DYNAMIC BIT ALLOCATION FOR TARGET TRACKING IN SENSOR NETWORKS WITH QUANTIZED MEASUREMENTS

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ABSTRACT

The problem of dynamic bit allocation for target tracking is investigated in this paper under a total sum rate constraint in sensor networks. Bits are dynamically allocated to sensors in such a way that a cost function, which is based on the Cramér-Rao lower bound evaluated at the predicted target state, is minimized. The optimal solution to this problem, namely joint bit allocation and local quantizer design, is computationally prohibitive and not realistic for real-time online implementation. Instead, a two-step optimization procedure is proposed. First, the best time independent quantizers are obtained offline by maximizing the average Fisher information about the signal amplitude, for different number of bits. With the time independent quantizers, the generalized Breiman, Friedman, Olshen, and Stone (BFOS) algorithm is employed to dynamically assign bits to sensors. Simulation results show that with the same or even less sum bit rate, the proposed dynamic bit allocation approach leads to significantly improved tracking performance, compared with the static bit allocation approach where each sensor is allocated with equal number of bits.

Index Terms— Bandwidth management, target tracking, quantized measurements.

1. INTRODUCTION

For an Ad Hoc sensor network that consists of a large number of spatially distributed sensors, it is desirable not to use all the sensors to track a target at each time, since there always exist constraints on computation, sensing range, communication bandwidth, and energy consumption. Thus a critical task is to select a subset of sensors to optimize system performance under these constraints. In [1] a method was proposed for dynamically selecting the optimal set of sensors to participate in target tracking in sensor networks. This method was later extended to deal with quantized local sensor data to further reduce the communication bandwidth required for target tracking [2].

In this paper, our goal is to find a bit allocation solution that optimizes the tracking performance under a sum bit rate constraint in the sensor network. This problem is more general than the sensor selection problem, because in bit allocation problems, each sensor could be assigned a different number of bits under a total sum bit rate constraint, while in sensor selection problems, either “1” or “0” is assigned to each sensor under a constraint on the total number of active sensors. Bit allocation has been studied in the context of vector quantization [3], communication systems [4], video coding [5], and detection in sensor networks [6]. But to the best of our knowledge, this paper presents the first research work on dynamic bit allocation for tracking in sensor networks.

There exist many sensor selection algorithms. One popular strategy is the information driven method [7]. The main idea is to select the sensors that can provide the most useful information, which is quantified by entropy or mutual information. However, if more than one sensor is involved in sensor selection or bit allocation, the complexity of the information theoretic measures increases exponentially with the number of sensors [8]. In this paper, we propose a CRLB based bit allocation approach. There are two main motivating reasons for our approach. First, the covariance for any unbiased target position estimator is bounded below by the CRLB. In this paper we construct a cost function based on the CRLB evaluated at the predicted target state. This predicted lower bound gives an indication of performance limits, so it can be used as a criterion for bit allocation or sensor selection. Second, the CRLB based cost function has a complexity that is linear in the number of sensors.

2. PROBLEM FORMULATION

The problem we seek to solve is tracking a moving target in a WSN environment where the sensors are deployed in a surveillance area. All the sensors report to a fusion center which sequentially estimates the target state, i.e., the position and the velocity of the target, based on received local sensor data. Although the deployment scheme of sensors is not an issue, throughout this work, without loss of generality the sensors are assumed to be grid deployed for simplicity.

