ABSTRACT
This paper considers a symmetric Gaussian interference game with incomplete information where players choose between frequency division multiplexing (FDM) and full spread (FS) of their transmit power. Previously, the only known Nash equilibrium point for this game was the point where players mutually choose FS and interfere with each other. This point may lead to undesirable outcome from global network point of view and even for each user individually. It happens when mutual FDM is better to both users than mutual FS.

In this paper, we show that if users agree to use different sub-bands in the case of FDM, then there exist a non pure-FS Nash equilibrium point, i.e. an equilibrium point where players choose FDM for some channel realizations and FS for the others. This Nash equilibrium point increases each user’s throughput and therefore improves the spectrum utilization. Furthermore, to reach this point, the only instantaneous channel state information (CSI) required by each user is its interference-to-signal ratio.

Index Terms— Bayesian games, interference channel, spectrum management

1. INTRODUCTION
Interference mitigation in wireless interference channels is an important issue under study [e.g. 1–9]. Consider a scenario of wireless users who share simultaneously the same band and are independent of each other and selfish in the sense that each one is only interested in maximizing its own utility. In this case, non cooperative game theory is the appropriate tool to analyze the interaction between the users (players). An important notion in game theory is Nash equilibrium which represent a study point the game, that is, a Nash equilibrium point increases each user’s throughput and therefore improves the spectrum utilization. Furthermore, to reach this point, the only instantaneous channel state information (CSI) required by each user is its interference-to-signal ratio.

In this paper we analyze a two user symmetric interference channel with incomplete information where each user knows the square magnitudes of its direct and impinging channel gains and its noise power spectrum density (PSD) but is unaware of its opponent's direct and impinging channel gains but only of its statistics (see Fig. 1). Based on their measurements, users choose between FDM and FS (see Fig. 2).

Most previous works in the field assumed complete information. This assumption however, is not always practical. Communicating channel gains between different users in a time varying channel within the channel coherence time may lead to large overhead. In this case, it is more suitable to consider each channel coherence time as a one-stage game where players are only aware of their own channel gains and their opponent's channel statistics (which vary slowly relative to the channels gains and therefore can be communicated [3]). The interaction between the players may be repeated but with different channel realization each time and therefore is not a repeated game. This motivates the use of games with incomplete information, also known as Bayesian games [10] which were first incorporated to wireless interference channels in [2]. For the case of symmetric one-time interaction with incomplete information, it was shown in [2] that FS is the only symmetric strategy profile which is a Nash equilibrium point.

Fig. 1. A wireless interference scenario with incomplete information. Each player knows the square magnitudes of its direct and impinging channel gains and the statistics of its opponent’s channel gains. E.g. player 1 knows $|h_{11}|^2$ and $|h_{12}|^2$ but knows only the statistics of $|h_{22}|^2$ and $|h_{21}|^2$.

A scenario where FS is the unique Nash equilibrium point, may lead to undesirable outcome from global network point of view and even for each user individually. This happens when mutual FDM is better to both users than mutual FS but the system operates in

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1By complete information, we mean that every user knows all direct and cross channel gains of all users in the network.

2In symmetric strategy profile users are restricted to identical strategies and therefore are not allowed to coordinate in advance to use different sub-bands in mutual FDM.
a mutual FS since the users are subject to the prisoner’s dilemma [11]. Thus, it is desirable to derive non pure-FS Nash equilibrium points which Pareto dominate (that is, component wise larger) the FS equilibrium point. Such points can serve as guidelines for designing a protocol that users would choose voluntarily to follow.

2. PROBLEM FORMULATION

Consider a flat-fading interference channel with two players where during a channel coherence time, the received signal is given by

\[ y_i(n) = h_{ii}x_i(n) + h_{ij}x_j(n) + v_i(n) \]  

where \( i, j \in \{1, 2\}, i \neq j, v_i(n) \) is a white Gaussian noise with variance \( \sigma^2 \) and \( h_{ij}, i, q \in \{1, 2\} \) are the channel fading coefficients which are random. Throughout this paper, the index \( j \) is always not equal to \( i \). Player \( i \)'s CSI at the transmitter side are the realized values of \( |h_{ij}|^2 \) for \( q = 1, 2 \) and a statistics of \( |h_{ij}|^2 \) for \( q = 1, 2 \), i.e. player \( i \) only knows the distribution of the ratio \( |h_{ij}|^2 / |h_{ij}|^2 \). We assume that during a single coherence period, players manage their spectrum only once based on their knowledge. Therefore, if the interaction is repeated it will be with different channel realization each time. This represents a case where the channel vary fast or a case where simplicity requirements enable only a single spectrum shaping every coherence period.

