ABSTRACT
Limited feedback beamforming improves link reliability with a small amount of feedback from the receiver to the transmitter. The performance of such a closed loop MIMO system is unknown in interference limited cellular environments, when the base stations have limited or no coordination. This paper establishes the degradation in throughput due to uncoordinated other cell interference and delay on the feedback channel. Under a Markov channel assumption, the paper shows that the throughput gain of cell edge users decays doubly exponentially as the delay increases. Numerical results illustrate how the decay rate decreases when the codebook size increases.

Index Terms—Limited feedback beamforming, MIMO systems, Markov model, other cell interference.

1. INTRODUCTION
Limited feedback enables the practical use of channel state information (CSI) in multiple input multiple output (MIMO) wireless communication systems. Using the limited feedback concept, channel state information is quantized at the receiver and used at the transmitter to design intelligent transmission strategies such as precoded spatial multiplexing and transmit beamforming [1]. The CSI delivered to the transmitter, however, is imperfect. It is prone to errors from channel estimation and quantization, as well as feedback delay [2]. Because of the time varying nature of the wireless channel, feedback delay can cause a mismatch between the CSI at the transmitter and the actual channel state. This mismatch decreases the performance of limited feedback beamforming systems.

MIMO cellular systems are interference limited. While multi-cell MIMO and base station cooperation techniques [3, 4] can mitigate the effect of interference, when the base stations share full or partial channel state and data information, the effect of interference is unknown for the case of limited or no coordination. Single cell limited feedback MIMO techniques are expected to lose much of their effectiveness in the presence of multi cell interference [5]. When each cell designs its channel state index independently of the other cell interference, the achievable sum rate of the limited feedback system decreases.

In this paper, we consider the combined effect of other cell interference and feedback delay on the achievable sum rate of limited feedback beamforming MIMO systems. In contrast to the block fading model, traditionally used in the literature of limited feedback, we use a Markov model of the channel to capture the effect of delay. The Markov chain model is used in [6, 7] to analyze the effect of the channel time evolution and consequently, the feedback delay. Both [6, 7] show that the throughput of MIMO systems degrades if the CSI at the transmitter is not updated in a timely manner. Other temporal correlation models and measurement results of the wireless channels are used in [8–10] to evaluate the effect of feedback delay. The authors in [7] show that the capacity gain is upper bounded by an exponential function of the delay. Prior work in [7] does not, however, account for the presence of the other cell interference at the receiver, which is the main contribution of this work.

2. SYSTEM OVERVIEW
In this section, we discuss the cellular setup, the limited feedback beamforming system, and the finite state Markov chain used in the analysis.

We consider the modified Wyner type [11] $N_B$-cell $K$-user per cell circular array cellular model. The base stations $B_i$, $i = 1, \ldots, N_B$, with $N_t$ transmit antennas each, serve mobile stations $M_k$ with $N_r$ receive antennas. The users are located at the edge of their cells, such that each user is reach-
able from the two closest base stations only. The base stations have limited or no coordination.

Each cell employs limited feedback beamforming. The system is discrete time where continuous time signals are sampled at the symbol rate $1/T_s$, with $T_s$ being the symbol duration. Consequently, each signal is represented by a sequence of samples with $n$ denoting the sample index. Assuming perfect synchronization between the base stations, matched filtering, and a narrowband channel, the $n$-th received data sample $y_1[n]$ for a single user of interest $M_1$ in base station $B_1$ can be written as

$$y_1[n] = \sqrt{P_1} H_1[n] f_1[n-D] s_1[n] + \sqrt{P_2} G_2[n] f_2[n-D] s_2[n] + v_1[n],$$

where $s_k[n], \ (k = 1, 2)$ is the transmitted signal from $B_k$ to $M_k$ with variance $E[|s_k[n]|^2] = 1$; $v_1[n] \in \mathbb{C}^{N_s \times 1}$ is $\mathbb{C}N(0, I)$, modeling the additive noise observed at $M_1$. $P_k$ is the transmit signal power at $B_k$. $f_k[n-D] \in \mathbb{C}^{N_r \times 1}$ denotes the delay transmit beamforming vector between $B_k$ and its designated $M_k$. $H_1[n] \in \mathbb{C}^{N_r \times N_s}$ is the $n$-th realization of the channel matrix between $M_1$ and $B_1$. $G_2[n] \in \mathbb{C}^{N_s \times N_s}$ represents the $n$-th realization of the MIMO channel between $B_2$ and $M_1$.

