that, except for very low values of the SNR, the exact version of the proposed blind TEQ essentially achieves the same performance of the nonblind MMSE channel shortener.

### 5. REFERENCES