TWO-STAGE SPECTRUM DETECTION IN COGNITIVE RADIO NETWORKS

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ABSTRACT

Cognitive radios try to exploit “blank spaces” in the licensed bands which are not being used by primary users at a particular place and time. In the absence of cooperation between the primary and the secondary networks, spectrum sensing enables secondary users to monitor a licensed band in order to find idle channels for opportunistic access. In this work, we propose a two-stage spectrum detection strategy that decreases the average channel search time by allowing the spectrum detector to focus on frequency channels which are more likely to be vacant. We show that the proposed detection strategy significantly outperforms the conventional single-stage strategy when the spectrum utilization is high.

Index Terms— Cognitive radio, opportunistic spectrum access, spectrum detection, channel search time.

1. INTRODUCTION

Inefficient spectrum usage across many frequency bands has recently been known as a major drawback of static spectrum assignment policies in current wireless networks. In order to meet the ever increasing spectrum demand, cognitive radio technology is proposed in the literature [1, 2].

Cognitive radios should be able to sense a wide range of frequencies (sometimes a multi-gigahertz wide bandwidth) and search for idle bands. Since they are usually considered lower priority or secondary users, they must ensure that no harmful interference is caused to primary users.

One approach to wideband spectrum sensing is to implement a wideband filter followed by a high rate A/D converter to process the full range of the available spectrum at once. In [3], a wideband spectrum sensing technique is proposed that jointly detects primary signals over multiple frequency bands.

The wideband signal received by the antenna of a cognitive radio is a superposition of primary signals received over different frequency bands. Many factors such as location of primary users as well as their operating power levels affect the signal strength in each frequency band. As a result, wideband spectrum sensing involves detection of extremely weak signals from widely separated or severely shadowed transmitters in the presence of strong signals from nearby transmitters [4]. Consequently, wideband spectrum sensing imposes stringent constraints on the linearity of the RF amplifier as well as the sampling rate and the accuracy of the A/D converter [4].

A more practical approach is to employ a tunable band-pass filter (BPF) to search one frequency band at a time. Based on a one-by-one search strategy, the optimal sensing time is obtained in [5] in order to minimize the average time required to find the first idle channel. However, when the spectrum utilization is medium to high, this approach results in a significantly long search time.

In this paper, we propose a two-stage spectrum detection strategy that significantly reduces the average channel search time when the spectrum utilization is medium to high.

2. REGULATORY CONSTRAINTS

In this paper, we adopt the system model proposed in [5], where the interference due to secondary transmissions is considered harmful if it causes the signal-to-interference ratio (SIR) at a primary receiver to fall below a predefined threshold \( \Gamma \). This threshold depends on many factors including the primary receivers robustness against interference, type of the primary service, and the characteristics of the interfering signal and should be determined by regulatory bodies. Given the threshold \( \Gamma \), the interference range of a secondary transmitter is defined as the maximum distance at which the resulting interference may still be harmful to primary receivers. Let \( P_P \) and \( P_s \) denote the transmit powers of the primary and secondary users, respectively. The interference range of a secondary transmitter \( D \) is then derived from the following relation

\[
\frac{P_P R^{-\alpha}}{P_s D^{-\alpha}} = \Gamma, \tag{1}
\]

where \( R \) is the maximum distance between a primary transmitter and its respective receiver, and \( \alpha \) is the path loss exponent. The detection sensitivity \( \gamma_{\text{min}} \) is defined as the minimum signal-to-noise ratio (SNR) at which a primary signal should still be detected by a secondary user. Since a secondary user can only detect primary transmitters (not primary receivers), it should be able to detect any active primary transmitter within a range of \( D + R \). Therefore, \( \gamma_{\text{min}} \) is given by

\[
\gamma_{\text{min}} = \frac{P_P (D + R)^{-\alpha}}{N_0 W}, \tag{2}
\]
where \( N_0 \) is the equivalent noise power spectral density, \( W \) is the signal bandwidth, and \( D \) is determined from (1).

### 3. ENERGY DETECTION

The binary hypothesis test for spectrum sensing at each channel is formulated as follows

\[
\mathcal{H}_0 : \quad x(n) = v(n) \\
\mathcal{H}_1 : \quad x(n) = h(s(n) + v(n)),
\]

where \( x(n) \) is the received baseband signal by the secondary user at time \( n \), \( s(n) \) is the primary transmitted signal, \( h \) is the channel gain between the primary transmitter and the secondary user, and \( v(n) \) is the additive background noise. In fact, background noise is an aggregation of various sources such as thermal noise, leakage of signals from other bands, and interference due to transmissions of primary users far away [6]. In this paper, we assume that \( v(n) \) can be approximated by a stationary white Gaussian noise with zero mean and a known variance \( \sigma_v^2 \).

If cognitive radios’ a priori knowledge is limited to local noise statistics, the optimal detector is an energy detector, where the received signal over each frequency band is squared and integrated over the observation interval [7]. We assume that each secondary user is equipped with a tunable bandpass filter (BPF) followed by an A/D converter to search one frequency channel at a time. The same BPF may also be used for signal reception when a secondary connection is already established.

