ABSTRACT
A novel distributed TDoA (Time-Difference-of-Arrival) estimation method for wireless sensor networks (WSN) is proposed in this letter. Linear frequency modulation (LFM) waves are emitted from two anchors simultaneously to create an interference field. Through the frequency measurement of local RSSI (Received Signal Strength Indication) signal, the TDoA is estimated independently at each sensor to detect range difference from the sensor to the two anchors. Furthermore, an extended method applied to mobile sensors is also presented. The proposed method only relies on radio transceivers and requires no time synchronization for sensors. Simulations results demonstrate the effectiveness of our method.

1. INTRODUCTION
Wireless sensor network (WSN) localization is an important area that attracted significant research interest. In many environmental monitoring applications, the measurement data are meaningless without knowing the location from where the data are obtained. The TDoA (Time-Difference-of-Arrival) is a commonly used method which utilizes the range difference to conduct positioning according to hyperbolic algorithm [1].

TDoA technique based on ultrasound needs the auxiliary ultrasound transceiver that adds to the cost and size of the platform, and also has the weakness of limited range and directionality constraints. Therefore, it cannot be applied to large-scale networks [2]. The UWB technique can be utilized to conduct TOA or TDOA estimation, which has the advantages of low-cost and penetrating ability, but also has the shortcoming of short-range [3][4]. In [5], we presents a TDoA estimation method which only relies on radio transceiver and the TDoA estimation is based on the Doppler Effect produced by a mobile anchor. Although the method has the advantages of low-cost and high precision, it requires the time-consuming movement of anchor and cannot achieve rapid positioning.

The author in [6] has proposed a range estimation method which utilizes two radio transmitters to create an interference signal with a low frequency envelop. The range estimation is based on the multiple measurement of the relative phase offset between two receivers at different carrier frequencies to combat the phase ambiguity effects, which greatly complicates the system operation. Moreover, the ranging method is hardly to be applied to mobile sensors.

In this paper, we propose a novel distributed TDoA estimation method for WSN. Two anchors emit linear frequency modulation (LFM) waves with a slight frequency difference to create an interference field. The frequency of RSSI (Received Signal Strength Indication) signal at each sensor is a function of the TDoA from the sensor to the two anchors and the frequency offset between the two emitted waves. Then the TDoA is estimated independently at each sensor through frequency measurement of RSSI signal. Furthermore, an extended method applied to mobile sensors is also presented. The proposed method only relies on radio transceiver and each sensor conducts TDoA estimation independently without any information exchange or time synchronization requirement, which is a distributed TDoA estimation method more suitable for WSN.

2. TDOA ESTIMATION FOR STATIONARY SENSOR
LFM signal can be utilized to conduct high-precision ranging, which has been widely used in radar system. LFMCW (linear frequency modulated continuous wave) radar emits a LFM wave to a fixed direction and receives the echo at the same time, then the distance from the target to radar can be obtained through the frequency offset between the echo and emitted wave.

![Fig. 1. Frequency-time change curves in the LFMCW radar system.](image_url)

Assume the LFM signal emitted by radar is $e^{j(2\pi ft + \phi)}$, where $f$ is the start sweep frequency, $\phi$ is the initial phase, $K$ denote the sweep slop. When the distance from the target to radar is $R$, the reflection signal will lag behind the baseline $\tau = 2R/c$, where $c$ is the light speed. Thus, the received echo signal at the radar can be expressed as

$$r(t) = \alpha \cdot e^{j(2\pi f(t-\tau) + \phi)} e^{j(sK\tau^2 + \phi)} e^{j(2\pi f(t+\tau) + \phi)} e^{-j2\pi K\tau t}$$

where $\alpha$ denote the amplitude of the attenuated signal. It is shown that at any time, the frequency offset between the echo and emitted wave is $K\tau$, which is always proportional to the distance from the target to the radar. Mixing the echo with the emitted wave and conducting spectral analysis for the beat signal, the distance can be obtained. However, this ranging method in radar system cannot be directly applied to WSN.