2.1. Target Dynamic Model

We consider a single target moving in a two-dimensional Cartesian coordinate plane. Target dynamics is defined by the 4-dimensional state vector $x_k = [x_{tk}, y_{tk}, x_{tk}', y_{tk}']^T$ where $x_{tk}$ and $y_{tk}$ denote the coordinates of the target in the horizontal and the vertical directions with the corresponding velocities $x_{tk}'$ and $y_{tk}'$, respectively, at time $k$. The superscript $T$ denotes the transpose operation. Target motion is defined by the following white noise acceleration model [9]:

$$x_k = F x_{k-1} + v_k$$  \hspace{1cm} (1)

where $F$ models the state dynamics and $v_k$ is the process noise which is assumed to be white, zero-mean and Gaussian with the following covariance matrix $Q$.

$$Q = \begin{bmatrix}
\Delta^2 & 0 & \Delta^2 & 0 \\
0 & \Delta^2 & 0 & \Delta^2 \\
\Delta^2 & 0 & \Delta & 0 \\
0 & \Delta^2 & 0 & \Delta
\end{bmatrix}$$

(2)
In (2), $\Delta$ and $q$ denote the time interval between adjacent sensor measurements and the process noise parameter, respectively. It is assumed that the fusion center has perfect information about the target state-space model (1) as well as the process noise statistics.

### 2.2. Sensor Measurement Model

The target is assumed to be any source that follows the power attenuation model provided below. At any given time $k$, the signal power received at a sensor is the following:

$$P_{ik} = \frac{P_0 d_0^n}{(d_{ik})^n}$$

(3)

where $P_0$ denotes the target signal power at a reference distance of $d_0$ from the target, $n$ is the signal decay exponent, and $d_{ik}$ is the distance between the target and the $i^{th}$ sensor, $d_{ik} = \sqrt{(x_i - x_{ik})^2 + (y_i - y_{ik})^2}$, where $(x_i, y_i)$ and $(x_{ik}, y_{ik})$ are the coordinates of the $i^{th}$ sensor and the target at time $k$, respectively. The received signal at each sensor is given by

$$s_{ik} = a_{ik} + n_{ik}$$

(4)

where $a_{ik} = \sqrt{P_{ik}}$ is the true measurement, and $n_{ik}$ is the noise term modeled as additive white Gaussian noise (AWGN), i.e., $n_{ik} \sim \mathcal{N}(0, \sigma_n^2)$, which represents the cumulative effects of sensor background noise and the modeling error of signal parameters. Without loss of generality, the reference distance $d_0$ and the signal decay exponent $n$ are assumed to be unity and 2, respectively. The fusion center is assumed to have complete information about $P_0$ and the noise statistics.

The received signal $s_{ik}$ at each sensor is locally quantized before being sent to the fusion center using a quantizer function $Q_i(.)$. The main reason for quantization is to decrease the amount of communication so that the energy consumption is reduced. The quantized observation model at sensor $i$ is given by

$$Q_i(s_{ik}) = m_{ik} = \begin{cases} 
0, & \gamma_{i0} < s_{ik} < \gamma_{i1} \\
L - 1, & \gamma_{i(L-1)} < s_{ik} < \gamma_{iL} 
\end{cases}$$

(5)

where $m_{ik}$ is the quantized measurement of the $i^{th}$ sensor and $\gamma_{i0}, \ldots, \gamma_{iL}$ are the predetermined thresholds for a $K = \log_2 L$ bit quantizer. Note that $\gamma_{i0} = -\infty$ and $\gamma_{iL} = \infty$. Based on (5), the transmitted observations from sensors to the fusion center can be denoted in vector form as $\mathbf{M}_k = [m_{1k}, m_{2k}, \ldots, m_{Nk}]^T$, where $N$ is the total number of sensors deployed in the area of interest.

### 2.3. Bandwidth Considerations

We assume that the network of $N$ sensors can transmit reliably at a maximum rate of $R$ bits per sampling time, where $R$ is an integer. At each time instant $k$, the quantizer $Q_i$, at sensor $i$ maps the measurement $s_{ik}$ to $m_{ik} \in \{0, \ldots, 2^R - 1\}$ where

$$\sum_{i=1}^N R_i = R$$

(6)

A generic system model is shown in Fig. 1. It is clear that the quantization process entails some performance loss. The main goal here is to jointly design $N$ quantizers at each time step such that we get the best tracking accuracy as possible with a sum rate constraint of $R$ bits per sampling time.

