ABSTRACT
The paper investigates linear beamforming techniques in relay networks with multiple independent sources, destinations and relay(s). The goal is to determine the beamforming matrix to minimize the sum transmit power at the relays while meeting signal-to-interference (SINR) requirements at the destinations. Two scenarios are considered: one is the case of single relay with multiple antennas in which the beamforming matrix can be of any form, while the other is the case of distributed single antenna relays, where the beamforming matrix is diagonal. An interference zero-forcing criterion is applied for the design of the beamforming matrix at the relays assuming that perfect channel state information is available. It is shown that the former problem is a constrained least square problem and the latter one can be solved by SemiDefinite Programming (SDP) with relaxation. Simulation results show that the proposed beamforming schemes can reliably support multiple parallel data streams with SINR requirements in multiuser relay systems.

Index Terms— Relay Networks, Beamforming, Zero-forcing, Convex Optimization, SINR Requirements

1. INTRODUCTION

There has been a lot of interest in cooperative (or relaying) approaches for wireless communications, due to their potential for significant performance improvement, such as coverage extension of cellular network, throughput enhancement and energy savings [1]. Amplify-and-Forward (AF) is one very popular relaying protocol because of its low complexity and implementation cost, since it only needs simple forwarding at the relays. Recently, the relaying research was extended to multiple antenna systems, which can greatly improve the throughput of relaying systems. Both capacity aspects and beamforming schemes at relays [2, 3] have been addressed for MIMO relay channels. A scenario involving multiple simultaneously transmitting single antenna sources, multiple relays and destinations was considered in [4] where a framework for AF relaying was proposed that achieves high throughput. The work of [5] builds on that of [4] by proposing optimum weight design for minimizing transmit power subject to a signal strength constraint at each destination.

In this paper, we consider a multiuser AF relaying network with QoS requirements at the destinations. Independent sources, each with a distinct destination, transmit their signals simultaneously in the first time slot. The relay(s) linearly process the received noisy signals by a beamforming matrix and then forward the processed signals to the destinations in the second slot. Both cases, i.e., single relay with multiple antennas, and distributed single antenna relays are considered. In the first case, the beamforming matrix is a full matrix, while in the latter case the beamforming matrix is a diagonal matrix as each relay can only forward an amplified version of the signal that it received. Perfect channel state information (CSI) of the relay(s) is assumed to be available for designing the beamforming matrix. The beamforming matrix is designed to minimize the power consumption at the relay(s) while satisfying the SINR requirements at all destinations. A zero-forcing (ZF) method [6] is applied to cancel the inter destination interference (IDI). We show that this QoS guarantee problem under zero-forcing beamforming (ZFBF) in the case of single relay with multi-antenna is a constrained least square problem. In the distributed case, this QoS meeting problem can be solved by SDP with relaxation. In the distributed relay case, ZFBF can lower the dimension of the problem thus enabling low complexity, while it requires comparably high transmit power at the relays. As compared to [5], the proposed work minimizes power subject to a different constraint, i.e., SINR instead of signal strength at each destination, which leads to a completely different optimization problem. Also, the proposed approach is different from that of [7] in that it considers multiple sources as opposed to one source used in [7]. Again, this leads to a different optimization problem.

2. SYSTEM MODEL

A multiuser AF relaying system with $M$ single antenna sources and $M$ single antenna destinations is considered. Totally $K$ relay antennas assist the sources to send data to the destinations. The $K$ antennas could belong to one relay, or are located on independent relays in the distributed relay case. The direct links between sources and destinations are assumed to be negligible due to large the path loss or shadowing. In every transmission period, the sources $\{1,...,M\}$ wish to send the baseband signal $s_1,...,s_M$ to destinations $\{1,...,M\}$, respectively. $s_i,i = 1,...,M$ are independent Gaussian variables with zero means and unit variances. Each source’s transmit power is fixed to $P_0$. All transmitted signals are narrowband and a frequency flat fading channel model is assumed. In the first slot of every transmission period, the received signal vector at the relays equals