Inspired by the LFMCW radar, we present a novel TDoA method applied to WSN based on LFM signal. Suppose that the
sensor network consists of numerous sensor nodes and three anchors (A, B, C). The sensor nodes with restricted energy supply are distributed randomly in the sensing field. We assume the anchors are not energy constrained and their communication ranges can cover the whole network. In addition, each anchor has a GPS receiver for self-localization.

![Fig. 2 Illustration of the TDoA estimation system.](image)

The anchors A and B emit LFM waves using a common sweep slope $K = BW/T$ simultaneously with a slight frequency difference: $s_A(t) = e^{j(2\pi f_A t + \pi K t^2 + \varphi_A)}$ and $s_B(t) = e^{j(2\pi f_B t + \pi K t^2 + \varphi_B)}$, producing an interference filed, where $f_A$ and $f_B$ are the start sweep frequencies of $A$ and $B$, $BW$ and $T$ are the sweep bandwidth and period, respectively. Then there is a constant frequency offset $f_A - f_B$ between the two emitted waves at the same time and we call it sweep frequency offset (SFO).

![Fig. 3 Frequency-time change curves in the interference filed.](image)

In the LOS (Line of Sight) environment, the base-band signal at a receiving node $n$ can be expressed as

$$r_n(t) = a_Ae^{j(2\pi f_A t + \pi K t^2 + \varphi_A)} + a_Be^{j(2\pi f_B t + \pi K t^2 + \varphi_B)}, \quad n = 1, 2, \ldots, n$$

(1)

where $a_A$ and $a_B$ denote the amplitudes of attenuated signals, $f_n$ is the LO (local oscillator) frequency of node $n$, $\tau_{An}$ and $\tau_{Bn}$ are the path delays from $A$ and $B$ to $n$.

$$\varphi_A = \pi K \tau_A^2 - 2\pi f_A \tau_A + \varphi_A - \varphi_n,$$

$$\varphi_B = \pi K \tau_B^2 - 2\pi f_B \tau_B + \varphi_B - \varphi_n.$$  

(1)

(1) indicates that the frequency offset between the two sweep waves at node $n$ is converted to $f_A - f_B + K(\tau_{Bn} - \tau_{An})$, where $f_A - f_B$ is the SFO introduced by the transmitting anchors, and $K(\tau_{Bn} - \tau_{An})$ is determined by the TDoA from $n$ to the two anchors. In Fig. 3, the black and gray curves represent the emitted waves of anchors and the received waves at the sensor node, respectively. It can be seen intuitively that the two sweep waves reach the receiver through different propagation delays $\tau_{An}$ and $\tau_{Bn}$, producing a frequency shift $K(\tau_{Bn} - \tau_{An})$ at the receiving node. Thereby, we can obtain the TDoA just through the measurement of this frequency shift.

The RSSI signal obtained through radio RSSI circuitry can be simply utilized to measure this frequency shift. By (1), the RSSI signal of node $n$ can be expressed as

$$||r_n(t)||^2 = a_A^2 + 2a_Aa_B\cos(2\pi(f_A - f_B)t + 2\pi K(\tau_{Bn} - \tau_{An}) + \varphi_A - \varphi_B)$$  

(2)

**Proof:** If the complex base-band signal of sensor is $a_Ae^{j\varphi_1} + a_Be^{j\varphi_2}$, then the RSSI signal can be denoted as

$$||a_Ae^{j\varphi_1} + a_Be^{j\varphi_2}||^2 = (a_Ae^{j\varphi_1} + a_Be^{j\varphi_2})(a_Ae^{-j\varphi_1} + a_Be^{-j\varphi_2}) = a_A^2 + a_B^2 + a_Aa_Be^{-j(\varphi_1 + \varphi_2)}$$

(2)

$$= a_A + a_B + 2a_Aa_B\cos(\varphi_1 - \varphi_2)$$

Let $\varphi_1 = 2\pi(f_A - f_B)t + \pi K t^2 - 2\pi K\tau_{An} + \varphi_A$ and $\varphi_2 = 2\pi(f_B - f_n)t + \pi K t^2 - 2\pi K\tau_{Bn} + \varphi_B$, then (2) holds. This completes the proof.

