Type-based multiple-access (TBMA) is a bandwidth efficient transmission scheme for decentralized estimation in wireless sensor networks (WSNs) over non-orthogonal multiple-access channels, whereas its performance degrades severely in zero-mean fading channels. In this paper, we improve its performance in the fading channels by using more resources for transmitting each type, which is named as TBMA with bandwidth extension (TBMA-BE). This transmission scheme needs no feedback of channel state information as existing solutions, thus it is feasible for large scale WSNs. We then develop an approximate maximum likelihood (ML) estimator, and derive the Cramér-Rao lower bound to reveal how the performance and the bandwidth efficiency are related. It is shown from the simulations that the TBMA-BE transmission scheme with the approximate ML estimator performs fairly well in non-orthogonal multiple-access channels with Rayleigh fading, and is even superior to the orthogonal multiple-access protocol at low communication signal to noise ratio (SNR). It also shows that the TBMA-BE is bandwidth efficient for the typical scenarios of WSNs where the communication SNR is low, since the total bandwidth consumption is proportional to the SNR.

Index Terms— Decentralized estimation, wireless sensor networks, type-based multiple-access

1. INTRODUCTION

Wireless sensor networks (WSNs) can be applied to estimate physical parameters in a decentralized manner. Typical WSNs for decentralized estimation usually consist of a large number of wireless sensors spatially scattered in a field to measure the parameters of interest. The sensors convert their observations into waveforms and then transmit them to a fusion center (FC) over multiple-access channels (MACs). The FC will generate the final estimation of the parameters. The decentralized estimation in WSNs is first studied under the assumption that the sensors transmit signals to the FC using the orthogonal MACs, e.g., [1] and references therein. Since WSNs are usually large scale networks, orthogonal multiple-access protocols are difficult to be implemented in practice and have low bandwidth efficiency.

Considering that the FC does not need to retrieve the original observation from each sensor, transmission schemes using non-orthogonal MACs have drawn considerable attention recently [2–5]. Several transmission schemes have been proposed for the decentralized estimation with non-orthogonal MACs, which allows the FC to recover the sufficient statistics from the received signals, then use them to estimate the parameter. The analog amplify-and-forward (AF) is such a transmission scheme first to be studied [3]. Then the type-based multiple-access (TBMA) with quantized observations, which assigns one orthogonal waveform to represent each type, is introduced for the decentralized estimation [2] and detection [4]. These transmission schemes are unified as the sufficient statistics based multiple-access (SSBMA) in [5].

It is shown that the asymptotic performances of the SSBMA schemes are superior to that of the orthogonal multiple-access transmission schemes in either ideal channels or additive white Gaussian noise (AWGN) channels [2, 6]. The estimates obtained with these schemes are still asymptotic efficient in non-zero mean fading channels, but their performance reduce dramatically in zero-mean fading channels. To improve the estimation performance in there, [7, 8] propose to feed the channel state information (CSI) back to the sensors, then each sensor compensates the impact of the channel according to the feedback information. To implement the CSI feedback in practice, orthogonal multiple-access protocols are still necessary for the sensors to transmit training sequences and for the FC to feed back the estimated CSI to the sensors, which also leads to low bandwidth efficiency. Without the CSI feedback, [9] finds the optimal transmission rate for the distributed detection in fading MACs.

In this paper, we study the decentralized estimation over non-orthogonal multiple-access fading channels with zero-mean. Differing from the CSI feedback approach, we allow the sensors to use more bandwidth resources for the transmission. The considered transmission scheme is named as TBMA with bandwidth extension (TBMA-BE), which assigns more than one orthogonal waveforms to represent a type, and the sensors select one waveform randomly for transmission. We develop an approximate maximum likelihood (ML) estimator, and analyze the bandwidth efficiency when TBMA-BE is used together with the ML estimator, which is shown to be higher than that using orthogonal MACs for low communication SNR. The simulations validate our analysis and show the superiority of the TBMA-BE to the TBMA transmission scheme in Rayleigh fading channels.

2. SYSTEM MODELS

Consider a WSN which consists of \( N \) sensors and a FC to measure an unknown deterministic parameter \( \theta \), where there are no inter-sensor communications.

To highlight the estimation methods, we consider a simple observation model. The observation for the unknown parameter provided by the \( i \)-th sensor is

\[
x_i = \theta + n_{s,i}, \quad i = 1, \cdots, N,
\]

where the observation noise \( n_{s,i} \) is independent identically distributed (i.i.d.), which subjects to Gaussian distribution with zero
mean and variance $\sigma^2$. Assume that $\theta$ is bounded with a dynamic range defined as $[-V, +V]$.