$$r = \sqrt{P_0}Fs + n$$

where $r = [r_1,...,r_K]^T$ with $(\cdot)^T$ denoting the operation of transpose, $s = [s_1,...,s_M]^T$, $n = [n_1,...,n_K]^T$ in which $n_i$ is a zero mean complex Gaussian noise with unit variance and $F = [f_1,...,f_M]$ with $f_i = [f_{i1},...,f_{iK}]^T, i = 1,...,M$ in which $f_{ki}$ is the channel coefficient between source $i$ and relay antenna $k$. In the second slot of each transmission period the relays forward to the destinations a linear transformation of $r$, i.e.,

$$t = Wr = \sqrt{P_0}WFs + Wn$$
where $W \in \mathbb{C}^{K \times K}$ and $t = [t_1, \ldots, t_K]^T$. The sum transmit power at the relays is expressed as

$$P_T = \mathbb{E}[\|t\|^2] = \text{Tr}(P_0WFW^H) + \text{Tr}(WW^H)$$  \hspace{1cm} (3)$$

where $\mathbb{E}$ and $(\cdot)^H$ represents the operation of expectation and conjugate transpose. Denoting the channel matrix from the relays to the destinations by $G$, the received signal at the destinations is

$$y = \sqrt{T}GWFs + GWn + z$$  \hspace{1cm} (4)$$

where $G = [g_1, \ldots, g_M]^T$ where $g_i = [g_{1i}, \ldots, g_{Ki}]^T$ is the channel vector from the relays to the destination $i$, and $z$ is a complex Gaussian vector in which each entry $z_i \sim \mathcal{CN}(0, 1)$.

The SINR at the destination $i$ equals

$$\text{SINR}_i = \frac{P_0|g_i^TWF_i|^2}{\sum_{j=1, j \neq i}^{M} P_0|g_i^TWF_j|^2 + \|g_i^TWF_i\|^2 + 1}$$  \hspace{1cm} (5)$$

Our goal is to design $W$ that meets a certain SINR at each destination, i.e., $\text{SINR}_i \geq \gamma_i$, while minimizing the sum transmit power at the relays, i.e.,

$$\min_{W \in \mathbb{C}^{K \times K}} P_T, \text{ s.t. } \text{SINR}_i \geq \gamma_i$$  \hspace{1cm} (6)$$

3. SINGLE RELAY WITH MULTIPLE ANTENNAS

In this case the relay processes the received signals from all antennas, thus the beamforming matrix $W \in \mathbb{C}^{K \times K}$ can have an arbitrary structure. To fulfill the zero-interference condition, $W$ needs to satisfy

$$g_i^TWF_j = 0, i, j = 1, \ldots, M, i \neq j$$  \hspace{1cm} (7)$$

which implies that GWF is a diagonal matrix. The following structure of $W$ is proposed based on this condition [2]:

$$W = G_0^H(GG_0^H)^{-1}g_R^H(FF_0^H)^{-1}F_0^H$$  \hspace{1cm} (8)$$

where $\Lambda_R = \text{diag}(\rho_1, \ldots, \rho_M)$ in which $\rho_i$ is a scaler. (8) requires that $\text{rank}(F) = M$ and $\text{rank}(G) = M$. The relay first uses $W$ to transform the received signal into parallel streams, then processes power allocation through $\Lambda_R$, and finally sends the signal precoded by $W_G$ to cancel the inter destination interference caused by $G$.

Substituting (8) in to (4), we rewrite the received signal vector at the destinations as

$$y = \sqrt{T}W_{\Lambda R}s + L_{RF}WF_{F} + z$$  \hspace{1cm} (9)$$

Let $W_F = [W_F^1, \ldots, W_F^M]^T$ and $\sigma_k^2 = \|W_F^k\|^2$. Then the SINR at the destination $i$ is expressed as

$$\text{SINR}_i = \frac{P_0|\rho_i|^2}{|\rho_i|^2\sigma_i^2 + 1}$$  \hspace{1cm} (10)$$

from which one can see that only $|\rho_i|^2$ controls the received SINR at the destination $i$. The transmit power under the ZF condition at the relays can be rewritten as