For each channel, the test statistic is the average received signal power during the observation interval \( T \) normalized by the noise power:

\[
Y = \frac{1}{M\sigma_v^2} \sum_{n=1}^{M} \| x(n) \|^2,
\]

where \( M \) is the number of samples, which is assumed to be equal to the time-bandwidth product, \( TW \) (Nyquist sampling). According to the central limit theorem [8], for large values of \( M \) (e.g., \( M \geq 50 \)), the test statistic \( Y \) is approximately normally distributed with mean

\[
E[Y] = \begin{cases} 
1 & \mathcal{H}_0 \\
\gamma + 1 & \mathcal{H}_1,
\end{cases}
\]

and variance

\[
\text{Var}[Y] = \begin{cases} 
\frac{2}{M} & \mathcal{H}_0 \\
\frac{2(2\gamma + 1)}{M} & \mathcal{H}_1,
\end{cases}
\]

where \( \gamma = \frac{Pvh^2}{\sigma_v^2} \).

As mentioned earlier, to avoid harmful interference to potential primary users, secondary users should be able to detect primary signals at SNR levels as low as \( \gamma_{\text{min}} \). Therefore, the corresponding false-alarm and missed detection probabilities are given by

\[
\begin{align*}
P_f(M) &= P(Y > \lambda | \mathcal{H}_0) = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{M}}{2} (\gamma - 1) \right) \quad (5) \\
P_m(M) &= P(Y < \lambda | \mathcal{H}_1) \\
&= 1 - \frac{1}{2} \text{erfc} \left( \frac{\sqrt{M} \lambda - (1 + \gamma_{\text{min}})}{2 \sqrt{2 \gamma_{\text{min}}}} \right), \quad (6)
\end{align*}
\]

where \( \lambda \) is the decision threshold, and \( \text{erfc}(\cdot) \) is the complementary error function. For a fixed number of samples \( M \), a lower probability of missed detection results in a higher probability of false alarm. The regulatory constraint on the reliability of primary user detection may be expressed as

\[
P_m |_{\gamma=\gamma_{\text{min}}} \leq \beta, \quad (7)
\]

where \( \beta \) is set by the regulator. Combining (5), (6), and (7), we have

\[
P_f(M) = \frac{1}{2} \text{erfc} \left( \text{erfc}^{-1}(2(1 - \beta)) \sqrt{2 \gamma_{\text{min}}} + 1 + \frac{\sqrt{M}}{2} \gamma_{\text{min}} \right). \quad (8)
\]

### 4. CHANNEL SEARCH TIME MINIMIZATION

We consider a primary communication system operating over a wideband spectrum which is divided into \( N \) frequency channels of bandwidth \( W \). At each time instant, a channel is busy with probability \( u \) and idle with probability \( 1 - u \), where \( u \) is the average utilization of the primary band. We also assume that the occupancy of each channel is independent of all other channels. When the number of primary channels \( N \) is sufficiently large (such that \( u^N \ll 1 \)), with high probability, at least one of the primary channels is idle during the channel search.

Before a connection can be established, a secondary user has to sense the primary channels sequentially looking for the first idle channel. The average length of the search interval \( M_{\text{search}} \) is equal to the number of samples taken per channel multiplied by the average number of channels sensed before an idle channel is found. While a shortened sensing time per channel allows a secondary users to search over more primary channels per unit of time, the corresponding higher false alarm probability results in a higher number of missed idle channels. In [5], the number of samples per channel \( M \) is optimized in order to minimize the delay incurred by the secondary user:

\[
M_{\text{search}} = \min_{M} \frac{M}{(1 - u)(1 - P_f(M))} \\
\quad \text{s. t. } P_f(M) \leq 1 - \frac{1 - \sqrt{\epsilon}}{1 - u}, \quad (9)
\]
where \( P_f (M) \) is given by (8) and \( \epsilon \) is set according to the QoS requirements of the secondary user. The constraint in (9) ensures that an idle primary channel is found at least with probability \( \epsilon \) before going through all the primary channels. As we will see in the next section, this approach results in a significantly long search time when the spectrum utilization is high.

5. TWO-STAGE SPECTRUM DETECTION

For large values of \( N \), we can approximate the term on the right hand side of the constraint in (9) to write it as

\[
P_f (M) \leq 1 - \frac{1 - \gamma/2}{1 - u} \approx 1 + \frac{\log_e(\epsilon)}{N(1 - u)} = 1 + \frac{\log_e(\epsilon)}{N_1}.
\]

where \( N_1 = N(1 - u) \) denotes the average number of idle channels. Hence the optimization problem in (9) can be written as

\[
\hat{M}_{\text{search}} = \min_M \frac{M}{(1 - u)(1 - P_f (M))}
\]

s. t. \( P_f (M) \leq 1 + \frac{\log_e(\epsilon)}{N_1}. \) (11)

As can be seen from (11), the higher the utilization factor \( u \), the higher the average length of the search interval. This is because, in general, a spectrum sensor has to go through more primary channels before successfully identifying an idle channel. In this section, we propose a two-stage spectrum detection strategy to adjust the focus on primary channels which are more likely to be idle.