Thus, when $n$ is stationary, the RSSI signal is a tone sine wave with frequency as follows

$$f_{RSSI,n} = f_A - f_B + K(\tau_{Bn} - \tau_{An})$$  

(3)

If the two anchors have perfect synchronization of time and frequency, a dedicated SFO $f_A - f_B$ can be exactly achieved. Then the TDoA can be directly obtained, given by

$$\tau_{Bn} - \tau_{An} = (f_{RSSI,n} - (f_A - f_B))/K$$

However, in practice, although we assume the energy supply and cost of the anchors are unrestricted and they can achieve more accurate time and frequency synchronization, the absolute synchronization is still unrealistic. Therefore, the time and frequency asynchronism should be taken into consideration, which means that a dedicated SFO can not be exactly achieved by the anchors. However, another anchor $C$ can be utilized to compensate this asynchronous error. In the same sweep period, anchor $C$ can also measure the frequency of its RSSI signal

$$f_{RSSI,C} = f_A - f_B + K(\tau_{BC} - \tau_{AC})$$  

(4)

where $\tau_{AC}$ and $\tau_{BC}$ are the path delays from $A$ to $C$ and $B$ to $C$, respectively. By its known location, the SFO can be estimated by $C$. Then anchor $C$ broadcasts SFO information to the network and each sensor can obtain its own frequency shift

$$f_{shift,n} = f_{RSSI,n} - (f_{RSSI,C} - K(\tau_{BC} - \tau_{AC})) = K(\tau_{Bn} - \tau_{An})$$  

(5)

Then combined with the sweep slope, the TDoA of each sensor can be obtained

$$\tau_{Bn} - \tau_{An} = f_{shift,n}/K$$  

(6)

Thus, due to the assistance of anchor $C$, our method has no requirement for the exact time or frequency synchronization between the two emitting anchors.

On the other hand, measuring the frequency of a given signal with sufficient accuracy becomes a challenge on resource-constrained hardware. A popular and efficient way to determine the frequency of the signals is frequency domain analysis. However, it has been shown in [7] that it would take approximately 15 seconds to calculate a 512-point FFT using an 8MHz processor typically available in many of the commercial sensor nodes. Therefore, a simple time-domain frequency measurement algorithm is adopted in our simulation, which computes the average period of the RSSI.
signal smoothed by a moving average filter. Here we should note that the high accuracy and stability oscillators can be equipped for anchors to achieve a little time and frequency asynchronism. Then the SFO $f_a - f_B$ can be controlled by anchors within a small range, which is suitable for the time-domain frequency measurement at sensors. The simulation results indicate that this simple method can achieve a considerable TDOA estimation performance.

Furthermore, since only one sweep period adopted for frequency measurement will result in limited precision which leads to poor TDOA estimation performance, several periods of sweep waves can be transmitted by anchors, and sensors can average the TDOA estimations over multiple periods.

As a note, by taking advantage of the de-ramp properties a LFM chirp, the work in [8] has proposed another time-difference measurement method for active RFID (radio frequency identification) system. However, this method implements the chirp generation in an active RFID user tag and uses the base stations as listen-only receivers. Thus, the TDOA estimation of each user is obtained at a pair of base stations. Thereby, this scheme is not distributive and unsuitable for large WSN. However, in this paper, we implements the chirp generation in a pair of anchor nodes to generate an interference field, and each sensor node can obtain its TDOA in a completely distributed manner, which is more practical to large networks.

3. TDOA ESTIMATION FOR MOBILE SENSOR

In many WSN scenarios, a fixed network structure cannot satisfy the requirements. The introduction of the mobile node has expanded the WSN applications while results in the failure of the many existing localization method for stationary node [9]. In this section, we will discuss the impact of sensor mobility on our proposed method and an extended method applied to mobile sensors is also presented.