The random processes $H_1[n]$ and $G_2[n]$ are assumed independent, stationary and temporally correlated. They are perfectly known at the receiver, thereby eliminating CSIR estimation error.

The CSI feedback is implemented as follows. The receiver estimates the channel state information sequence $H[n]$ using pilot symbols sent by the base station. The CSI quantizer efficiently quantizes the sequence by means of a Grassmannian codebook. To maximize the received signal-to-noise ratio, the quantizer function $Q$ at the receiver maps the channel $H[n]$ to a beamforming vector $f[n]$ from a finite set of possible beamforming vectors $F$ such that

$$f[n] = \arg \max_{v_\ell \in F} \|H[n] v_\ell\|^2, \quad 1 \leq \ell \leq N.$$

The receiver sends the index $I_n$ of the beamforming vector to the transmitter over a finite rate feedback channel, assumed error free, subject to a fixed delay $D$. Due to the presence of delay, the beamformer index at the transmitter $I_{n-D}$ lags behind the index corresponding to the current state $I_n$ at the receiver by $D$ samples.

We model the temporal evolution of the MIMO channels $H[n]$ as a discrete time first order Markov process [12]. Since the feedback state index $I_n$ is closely related to the channel $H[n]$, we model the time variation of the feedback state $I_n$ by a first order finite state Markov chain. This Markov chain \{\!\{I_n\}\!\}, mapped from a stationary channel $H[n]$, is stationary and has the finite state space $\mathcal{I} = \{1, 2, 3, \cdots, N\}$, where $N$ is the size of the codebook. The probability of transition from state $I_{n-1} = m$ to state $I_n = l$ is given by $P_{ml}$. The stochastic matrix is thus $P$, with $[P]_{ml} = P_{ml}$. The Markov chain is assumed ergodic with a uniform stationary distribution vector $\pi$, where $\Pr\{I_n = i\} = \pi_i = \frac{1}{N}$.

Subsequently, the joint probability mass function between $H_k[n]$ and $H_k[n-D]$ is

$$\Pr[H_k[n] \in \mathcal{V}_{k_{1\ell}}, H_k[n-D] \in \mathcal{V}_{k_{1\ell}},] = [P^D]_{k_{1\ell}k_{\ell1}} \pi_{k_{1\ell}},$$

(2)

where $H_k[n]$ belongs to the Voronoi cell $\mathcal{V}_{k_{1\ell}}$ and $H_k[n-D]$ belongs to the Voronoi cell $\mathcal{V}_{k_{1\ell}}$.

3. THE FEEDBACK THROUGHPUT GAIN

The instantaneous mutual information at the mobile stations, assuming with-cell interference, assuming the transmitted signals at the base stations are Gaussian distributed, is expressed as

$$R[n] = \log_2 (1 + \text{SINR}[n]),$$

(3)

where the signal-to-interference-noise ratio SINR$[n]$, assuming MRC combining vector $w[n] = \frac{H_1[n] f_1[n-D]}{\|H_1[n] f_1[n-D]\|}$ at the mobile station $M_1$, is given by

$$\text{SINR}[n] = \frac{P_1 \|H_1[n] f_1[n-D]\|^2}{P_2 \|w^* \|G_2[n] f_2[n-D]\|^2 + \|w^* v_1[n]\|^2}.$$ .