### 3. BANDWIDTH MANAGEMENT

We quantify the tracking accuracy at time $k$ by defining a cost function as the Cramér-Rao lower bound (CRLB) evaluated at the predicted target state. The bandwidth management problem, similar to [6], can be stated as follows: At each time step $k - 1$, find an optimum partition of $R$ into $N$ nonnegative integers $R_1, R_2, \ldots, R_N$ and associated with each $R_i$ find quantizer functions $Q_1, Q_2, \ldots, Q_N$ such that the CRLB evaluated at the predicted target position, $\hat{x}_{i(k|k-1)}$ and $\hat{y}_{i(k|k-1)}$, is minimized.

The above problem can be solved by using a brute-force approach where a search of all possible bandwidth allocation between sensors is carried out and the corresponding optimal quantizer functions are found which provide the minimum predicted CRLB. However, for a network of $N$ sensors and a total of $R$ bits, there are a total of $(N+R-1)!$ possible distributions of bandwidth among sensors. For a tracking problem where the optimization needs to be carried out in real time, such a brute-force approach is not feasible. Therefore, a sub-optimal yet more efficient and fast algorithm is needed to solve the bandwidth management problem. We propose to combine two different approaches to solve our joint optimization problem. If each sensor $i$ has a predefined time-independent set of stored quantizers corresponding to $0, 1, \ldots, R$ bits, i.e., $\{Q_i^0, Q_i^1, \ldots, Q_i^R\}$, then the problem reduces to finding the best possible bandwidth distribution among sensors. We propose a two-step optimization procedure: 1) Find the best time independent quantizers offline for each sensor corresponding to all possible bandwidth distributions and store them. 2) Using the quantizers found in the first step, find the best bandwidth distribution. Our approach is explained below.

### 3.1. Offline Quantizer Design

For the offline quantizer design problem, we adopt the Fisher information based approach proposed in [10] for the target localization problem. Here, we give a brief description of the approach and refer the interested reader to [10] for details. Based on the fact that all the information about $\theta = [P_0, x_t, y_t]^T$ is contained in sensors’ signal amplitudes ($a_i$s), intuitively if all the signal amplitudes, $a_i$ ($i = 1, \ldots, N$) can be accurately recovered from their corresponding quantized data $m_i$ ($i = 1, \ldots, N$), an accurate estimate of $\theta$ can be obtained. Assuming that the target location and the sensor positions all follow a uniform distribution over the sensor field, i.e., $x_i, x_t, y_i, y_t \sim \mathcal{U}[-b/2,b/2]$ where $b$ is the length of the sensor field, and the target signal power $P_0 \sim \mathcal{U}[0, P_m]$, the probability distribution function (pdf) of the signal amplitude, $f(a)$ measured at
a random location can be calculated [10] as

\[
f(α) = \frac{2}{α P_m} \begin{cases} 
\frac{α^2}{3} - \frac{β}{2} a, & 0 < a \leq \frac{\sqrt{2P_m}}{\sqrt{b}} \\
\frac{α^2}{3} - \frac{β}{2} a + \frac{α^2}{3} - \frac{α^2}{3} a^{-1} - βa, & \frac{\sqrt{2P_m}}{\sqrt{b}} < a \leq \frac{2P_m}{3b} \\
& \text{o.w.}
\end{cases}
\]  

where

\[
α = 1 + \frac{8d_0^3}{3b^3} - \frac{d_0}{b^2} - \frac{d_0^2}{3b^3}
\]

which is the probability that a sensor is at least \(d_0\) \(m\) away from the target,

\[
β = \frac{π}{2b} d_0^2 + \frac{α_0^2}{3b^3} - \frac{8d_0^3}{3b^3}
\]

and the function \(g(t)\) is defined as

\[
g(t) = \frac{t^2}{2\sigma^2} \arcsin \left( \frac{2t^2}{\sigma} - 1 \right) - \frac{t^2}{3\sigma^3} + \frac{t^2}{\sigma^5} \sqrt{1 - \frac{t^2}{\sigma^2}} (12t^2 + 28t + 20b^4).
\]