The channel is divided into two equal sub-bands (see Fig. 1) and players 1 and 2 actions are given by

\[ p_1(\alpha_1) = \overline{P}[\alpha_1, 1 - \alpha_1]^T \]
\[ p_2(\alpha_2) = \overline{P}[1 - \alpha_2, \alpha_2]^T \]

respectively (see Fig. 2), where \( \alpha_i \in S_i = \{1, 1/2\} \) and \( \overline{P} \) is the total power constraint. The actions \( \alpha = 1 \) and \( \alpha = 1/2 \) correspond to FDM and FS respectively. This formalism implies that both players agree which part of the band will be used by each one if he chooses FDM. We denote \( SNR_i = |h_{ii}|^2 \overline{P} / \sigma^2 \) and \( INR_i = |h_{ij}|^2 \overline{P} / \sigma^2 \) and \( \tau_i = [SNR_i, INR_i] \). The game state is determined by \( (\tau_1, \tau_2) \) where each vector \( \tau_i \) represents player \( i \)'s information also called in game theory player \( i \)'s observed signal. Players \( i \)'s payoff \( u_i \), is given in Table 1.

**Definition 1** A pure strategy \( s_i(\tau_i) \), is a function who maps each signal value to an action, i.e. \( s_i : \text{Range}(\tau_i) \rightarrow S_i \).

**Definition 2** The Bayesian interference game (BIG) is defined by the following

1. set of players \( \{1, 2\} \)
2. actions given in (2) with \( \alpha_i \in \{1, 1/2\} \) where \( \alpha_i = 1 \) corresponds to FDM and \( \alpha_i = 1/2 \) corresponds to FS
3. probability density function \( f_{\tau}(\tau) \), where \( \tau = [\tau_1, \tau_2]^T \)
4. an utility function \( u_i(s_i(\tau_i), s_j(\tau_j)) \) given in Table 1

In the Bayesian game, player \( i \)'s objective is to maximize its expected payoff given the signal \( \tau_i \), i.e.

\[ \pi_i(s_i, s_j) = E\{u_i(s_i, s_j)|\tau_i\} \]

**Definition 3** The BIG is symmetric if \( SNR_i, INR_i \) are i.i.d. random variables for each \( i = 1, 2 \).

**Definition 4** a Nash equilibrium point of the Bayesian game is a strategy profile \( s = (s_1, s_2) \) such that

\[ \pi_i(s_1, s_{-1}) \geq \pi_i(s_{\hat{i}}, s_{-1}) \forall \hat{i} \neq i \]

Since the action space is binary, a strategy \( s_i(\tau_i) \) in the BIG is equivalent to a decision region \( D_i \subseteq \text{Range}(\tau_i) \) such that \( s_i(\tau_i) = 1 \) (i.e. FDM) if \( \tau_i \in D_i \) and \( s_i(\tau_i) = 0.5 \) if \( \tau_i \notin D_i \). Recall that \( s_i(\tau_i), i = 1, 2 \) are pure strategies, i.e the action is chosen deterministically given \( \tau_i \). In [12] it is shown that such strategies capture all Nash equilibrium points of the BIG. Thus, we don’t consider mixed strategies where \( s_i(\tau_i) \) is a conditional probability (conditioned on \( \tau_i \)) distribution for choosing actions.

3. GAME ANALYSIS AND BEST RESPONSE FUNCTION

In the sequel, player \( i \)'s best response is derived. To this end we define for every \( i \) the following game outcomes and discuss their relationship for different channel values:

- \( T_i \) (Temptation) is player \( i \)'s payoff (see Table 1) for choosing FS while the other player chooses FDM.
- \( R_i \) (Reward) is player \( i \)'s payoff if he and player \( j \) mutually choose FDM.
- \( P_i \) (Penalty) is player \( i \)'s payoff if he and player \( j \) mutually choose FS.
- \( N_i \) (Naive) is player \( i \)'s payoff for choosing FDM while player \( j \) chooses FS.