We define the ergodic system throughput as $\bar{R}(D) = \mathbb{E}[R[n]]$, where the expectation is taken over all the channels $H_k[n], G_k[n]$ in the instantaneous rate expression.

Following (2), and depending on the interference strength, $\bar{R}(D)$ is bounded in terms of the transition probabilities of the Markov chain. Worst case interference at the mobile user occurs when the beamforming vector $f_2[n-D]$ is chosen to maximize $\|G_2[n] f_2[n-D]\|^2$,

$$\bar{R}_1(D) = \mathbb{E}[H_1[n-D] \|H_1[n-D]\|^2] = \sum_{k_{1\ell}k_{\ell1}} \sum_{k_{2\ell}} R_{k_{1\ell}k_{\ell1}} \sum_{k_{2\ell}} R_{k_{2\ell}k_{\ell1}} \|P^D\|_{k_{1\ell}k_{\ell1}} \bar{R}_2(D),$$

(4)

where $R_{k_{1\ell}k_{\ell1}}$ is the instantaneous rate, $R[n]$, with $H_k[n-D] \in \mathcal{V}_{k_{1\ell}}, H_k[n-D] \in \mathcal{V}_{k_{1\ell}}, \text{for } k \in \{1, 2\}$.

If, however, the codebook index does not maximize $\|G_2[n] f_2[n-D]\|^2$, the ergodic throughput is simply

$$\bar{R}_2(D) = \sum_{k_{1\ell}k_{\ell1}} \sum_{k_{2\ell}} R_{k_{1\ell}k_{\ell1}} \|P^D\|_{k_{1\ell}k_{\ell1}} \pi_{k_{1\ell}}^3,$$

(5)

where the joint probability mass function of the random processes $G_2[n]$ and $f_2[n-D]$ is given by the product of the stationary probability of each random process individually. Consequently, $\bar{R}(D)$ is written in terms of the probability $r$ of the worst case interference

$$\bar{R}(D) = r\bar{R}_1(D) + (1 - r)\bar{R}_2(D),$$

(6)
where \( r = \frac{1}{N} \) is the probability of \( G_2[n] \) being in the same Voronoi cell as that of the channel \( H_2[n] \). It decreases with increasing codebook size.

To capture the effect of increasing feedback delay on the ergodic system throughput, we use the notion of throughput CSI as opposed to not receiving any CSI at the transmitter. When the delay goes to infinity, the feedback information becomes obsolete and thus irrelevant. The throughput gain is, [7],

\[
\Delta R(D) = \bar{R}(D) - \bar{R}(\infty),
\]

where \( \bar{R}(\infty) = \sum_{k_{10}, k_{1D}} \sum_{k_{20}, k_{2D}} R_{k_{1D}k_{10}} \pi^4 \).

This follows from the fact that, as \( D \to \infty \), the channel state Markov chain converges to the stationary distribution \( P^D \to [\pi, \cdots, \pi] \).

The throughput gain allows us to analyze the effect of increasing feedback delay on the multicell system, and draw conclusions as to when closed loop limited feedback MIMO systems are feasible. We derive an upper bound on the ergodic throughput gain \( \Delta \bar{R}(D) \), based on the Markov chain convergence rate [13].

**Proposition 1** For fixed feedback delay of \( D \) samples, the feedback throughput gain can be bounded as

\[
\Delta \bar{R}(D) \leq a \left( \sqrt{\lambda} \right)^{2D} + b \left( \sqrt{\lambda} \right)^D, \tag{8}
\]

where \( a = r \sum_{k_{1D}, k_{2D}} \max_{k_{10}, k_{20}} R_{k_{1D}k_{10}} \pi \), and

\[
b = r \sum_{k_{1D}, k_{10}, k_{2D}} \max_{k_{20}} R_{k_{1D}k_{10}} \pi^2 \sqrt{\pi} + \sum_{k_{1D}, k_{10}, k_{2D}} \max_{k_{20}} R_{k_{1D}k_{10}} \pi^2 \sqrt{\pi}.
\]