For a given number of bits \(r\), the average Fisher information, \(F(Γ^r)\) contained in the sensor data \(m_i\) about the signal amplitude \(α_0\) can be calculated, where \(Γ^r = [γ_0, \ldots, γ_2]\) denotes the set of quantization thresholds for each sensor [10]:

\[
F(Γ^r) = \frac{1}{2π} \int_0^{2π} \sum_{l=0}^{r-1} \frac{e^{-\frac{(γ_l-a)^2}{2σ^2}} - e^{-\frac{(γ_{l+1}-a)^2}{2σ^2}}}{2πσ^2} \left( Q \left( \frac{γ_l-a}{σ} \right) - Q \left( \frac{γ_{l+1}-a}{σ} \right) \right) f(α)da.
\]

Then the optimum quantization thresholds can be found by maximizing the average Fisher information about \(α\)

\[
\max_{Γ^r} F(Γ^r).
\]

Note that this method does not require the knowledge of the sensor locations, the target location, or the signal position, distance \(d_0\) (\(P_0\)). So the thresholds can be set before deployment, as long as there is some prior knowledge about the sensor field (\(b\)), and the possible range of \(P_0\), i.e., \(P_0\). Therefore, we can calculate the set of optimum quantizers for each sensor corresponding to bandwidth distributions 1, \ldots, \(R\) and store them before deployment.

### 3.2. Bandwidth Distribution

Since the sensors now have the predefined quantizers, the remaining problem is how to find the best bandwidth distribution among sensors such that the CRLB evaluated at the predicted target position is minimized. In this case, a brute-force approach is still not feasible especially when the number of sensors \(N\) and the total number of bits \(R\) are large. Here, similar to the methodology in [11], we propose to use the generalized BFOS (GBFOS) algorithm [3] along with the previously stored quantizers designed in Section 3.1. This algorithm starts by assigning the maximum number of bits, \(R\) to each sensor in the network and then reduce the number of bits one bit at a time until the sum rate constraint is satisfied, i.e., \(\sum_{r=1}^{N} R_i = R\). Reduction of the bits is carried out by minimizing a distoration function which is the CRLB evaluated at the predicted target position for our problem. Let \(F(x_i(k), t_i(k-1), y_i(k), t_i(k-1), \hat{R})\) denote the Fisher information matrix (FIM) about the target position, where \(\hat{R} = [R_1, R_2, \ldots, R_N]\), then we can define our distortion (cost) function as

\[
C(\hat{R}) = F_{11}^{-1} + F_{22}^{-1} F(x_i(k), t_i(k-1), y_i(k), t_i(k-1), \hat{R})
\]

has been derived in [10] and its elements are given as follows:

\[
\begin{align*}
J_{11} &= n^2 \sum_i \kappa_i a_i^2 d_i^{-4} (x_i - x_i)^2 \\
J_{12} &= J_{21} = \kappa_i a_i^2 d_i^{-4} (y_i - y_i)^2 \\
J_{22} &= \kappa_i a_i^2 d_i^{-4} (y_i - y_i)^2
\end{align*}
\]

Then the bandwidth distribution algorithm can be stated as in Algorithm 1.

#### Algorithm 1 Bandwidth distribution algorithm

1. For \(i = 1, 2, \ldots, N\), set \(R_i = R\).
2. For \(i = 1, 2, \ldots, N\), calculate \(C(\hat{R})\) where \(\hat{R} = [R_1, R_2, \ldots, R_i, R_i - 1, \ldots, R_N = R_N]\).
3. Determine the sensor \(i\) for which \(C(\hat{R})\) is the lowest and assign index \(i\), i.e., \(\hat{R} = \arg\min_{R_i} C(\hat{R})\). Set \(R_i = R_i - 1\).
4. Check if \(\sum_{i=1}^{N} R_i = R\); if so, stop.
5. Go to step 2.