The sensors quantize their observations into several levels, where each level can be considered as a type observed by the sensors. Uniform quantizer is used because it is optimal for the deterministic parameter. Denote the number of the types by $M$ and the dynamic range of the quantizer by $[-W, +W]$. The observation of the $i$-th sensor is quantized to the $m$-th ($1 \leq m \leq M$) type.

After the quantization, the sensors generate the transmitted signals based on the types they observed. Denote the transmitted signals of the $i$-th sensor as $s_i(t)$ ($0 \leq t \leq T_s$), where $T_s$ is the duration of the transmitted signals.

We consider the synchronous MAC as in [2] with block fading, where the time delay of the received signals only affects the phase of the channel coefficients, and the channel coefficients are invariant during the period that sensors transmit the signals representing one observation. The receive signals at the FC is then

$$y(t) = \sum_{i=1}^{N} h_i s_i(t) + n(t), \quad i = 1, \ldots, N,$$

where $h_i$ is the channel coefficient, which subjects to complex Gaussian distribution with zero mean and unit variance, say, $h_i \sim \mathcal{CN}(0,1)$, and $n(t) \sim \mathcal{CN}(0, \sigma^2)$ is the thermal noise of the receiver.

3. TBMA WITH BANDWIDTH EXTENSION

When the TBMA transmission [2] is used, each sensor generates one orthogonal waveform to represent the type it observed. In total $M$ orthogonal waveforms are required.

We consider a more general scheme wherein the sensors use $K_m$, $m = 1, \ldots, M$, orthogonal waveforms to represent the $m$-th type. Then $B_m = \sum_{m=1}^{M} K_m$ orthogonal waveforms are required.

Since the scheme usually needs more orthogonal waveforms than TBMA, it is called as TBMA with bandwidth extension. When $K_m = 1$, $\forall m = 1, \ldots, M$, the TBMA-BE degenerates into the TBMA.

Define $\phi_{m,k}(t)$ ($0 \leq t \leq T_s$) as the waveforms used by TBMA-BE transmission, where $m = 1, \ldots, M$, and $k = 1, \ldots, K_m$. All $\phi_{m,k}(t)$s are orthogonal to each other. To simplify the analysis, the energy of $\phi_{m,k}(t)$ is normalized to one,

$$\langle \phi_{m,k}(t), \phi_{m,k}(t) \rangle = 1, \quad \forall m, k,$$

where

$$\langle f(t), g(t) \rangle = \int_0^{T_s} f^*(t)g(t)dt,$$

(4)

is the inner product of $f(t)$ and $g(t)$, and $^*$ denotes the complex conjugate.

When the $i$-th sensor observes the $m_i$-th type, $m_i \in [1, M]$, and selects the $k_i$-th waveform, $k_i \in [1, K_m]$, for transmission, the transmitted signal of this sensor can be expressed as

$$s_i(t) = \sqrt{\frac{\mathcal{E}_d}{M}} \sum_{m=1}^{M} \sum_{k=1}^{K_m} E_i(m,k) \phi_{m,k}(t),$$

where $\mathcal{E}_d$ is the energy used by each sensor to transmit one observation, and $E_i(m,k)$ is defined as

$$E_i(m,k) = \begin{cases} 1, & m = m_i, k = k_i \\ 0, & \text{otherwise} \end{cases}.
$$

Without loss of generality, we assume that the sensor selects the waveform uniformly, thus $k_i$ is uniformly distributed within $[1, K_m]$.

4. APPROXIMATE ML ESTIMATOR

4.1. Likelihood Function

After the waveform matched filter, the discrete received signal at the FC is

$$y_{m,k} = \langle y(t), \phi_{m,k}(t) \rangle = \sqrt{\frac{\mathcal{E}_d}{M}} \sum_{i=1}^{N} h_i E_i(m,k) + n_{m,k},$$

(7)

where $n_{m,k} \sim \mathcal{CN}(0, \sigma^2)$ is i.i.d. noise.