$$P_T = P_0 \sum_{i=1}^{M} |[W_G\Lambda_R]_{pi}|^2 + \sum_{p,q=1}^{K} |[W_G\Lambda_RWF_p]_{pq}|^2$$  \hspace{1cm} (11)$$

where $[\cdot]_{ij}$ denotes the $i, j$ element of a matrix. Note that $[W_G\Lambda_RWF_p]_{pq} = \sum_{i=1}^{M} [W_G]_{pi}WF_p \frac{\rho_i}{\|W_F^i\|^2}x$, where $a_{pq} = \frac{\|W_G\|_pWF_p \frac{\rho_i}{\|W_F^i\|^2}}{[W_G]_{pi}WF_p \frac{\rho_i}{\|W_F^i\|^2}x}$. We can rewrite the second term of the RHS of (11) as:

$$\sum_{p,q=1}^{K} |[W_G\Lambda_RWF_p]_{pq}|^2 = |[Ax]|^2$$  \hspace{1cm} (12)$$

where $A = [a_1, \ldots, a_M]^T \in \mathbb{C}^{K \times M}$. Similarly, defining $b_{pi,j} = \frac{1}{\|W_F^i\|^2}[W_G]_{pi}WF_p \frac{\rho_i}{\|W_F^i\|^2}x$, we can write the tightening version of the transmit power at the relays as

$$P_T = x^HQx$$  \hspace{1cm} (14)$$

where $Q = \sqrt{P_0B}^H\Lambda_0^H|\sqrt{1B}^H\Lambda_0^H|A_0^T$. Substituting (14) into (6), the original sum transmit power optimization problem under SINR constraints becomes:

$$\min_{x \in \mathbb{C}^{M \times 1}} x^HQx, \text{ s.t. } |[x_i]|^2 \geq \psi_i^2, \ i = 1, \ldots, M$$  \hspace{1cm} (15)$$

where $\psi_i = \sqrt{\gamma_i(P_0 - \gamma_i|\sigma_i|^2)}$. It is required that $P_0 > \gamma_i|\sigma_i|^2$ to make this problem feasible. However, the feasible set of this problem is non-convex if $[x_i]$ takes complex values. To make this problem tractable we switch to a constrained version of this problem, i.e. $x$ is a real vector. Under this assumption, the following property is used to convert (15) into a convex optimization problem.

$$x^HQx = x^HQx = x^H\Re(Q)x$$  \hspace{1cm} (16)$$

where $\Re[\cdot]$ and $\Im[\cdot]$ denotes the real and imaginary part of $\cdot$. $\Re(Q)$ is a skew-Hermitian real matrix thus $x^H\Re(Q)x = 0$, which justifies (16).

Now using the (16), we can write the tightening version of the (15) into a constrained least-squares problem:

$$\min_{x \in \mathbb{R}^{M \times 1}} \|\Re(Q)x\|^2 \text{ s.t. } |[x_i]|^2 \geq \psi_i^2, \ i = 1, \ldots, M$$  \hspace{1cm} (17)$$

where $\|\cdot\|$ denotes element wise 'larger or equal'. Problem (17) can be effectively solved using convex optimization tools, for example, Matlab function lsqlin.

Remarks: Both useful signal and noise at the relays contribute to the sum transmit power of (11). It’s obvious that the minimization of the first term of the RHS of (11) is achieved at $x = \Psi = [\psi_1, \ldots, \psi_M]^T$, except that the second term of the RHS of (11) is not necessarily minimized at $x = \Psi^{-1}$. However, when the source transmit power $P_0$ is fairly large, the signal part i.e., the first term of the RHS of (11) dominates the sum transmit power of the relays so that the minimization point is almost $x = \Psi$, which yields low beamforming matrix computation complexity. In this case, the minimization of the sum transmit power at the relays is $\Psi^{-1}Q\Psi$.