### 4. NUMERICAL EXAMPLES

In this section, we provide some numerical results where we evaluate the performance of our bandwidth management approach developed in in Section 3. Sensors are assumed to be grid deployed in a 200m \(\times\) 200m surveillance area. \(P_0 = 25000\) and sensor background noise is assumed to have unit power, i.e., \(\sigma^2 = 1\). For the following target dynamic model parameters, all units are in meters, seconds and meters per second corresponding to distance, time and velocity measurements, respectively. The initial state distribution of the target \(p(x_0)\) is assumed to be Gaussian with mean \(μ_x = [-80 \quad -80 \quad 2\]T and covariance \(Σ_{x_0} = Q \times \text{diag} [1\ 10\ .5].\) The target motion follows a near constant velocity model with a process noise parameter, \(q = 2.5 \times 10^{-3}\). Measurements are assumed to be taken at regular intervals of 1 s, i.e., \(Δ = 1\), and the observation length is 60 s. We employ a sampling importance resampling (SIR) particle filter to track the target [12]. The number of particles used for the particle filter is 1000.

In Fig. 2, we provide an example track where we compare our algorithm with a static bit allocation scenario. There are 9 sensors and 9 bits in each scenario. Our algorithm dynamically distributes the bandwidth (dynamic bit allocation) whereas the static scenario assigns equal number of bits (one bit in this case) to each sensor at all time instants. As the example track in Fig. 2 indicates, the dynamic bandwidth distribution approach that we propose improves the tracking performance.

Snapshots of bandwidth distributions assigned using our dynamic bit allocation algorithm at different time instants within an example track is shown in Fig. 3. There are again 9 sensors and
9 bits. It is clear from the figure that the dynamic bit allocation algorithm inherently deactivates some sensors depending on where the target is located, and assigns larger number of bits to sensors with higher measurement signal-to-noise ratios (SNRs) which is an intuitive result.

Finally, we provide Table 1 where we evaluate the tracking performance achieved by our dynamic bit allocation algorithm under different scenarios. Root mean square error (RMSE) of the target location error over the complete track is considered as the performance criterion. All simulation results are based on 100 Monte Carlo trials. It is clear from Table 1 that the developed dynamic bit allocation approach significantly achieves better tracking accuracy than the static bit allocation scenario especially when the number of sensors (degrees of freedom) increases. Furthermore, it is very promising to see from the 16 sensor case that the dynamic bit allocation approach can help save bandwidth while improving performance. 9 bits allocated dynamically between 16 sensors provides better tracking performance compared to having 16 bits allocated equally among 16 sensors.

Table 1. RMSE comparison for dynamic and static bandwidth distribution

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<thead>
<tr>
<th></th>
<th>Dynamic (RMSE)</th>
<th>Static (RMSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 sensors</td>
<td>18.3411 (4 bits)</td>
<td>19.9615 (4 bits)</td>
</tr>
<tr>
<td>9 sensors</td>
<td>9.4395 (9 bits)</td>
<td>15.0877 (9 bits)</td>
</tr>
<tr>
<td>16 sensors</td>
<td>6.9315 (9 bits)</td>
<td>13.2145 (16 bits)</td>
</tr>
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5. CONCLUSIONS

In this paper, we developed a dynamic bit allocation approach for tracking a single target in a resource-constrained sensor network with quantized measurements. Under a sum rate constraint, a GBFOS algorithm is used to allocate the bits to sensors, to minimize a cost function that is established on the CRLB evaluated at the predicted target state by the tracker. We compared our method with a static one that assigns the bits equally to all the sensors. Simulation results demonstrated that even under a stringent bandwidth constraint, the proposed dynamical bit allocation solution exhibits significantly improved performance. In the future, the proposed approach will be extended to multi-target tracking scenarios. The GBFOS algorithm, a greedy optimization method, is sub-optimal. We will study its optimality gap and develop alternative optimization approaches.

6. REFERENCES