Because each sensor observes the parameter and processes the observation independently, $E_i(m,k)$, $i = 1, \cdots, N$, are i.i.d. stochastic variables given $\theta$. Following the Central Limit Theorem, $y_{m,k}$, $\forall m, k$, are approximately complex Gaussian distributed with zero mean. The covariances can be derived as

$$E[|\hat{y}_{m,k}|^2] = \frac{\mathcal{E}_d}{M} \sigma^2, \quad \forall m = n, k = l,$$

(8)

where $p(m|\theta)$ is the probability that the $m$-th type is observed by the sensor given $\theta$. Based on the observation and quantization model considered, we have,

$$p(m|\theta) = \frac{1}{M} \sum_{m=1}^{M} \exp \left( -\frac{(x - \theta)^2}{2\sigma^2} \right) dx,$$

(9)

where $I_m$ is the quantization interval of $m$-th type.

Define vector $y_m = [y_{m,1}, \cdots, y_{m,K}]^T$ and vector $y = [y_1^T, \cdots, y_M^T]^T$. We can obtain the likelihood function as

$$p(y|\theta) = \frac{1}{\sqrt{(2\pi)^{K_m} \det R_y(\theta)}} \exp \left( -\frac{1}{2} y^H R_y(\theta)^{-1} y \right),$$

(10)

where $R_y(\theta)$ is the covariance matrix of $y$. According to (8), $R_y(\theta)$ has the form as

$$R_y(\theta) = \frac{\mathcal{E}_d}{M} \text{diag}[\pi(1)I_{K_1}, \cdots, \pi(M|\theta)I_{K_M}] + \sigma^2 I_{M},$$

(11)

where $\text{diag}[\cdots]$ denotes to the diagonal matrix, and $I_n$ is $n \times n$ identical matrix.

Both the determinant and inverse of $R_y(\theta)$ can be easily computed since $R_y(\theta)$ is diagonal. Then the log-likelihood function can be derived as

$$\log p(y|\theta) = -\sum_{m=1}^{M} \left( K_m \log \left( \frac{\mathcal{E}_d \pi(1)}{K_m} + \sigma^2 \right) + \frac{\|y_{m,k}\|^2}{\mathcal{E}_d \pi(1)} \right) + \alpha,$$

(12)

where $\| \cdot \|_2$ is $l_2$-norm of a vector, and $\alpha$ is a constant which gives no effect on the estimation.

It is hard to find the maximum of the log-likelihood function because it is non-concave and has multiple extrema. Therefore, to estimate $\theta$, we need to use a searching algorithm to find all the extrema. The exhaustive search is used in the simulations to evaluate the performance of the ML estimator.
4.2. Cramér-Rao Lower Bound and Optimal $K_m$

The Cramér-Rao lower bound (CRLB) of the approximate ML estimator can be derived as

$$\text{Var}[\hat{\theta}] \geq \left( -\mathbb{E} \left[ \frac{\partial^2 \log p(y|\theta)}{\partial \theta^2} \right] \right)^{-1} = \left( \sum_{m=1}^{M} \frac{\mathbb{E}_{d}[N^2 K_m]}{\mathbb{E}[p(m|\theta)]} \right)^{-1}. \quad (13)$$

It shows that the CRLB is not a monotonic function of $K_m$. By letting its derivative to zero, we can obtain its minimum as

$$\text{Var}[\hat{\theta}] \geq \frac{4}{N \gamma_c} \left( \sum_{m=1}^{M} \frac{\mathbb{E}_{d}[N^2 K_m]}{\mathbb{E}[p(m|\theta)]} \right)^{-1}, \quad (14)$$

which can be achieved when

$$K_m^* = N \gamma_c p(m|\theta), \quad (15)$$

where $\gamma_c = \mathbb{E}_{d}/\sigma^2$ is the communication SNR.

Because $\sum_{m=1}^{M} p(m|\theta) = 1$, the total number of the orthogonal waveforms required to minimize the CRLB is

$$B_w^* = \sum_{m=1}^{M} K_m^* = N \gamma_c, \quad (16)$$

which is the total bandwidth normalized by the bandwidth of each waveform.

Note that the total normalized bandwidth required by the orthogonal multiple-access protocols with the near-optimal transmission codebook studied in our previous paper [10] is $B_w = N M$. This means that the TBMA-BE transmission will be more bandwidth efficient than the orthogonal multiple-access protocols for low communication SNR.

Unfortunately, $K_m$ cannot be designed according to (15) if the sensors have no prior knowledge about the distribution of $\theta$. To estimate an unknown deterministic parameter, the sensors can assume that the parameter is uniformly distributed in its dynamic range. Thus the uniform bandwidth allocation for types, i.e., let $K_m = K$, $\forall m$, is the optimal allocation in practice. In this case, we only need to design one factor $K$.