\footnote{A simple case $\Re(Q) = \begin{bmatrix} 1.05 & -0.95 \\ -0.95 & 1.05 \end{bmatrix}, x \geq [1, 2]^T$. The minimization is achieved at $x = [1.8, 2]^T$, rather than $x = [1, 2]^T$.}
4. DISTRIBUTED RELAY CASE

In the case that all relays are distributed the beamforming matrix $W$ is diagonal.

4.1. Optimal Beamforming

Using this diagonal property, the sum transmit power in (3) at the relays is written as

$$P_T = w^H R_T w$$

where $w = \begin{bmatrix} [W]_{11}, \ldots, [W]_{K} \end{bmatrix}^T$ with $[W]_{pp}, p = 1, \ldots, K$ denotes the forwarding weight of relay $p$ in the second slot and $R_T = \text{diag}\left\{1 + P_0 \sum_{i=1}^{M} |f_{1i}|^2, \ldots, 1 + P_0 \sum_{i=1}^{M} |f_{Ki}|^2\right\}$.

Denoting $a \otimes b$ by the pointwise product of two matrices(vectors), we can rewrite the SINR expressions (5) as

$$\text{SINR}_d = \frac{P_0 w^H R^{(i)}_T w}{P_0 w^H R_T w + w^H R^{(i)}_T w + 1}$$

where $R^{(i)}_T = (g^{T}_i \otimes f^{*_i}_i)^H(g^{T}_i \otimes f^{*_i}_i)$, $R^{(i)}_T = \sum_{j=1}^{K} (g^{T}_j \otimes f^{*_j}_j)$ and $R^{(i)}_T = \text{diag}\{[G^{[i]}_1]^2, \ldots, [G^{[i]}_M]^2\}$. Under these conditions, the problem of meeting QoS constraints with minimal relay transmit power is rewritten as

$$\min_w w^H R_T w, \text{ s.t. } w^H Q^{(i)}_T w \geq 1, i = 1, \ldots, M$$

where $Q^{(i)}_T = \gamma^{-1}_i[P_0 R^{(i)}_T - \gamma_i R^{(i)}_T - \gamma_i R^{(i)}_T]$.

Let $X \triangleq \text{ww}^H$. The above optimization problem is equivalent to:

$$\min_X \text{Tr}(XR_T)$$

s.t. $\text{Tr}(XQ^{(i)}_T) \geq 1, i = 1, \ldots, M; X \succeq 0; \text{rank}(X) = 1$

Here $X \succeq 0$ means that $X$ is semi-positive definite. The feasible set of the above problem is not convex because the rank constraint. Therefore, we switch to consider a relaxed problem by dropping the rank constraint [7,8]:

$$\min_X \text{Tr}(XR_T)$$

s.t. $\text{Tr}(XQ^{(i)}_T) \geq 1, i = 1, \ldots, M, X \succeq 0$

which can be solved by SDP tools like SeDuMi [9]. If the rank of the solution $X^*$ of the above problem is not equal to one, some randomization algorithms can be used to obtain a suboptimal rank one solution (see [8] and references therein).

4.2. Zero-forcing Beamforming

When the number of relays ($K$) is large, we have to process the optimization over a long vector $w \in \mathbb{C}^{K \times 1}$ which yields high complexity. Next we apply the ZF concept into the beamforming matrix design to completely cancel IDI. The ZF operation also projects the beamforming vector to a low dimensional subspace so that the number of variables for optimization is smaller, thus lowering complexity.

To completely cancel the interference at each destination, the weight vector needs to satisfy $[5,10]$

$$(g^{T}_j \otimes f^{*_j}_j)w = 0, i, j = 1, \ldots, M, i \neq j$$

Note that $K > M(M-1)$ is needed to guarantee non-zero solution. Denoting $C_q = [e_1, \ldots, e_M]^T$ with $e_i = [f_i \otimes R_1, \ldots, f_i \otimes R_{K-1}, f_i \otimes R_{K+1}, \ldots, f_i \otimes R_M]$, the SVD of $C_q$ is expressed as