Note that \( R_i > N_i \) since log is a monotonic function, furthermore, due to Jensen’s inequity, \( P_i > N_i \). Thus, the following situations are possible:

- \( A_i \) is the case in which \( R_i \geq T_i \) which is equivalent to \( INR_i > SNR_i/2 \).
- \( B_i \) is the case in which \( T_i \geq R_i \) which is equivalent to \( INR_i \leq SNR_i/2 \).

Recall that player \( i \) is not aware of the states of its opponent (\( A_j \) or \( B_j \)) but only of their probabilities (conditioned on \( \tau_i \)).

If player \( i \) experiences the situation \( B_i \) (which is \( INR_i \leq 1/2SNR_i \)), then FS is its best response. This is because FS is a strongly dominating action, that is, it produces a higher payoff to player \( i \) given any action of its opponent.
Table 1. User’s i payoff \( u_i \)

<table>
<thead>
<tr>
<th>player i chooses FDM ((\alpha_i = 1))</th>
<th>player j chooses FDM ((\alpha_j = 1))</th>
<th>player j chooses FS ((\alpha_j = 1/2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_i = \frac{1}{2} \log_2 (1 + SNR_i) )</td>
<td>( N_i = \frac{1}{2} \log_2 \left( 1 + \frac{SNR_i}{1 + INR_i} \right) )</td>
<td>( P_i = \log_2 \left( 1 + \frac{SNR_i/2}{1 + INR_i/2} \right) )</td>
</tr>
<tr>
<td>( T_i = \frac{1}{2} \log_2 (1 + SNR_i + P_i) )</td>
<td>( N_i = \frac{1}{2} \log_2 \left( 1 + \frac{SNR_i}{1 + INR_i} \right) )</td>
<td>( P_i = \log_2 \left( 1 + \frac{SNR_i/2}{1 + INR_i/2} \right) )</td>
</tr>
</tbody>
</table>

It remains to find player i’s best response for the situation \( A_i \), i.e. in case where \( INR_1 > 1/2SNR_i \), that is, strong interference. Let \( a_j = P(D_j) \), then player i’s expected payoff is given by

\[
\pi_i(FDM_i|\tau_i; a_j) = a_j R_i(\tau_i) + (1 - a_j) N_i(\tau_i) \quad (5)
\]

\[
\pi_i(FS_i|\tau_i; a_j) = a_j T_i(\tau_i) + (1 - a_j) P_i(\tau_i) \quad (6)
\]

Thus, player i’s best response is

\[
\hat{s}_i(\tau_i, a_j) = \begin{cases} 
\alpha_i = 0, & \text{if } a_j R_i(\tau_i) + (1 - a_j) N_i(\tau_i) > 1 \\
\alpha_i = 1/2, & \text{otherwise}
\end{cases} \quad (7)
\]

Observe that player i’s best response depends on its opponent strategy and channel distribution only via \( a_j \), the probability for choosing FDM. Thus, player i’s best response is invariant to decision regions with equal probability measure.

3.1. Nash equilibrium points

We now analyze a two-user symmetric game, that is, \( SNR_1, INR_1, SNR_2 \) and \( INR_2 \) are i.i.d. The same game was analyzed in [2] but with different class of action set, i.e. the users were not allowed to coordinate in advance to use disjoint subbands in the case of FDM. In this case, it was shown in [2] that FS is the only Nash equilibrium point and no FDM is possible. This is a very pessimistic result. In this work we modify the model used in [2].

In the sequel it is shown that if users are allowed to coordinate in advance to use disjoint subbands in the case of FDM (as implied in (2)), FDM is possible from game theoretic point of view and also increases the total system throughput as well as the individual throughput. Such a coordination can be carried out via Carrier Sense Multiple Access (CSMA) techniques (see e.g. [13]) where each player randomly choose a subband and perform a random power backoff in case of collision. This is done at the first interaction when users exchange information (channel statistics).

Theorem 1 The strategy profile

\[
\hat{s}_i(\tau_i) = \begin{cases} 
\alpha_i = 0, & \text{if } INR_i > SNR_i \\
\alpha_i = 1/2, & \text{otherwise}
\end{cases} \quad (8)
\]

for \( i = 1, 2 \) is a Nash equilibrium point of the BIG. Furthermore, this non pure-FS Nash equilibrium point Pareto dominates the pure-FS Nash equilibrium point.