**Proof:** The proof is based on the following inequality for the ergodic channel state Markov chain [13],

\[
\left( \sum_{m=1}^{N} \left| [P^D]_{lm} - \pi_m \right| \right)^2 \leq \lambda^D \pi_l, \quad 1 \leq l \leq N.
\]

\( \lambda \in [0, 1] \) is the second largest eigenvalue of the matrix \( \bar{P} \), and \( \bar{P} \) is defined as the time reversal of the stochastic matrix \( P \). The details of the proof are included in [14]. \( \square \)

The coefficients \( a \) and \( b \) depend on the instantaneous rate \( R_{k_{1D}k_{10}} \) and the stationary probability distribution \( \pi \) of the Markov chain. \( a \) decreases with \( r \) such that, as the codebook size increases, \( a \to 0 \). This causes the rate of decay of the throughput gain to approach that of the noise limited environment [7], with \( b \to \sum_{k_{1D}, k_{10}, k_{2D}} \max_{k_{20}} R_{k_{1D}k_{10}} \pi^2 \sqrt{\pi} \).

The feedback gain thus decreases at least exponentially with the feedback delay. The decreasing rate is \( \lambda \) or \( \sqrt{\lambda} \), depending on the values of the coefficients \( a \) and \( b \). These are determined by the channel coherence time and the size of the codebooks used. The eigenvalue \( \lambda \) is a key parameter in characterizing the behavior of the system. A larger value of \( \lambda \) indicates longer channel coherence time and larger codebook size and vice versa. As the delay \( D \) grows larger, the term \( (\sqrt{\lambda})^{2D} \) decreases faster than \( (\sqrt{\lambda})^D \), bringing the exponential decay in the throughput gain closer to that of the single cell environment.

### 4. Simulation Results

For our simulations, we assume that the scattering environment is uniform such that the channel coefficients are \( CN(0, 1) \) and the temporal correlation follows Clarke’s model and is characterized by the first order Bessel function \( J_0(2\pi f_DT_s) \), where \( f_D \) is the Doppler frequency and \( T_s \) is the sampling period. An autoregressive model is used, in the simulations, to model the Rayleigh fading of the channel.

We compare the feedback rate gain of the interference limited system with that of the noise limited system [7]. Figure 1 plots the throughput gain of the systems versus the feedback delay \( D \). The results show that the exponential rate of decrease of the multi-cell system decays more quickly than that of the single-cell for the cases of small to moderate delay (up to \( D = 15 \)), as predicted by the theory. This agrees with the analytical results that show that the rate of decrease in the multi-cell environment is faster than that of the single cell environment, especially when the codebook size \( N \) is small. This rate of decrease in the multi-cell environment however changes to \( \sqrt{\lambda} \), as observed in Figure 1 in the change in the slope. This is due to the increasing delay, which causes the \( \sqrt{\lambda} \) term to become dominant.

The performance of the multi-cell system in the presence of delay with varying codebook sizes is shown in Figure 2 for a two transmit, two receive MIMO system and codebook...
sizes 4, 8 and 16. One can clearly see that the rate of decay of the throughput gain decreases as the codebook size increases, it approaches that of the single cell system for larger codebook sizes. This is due to the fact that, as the codebook size increases, the coefficient $a$ goes to zero, and the rate of decay approaches that of the noise limited environment [7]. It suggests a tradeoff between the feedback rate and the throughput gain in interference limited scenarios.

5. CONCLUSION

In this paper, we derived an upper bound for the rate of decay of the throughput gain for limited feedback systems in the presence of other cell interference. We showed that the exponential rate of decay of the ergodic feedback gain doubles, when compared to the noise-limited single cell scenario, especially at low to moderate feedback delay values. Furthermore, numerical results showed that a smaller codebook size leads to a faster reduction of the capacity gain with the feedback delay.

6. REFERENCES