$$C_q = U \Sigma V^H$$

where $V = [V_s, V_{null}]$, in which $V_s \in \mathbb{C}^{K \times (M(M-1))}$ and $V_{null} \in \mathbb{C}^{K \times (K(M-1))}$. To satisfy (22), it is sufficient and necessary for $w$ to have the following structure:

$$w = V_{null} \tilde{w}$$

where $\tilde{w} \in \mathbb{C}^{K - M(M-1) \times 1}$. Substituting (22) and (24) into (20), the transmit power minimization problem under ZF conditions in the distributed relay case is formulated as

$$\min_{\tilde{w}} \tilde{w}^H \tilde{R}^H \tilde{R}_T \tilde{w}, \text{ s.t. } \tilde{w}^H \tilde{Q}^{(i)}_T \tilde{w} \geq 1, i = 1, \ldots, M$$

where $\tilde{Q}^{(i)}_T = \gamma^{-1}_i[V_{null}^T P_0 R^{(i)}_T - \gamma_i R^{(i)}_T]V_{null}$ and $\tilde{R}_T = V_{null} R_T V_{null}$. This problem can also be solved by SDP with relaxation similarly as in the optimal beamforming case.

5. SIMULATION RESULTS AND DISCUSSIONS

A three-source three-destination system is considered. The channel coefficients of source-relay-antenna pairs or relay-antenna-destination pair are i.i.d. zero mean complex Gaussian random variables with unit variance. Three schemes are evaluated in the simulations: the ZFBF with single multi-antenna relay, as given in (17), the optimal beamforming with distributed relays as given in (20), and ZFBF with distributed relays as in (25). Each figure is based on 5,000 channel realizations.

Figure 2 demonstrates the average relay power needed to meet the SINR requirement of at least one destination, no matter how large the power the relays consume. With larger source power $P_0$, the outage probabilities decrease in all cases, while the ZFBF approach with a single multi-antenna relay outperforms the other two approaches. With increasing source power the gap among the three outage probabilities decrease.

Figure 2 shows the outage probability of the three cases. Outage occurs when the relay(s) fail to meet the SINR requirement of at least one destination, no matter how large power the relays consume. With larger source power $P_0$, the outage probabilities decrease in all cases, while the ZFBF approach with a single multi-antenna relay outperforms the other two approaches. With increasing source power the gap among the three outage probabilities decrease.

Figure 3 demonstrates the average relay power needed to meet the SINR requirement of each destination over all their non-outage transmission periods. However, if one scheme’s outage probability in simulation exceeds 10%, the power consumed at relays will be set to $+\infty$ which indicates that the source-relay-destination link cannot support reliable communications. Again, the ZFBF in the single multi-antenna relay case yields the smallest required transmit power at the relays. Also, the performance differences among the three methods are smaller with higher source power.

The effect of the relay number on the optimal transmit power at the relays is shown in Fig. 3. With $K < M(M-1) + 1 = 7$, the ZFBF in distributed relay case does not work because the ZF conditions cannot be satisfied. With more relays, all three schemes need less power to satisfy the SINR requirements at the destinations. With fewer relay antennas, the ZFBF in the distributed relay case lacks enough dimensions to process the optimization ($w \in \mathbb{C}^{K - M(M-1) \times 1}$), and that causes the required sum transmit power at the relays to be very high. With more relays, the minimized required power in the ZF scheme is very close to that of the optimal beamforming with distributed relays. One can also see that in Fig.3, the sum transmit power of relays is all under 25dB for providing at least 10dB SINR at the destinations. This affordable power consumption
6. CONCLUSIONS

In this paper we focus on the relay systems with multiple sources and destinations. Both the case of single relay with multi-antenna and distributed single antenna relays are considered. Three beamforming schemes are proposed to maximize the power efficiency of relay(s) and to meet SINR requirements at the destinations under perfectly known CSI. Moreover, the simulation results provide guidelines for choosing the beamforming method, which is determined by whether single or distributed relay(s) are available, the number of destination nodes and the number of antennas at the relays. Future work will include user scheduling algorithms for the case in which the communications of some source-destination pairs cannot be supported and analysis of queuing delays occurring due to outage.

7. REFERENCES