Since the above Nash equilibrium point Pareto dominates the pure-FS equilibrium, rational users would agree to operate in this point rather than pure-FS one.

Proof: To show that (8) is a Nash equilibrium, we need to show that it is the best response of each player if the opponent use this strategy profile. Assume that player 2 uses the strategy (8) and therefore \( a_2 = 1/2 \), we would like to show that in this case (8) is player 1’s best response. Based on (7), player 1’s best response is to choose FDM if

\[
\frac{R_1(\tau_1) + N_1(\tau_1)}{T_1(\tau_1) + P_1(\tau_1)} > 1
\]

and to choose FS otherwise. By substituting the expressions for \( N_1, R_1, T_1, P_1 \) from Table 1 into (9) we obtain

\[
\log_2(1 + SNR_1) + \frac{1}{2} \log_2 \left( 1 + \frac{SNR_1}{1 + INR_1/2} \right) \\
\geq \log_2(1 + SNR_1/2) + \frac{1}{2} \log_2 \left( 1 + \frac{SNR_1/2}{1 + INR_1/2} \right)
\]

By substituting \( INR_1 = bSNR_1 \) and simplifying we obtain that player 1’s best response is to choose FDM if

\[
(b - 1) \left( (2b^2 + 5b + 1)SNR_1^2 + 8SNR_1 + 8 \right) \geq 0
\]

and to choose FS otherwise. Note that

\[
(2b^2 + 5b + 1)SNR_1^2 + 8SNR_1 + 8 \geq 0
\]

\[
\forall b \geq 0, \quad SNR_1 > 0
\]

which can be verified by observing that \( (2b^2 + 5b + 1) \) \( \forall b \geq 0 \) and that the equation

\[
(2b^2 + 5b + 1)SNR_1^2 + 8SNR_1 + 8 = 0
\]

either has no real roots or only negative roots for non-negative values of \( b \). Thus, (11) is true for \( b \geq 1 \) and false otherwise and therefore, player 1’s best response is given by (8). This establishes that (8) is an equilibrium point.

It remains to show that the resulting non pure-FS Nash equilbrium point Pareto dominates the pure-FS one. Player 1’s conditional expected payoff is

\[
\pi_1(\tau_1) = \max \{ 0.5R_1(\tau_1) + (1 - 0.5)N_1(\tau_1), 0.5T_1(\tau_1) + (1 - 0.5)P_1(\tau_1) \}
\]

Thus, it is sufficient to show that

\[
0.5T_1(\tau_1) + 0.5P_1(\tau_1) \geq P_1(\tau_1)
\]

This is shown straightforwardly by substituting the expressions for \( N_1, R_1, T_1, P_1 \) from Table 1 into (15).

Figure 3 shows the performance gain of both users when operating in the non pure-FS Nash equilibrium point of Theorem 1 rather than the pure-FS Nash equilibrium point.

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4. DISCUSSION

In this paper we considered a symmetric Gaussian interference game with incomplete information. Prior to this work, it was known that there exist a unique pure-FS Nash equilibrium point. This is a very pessimistic results. It means that users will interfere with each other even in strong interference scenarios where FDM is beneficial to both of them.

The motivations of this work was to derive a non pure-FS Nash equilibrium point to improve both total and individual system throughput in the case of selfish and rational users with incomplete information. It was shown that if users agree to use different subbands in the case of FDM then there exist a non pure-FS Nash equilibrium point, i.e. an equilibrium point where players chooses FDM for some channel realizations and FS for the others. The choice between FDM and FS is determined only by the interference to signal ratio (ISR) which is given by $\text{ISR} = \text{INR} / \text{SNR}$. This is the only required instantaneous CSI. Each player chooses FDM if $\text{ISR} > 1$ and choose FS otherwise. Intuitively, if a user experiences strong interferences, he should choose FDM even if there is a risk (50%) that its opponent would choose FS. The expected payoff is higher.

The use of Bayesian games for the interference channel opens a new direction. Additional question which naturally arise is the uniqueness of non-FS Nash equilibrium points. Furthermore, it will be very useful if the results of this paper be extended to the case of arbitrary channel distributions which cover scenarios involving weak and strong users, different fading effects (e.g. Rayleigh, Rician and Nakagami) in the direct and in the interfering paths, and to extend to results to arbitrary number of users. These question are addressed in [12].

References