First, each channel is sensed for an interval of \( M_1 \) samples. Based on the average received power in the first stage \( Y_1 \), the spectrum sensor decides either to proceed to the second stage, where the channel is sensed for \( M_2 \) more samples, or to move to the next channel. This process continues until an idle primary channel is found. In fact, the first stage aims to decrease the effective utilization factor for the channels proceeded to the second stage. The decision threshold in the second stage is set according to the regulatory constraint in (7).

If we assume that the active primary transmitters in the proximity of the secondary user are uniformly distributed in a circle of radius \( D + R \), the probability distribution functions of the average received power in the first stage under hypotheses \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) are given by

\[
f_{M_1}(Y_1, \text{dB} = y | \mathcal{H}_0) = 0.23 \times 10^{y/10} \\
\times e^{-y^{2}/(4/M_1)},
\]

and

\[
f_{M_1}(Y_1, \text{dB} = y | \mathcal{H}_1) = 0.23 \times 10^{y/10} \\
\times \int_0^1 2 r e^{-(r(\gamma_{\text{min}} r^{-\alpha} + 1))} (4/M_1)(2\gamma_{\text{min}} r^{-\alpha} + 1) \\
\times \sqrt{4\pi(2\gamma_{\text{min}} r^{-\alpha} + 1)/M_1} \\
\times \frac{r^{-\alpha} e^{-(\gamma_{\text{min}} r^{-\alpha} + 1)}/(M_1)(2\gamma_{\text{min}} r^{-\alpha} + 1)}{\sqrt{4\pi(2\gamma_{\text{min}} r^{-\alpha} + 1)/M_1}} dr,
\]

respectively.

In the first stage, we choose the decision threshold \( \lambda_1 \) to be equal to 0 dB so that an idle channel would proceed to the second stage with probability 1/2. The effective utilization factor for the channels proceeded to the second stage is given by

\[
u_1(M_1) = \frac{u F_{M_1}(0 \text{ dB} | \mathcal{H}_1)}{u F_{M_1}(0 \text{ dB} | \mathcal{H}_0) + (1 - u)(1/2)},
\]

where \( F_{M_1}(\cdot | \mathcal{H}_0) \) and \( F_{M_1}(\cdot | \mathcal{H}_1) \) are the cumulative distribution functions of the average received power in the first stage under \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \), respectively.

The average length of the search interval is given by\(^1\)

\[
M_{\text{search}} = \frac{M_1}{(1/2)(1 - u)(1 - u_1(M_1))(1 - P_f (M_2))} \\
+ \frac{M_2}{(1 - u_1(M_1))(1 - P_f (M_2))},
\]

where the first and the second terms represent the aggregate time spent in the first stage and second stage detections, respectively. Note that the second term in (15) is similar to the objective function in (11) except the fact that the utilization factor \( u \) is replaced with \( u_1(M_1) \). The number of samples in the first and second stage, \( M_1 \) and \( M_2 \), have to be optimized in order to achieve the minimum length of the search interval. The optimal value of \( M_2 \) can be obtained by solving the following optimization problem

\[
\hat{M}_2 = \arg \min_{M_2} \frac{M_2}{(1 - P_f (M_2))}
\]

s. t. \( P_f (M_2) \leq 1 + \frac{\log_e(\epsilon)}{(1/2)N_1}. \) (16)

Again, the constraint in (16) ensures that an idle primary channel is found at least with probability \( \epsilon \). Therefor, we have

\[
\hat{M}_{\text{search}} = \min_{M_1} \frac{\hat{M}_2 + 2 M_1/(1 - u)}{(1 - u_1(M_1))(1 - P_f (M_2))},
\]

In our numerical example, we set \( N = 200 \), \( \alpha = 3 \), \( \beta = 0.001 \), and \( \epsilon = 0.01 \). Fig. 1 illustrates the average length

\(^1\)The derivation is similar to that in [5] and is omitted due to space limitations.
of the search interval (the objective function in (17)) versus $M_1$ for different values of $\gamma_{\text{min}}$, where $M_2$ is given by (16), and $u = 0.8$. As can be seen from this figure, increasing the number of samples taken in the first stage $M_1$ reduces the overall search time due to the decreased utilization factor in the second stage $u_1(M_1)$. However, at some point, this gain is canceled out by the long time spent during the first stage.

Fig. 2 illustrates the average length of the search interval $\hat{M}_{\text{search}}$ as a function of the utilization factor $u$ for the single-stage and two-stage detection strategies, where $\gamma_{\text{min}} = -10$ dB. As can be seen from this figure, the proposed two-stage detection strategy significantly outperforms the conventional single-stage strategy when the spectrum utilization is high.

6. REFERENCES