In simulations, the manner how $K$ and the estimation accuracy are related will be explored, and the optimal $K$ which minimizes the mean square error (MSE) will be obtained. In practical systems, the optimal $K$ for different communication SNR and number of the sensors can be obtained off-line and provided to the sensors before the deployment of the networks, with which the sensors can design the bandwidth allocation.

It is noteworthy that the CRLB cannot be achieved since we approximate the distribution of $\gamma$ with the Central Limit Theorem. Nevertheless, we will show in the next section that the optimal $K$ obtained by simulations is linear in $N$ and $\gamma_c$, which is consistent with the result derived from the CRLB.

5. SIMULATIONS

We evaluate the MSE of estimating $\theta$ using the TBMA-BE transmission and the approximate ML estimator by simulations. The uniform bandwidth allocation, where $K_m = K$, $\forall m$ and $B_w = M K$, is considered for comparison.

The observation SNR is defined as $\gamma_c = \log_{10}(W^2/\sigma^2)$. We set it to be $20dB$ in the simulations, and set the quantization levels to be 16 according to the results in [11].

We consider the Rayleigh fading channels in simulations, where the channel coefficients subject to the complex Gaussian distribution with zero mean and unit variance.

For comparison, we also evaluate the MSE of decentralized estimation over orthogonal MACs (marked as “Orth-MAC Opt” in the legends). This curve is obtained under the assumption that the sensors use ideal multiple-access protocols, i.e., the transmitted signals is orthogonal without multi-user interferences and the multiple-access protocols need no control frame. The transmission codebook used in the simulation is the near-optimal codebook designed in [10], and the estimator is the ML estimator [10].

![Fig. 1. MSE versus $\gamma_c$ when $N = 10$.](image)

Figure 1 shows the MSEs of the approximate ML estimator as a function of the communication SNR when using TBMA and TBMA-BE transmission schemes, which is marked as “TBMA” and “TBMA-BE” with different $K$ in the legend. We also search the minimal MSE when the bandwidth is uniformly allocated with different $K$ in the simulations, and plot it with the mark “TBMA-BE optimal $K$”. As expected, it is shown that the TBMA-BE outperforms the TBMA.

It is also shown that the estimator with TBMA-BE transmission outperforms that using orthogonal MACs at low communication SNR. Through simulations, we find that the optimal $K$ is much smaller than $N$ for low $\gamma_c$. Therefore the total bandwidth consumed by the TBMA-BE transmission ($B_w = M K$) is much lower than that by the orthogonal MACs ($B_w = MN$). It means that the orthogonal multiple-access protocols are inefficient for the decentralized estimation at low SNR level, which is the typical operating point of WSNs.

It is shown in (15) that the optimal total bandwidth $B_w^*$ minimizing the CRLB is proportional to $\gamma_c$ and $N$. In the sequel, we will validate the analysis through simulations.

Figure 2 shows the optimal total bandwidth obtained by searching the minimal MSE through simulations when the bandwidth is allocated uniformly. In order to verify the linear relationship between total bandwidth and $\gamma_c$, we use the linear regression (LR) method to fit the simulation data, and we show the LR curve for comparison. These two curves are marked as “TBMA-BE $B_w^*$ Simu” and “TBMA-BE Optimal $K$ Simu”. The linear regression result is shown in Table 1.
6. CONCLUSION

We consider the decentralized estimation over non-orthogonal MACs with zero mean in this paper. A TBMA-BE transmission scheme has been introduced, and a corresponding approximate ML estimator has been proposed and analyzed.

It is shown by analyses and simulations that the TBMA-BE transmission can improve the estimation accuracy by extending the bandwidth with two or three times in zero-mean fading channels compared with the performance of TBMA. It even outperforms the transmission scheme with orthogonal multiple-access protocols with much lower bandwidth requirement at low communication SNR which is typical in WSNs. Although both the transmission scheme in orthogonal MACs and the one in non-orthogonal MACs require the linear increase of the bandwidth with the number of the sensors, the slope of the increase for TBMA-BE transmission is lower than that for orthogonal multiple-access protocols at low communication SNR levels, which indicates that the TBMA-BE is more bandwidth efficient than the transmission schemes in orthogonal MACs.

7. REFERENCES