![Fig. 4. Illustration of the mobile wireless sensor network.](image)

Consider a mobile scenario where anchor $A$ and $B$ emit sweep waves simultaneously. For a mobile sensor $n$, we have $d(\tau_{X,n})/dt = v_{X,n}/c$, where anchor $X = A, B$, $c$ is light speed, and $v_{X,n}$ is the radial velocity between $n$ and $X$. By (2), the RSSI frequency at $n$ can be expressed as

$$f_{RSSI,n} = f_A - f_B + K(\tau_{An} - \tau_{An}) + f_B v_{B,n} - f_A v_{A,n} \frac{K}{\lambda_B - \lambda_A}$$

(7)

Compared with (3), the RSSI frequency is shifted by three parts. $f_B v_{B,n} - f_A v_{A,n}/c$ is the Doppler shift. If radio frequency is 300MHz, taking $v_{B,n} - v_{A,n} = 1m/s$ and $K = 1MHz/100ms = 10^7Hz/s$ as an example, the introduced TDOA error is $10^{-5}$s and the corresponding range error is 30m. Thus, this component cannot be ignored. In contrast, the other two parts are rather small and negligible. Then rewrite (7)

$$f_{RSSI,n} = f_A - f_B + K(\tau_{Bn} - \tau_{An}) + \frac{v_{B,n}}{\lambda_B} - \frac{v_{A,n}}{\lambda_A}$$

(8)

where $\lambda_A = c/f_A$ and $\lambda_B = c/f_B$. Thus, in order to obtain the correct TDoA estimation, the Doppler Effect must be eliminated. Fortunately, due to the mobile inertia, the Doppler shift will remain basically unchanged in a short time slot. Then when sweep period is $T$, the RSSI frequencies at $n$ and anchor $C$ can be expressed as

$$\left\{ \begin{array}{l} f_{RSSI,A,K} = f_A - f_B + K(\tau_{Bn} - \tau_{An}) + \frac{v_{B,n}}{\lambda_B} - \frac{v_{A,n}}{\lambda_A} \\ f_{RSSI,C,K} = f_A - f_B + K(\tau_{BC} - \tau_{AC}) \end{array} \right.$$ 

(9)

By (5), the frequency shift obtained by sensor is

$$f_{shift,n,K} = K(\tau_{Bn} - \tau_{An}) + \frac{v_{B,n}}{\lambda_B} - \frac{v_{A,n}}{\lambda_A}$$

(10)

Next, when the sweep period is shorten to $T/2$ with same sweep bandwidth, we have

$$\left\{ \begin{array}{l} f_{RSSI,A,2K} = f_A - f_B + 2K(\tau_{Bn} - \tau_{An}) + \frac{v_{B,n}}{\lambda_B} - \frac{v_{A,n}}{\lambda_A} \\ f_{RSSI,C,2K} = f_A - f_B + 2K(\tau_{BC} - \tau_{AC}) \end{array} \right.$$ 

(11)

Then, the frequency shift is

$$f_{shift,n,2K} = 2K(\tau_{Bn} - \tau_{An}) + \frac{v_{B,n}}{\lambda_B} - \frac{v_{A,n}}{\lambda_A}$$

(12)

Hence, when sweep period is halved, the Doppler shift remains unchanged, while the frequency shift caused by sweep increases doubly. Then through the difference of above two frequency shifts, the correct TDOA can be obtained

$$\tau_{Bn} - \tau_{An} = (f_{shift,n,2K} - f_{shift,n,K})/\lambda$$

(13)

Furthermore, similar to section II, considering the limited moving distance of mobile sensor in a short time slot, the TDOA can be averaged over multiple continuous sweep periods in order to improve the estimation accuracy.

4. SIMULATION

In simulations, we assume the SFO obeys uniform distribution: $f_A - f_B \sim U(180Hz, 220Hz)$, the sweep period is 100ms, and the sample rate of RSSI is 16KHz.

4.1. TDOA Estimation Performance for Stationary Sensor

In the stationary scenario, the distance between sensor and anchors obeys uniform distribution: $\tau_{An}, \tau_{Bn} \sim U(0, 500m/c)$. We adopt the average range error obtained by TDOA to evaluate the estimation performance.

The average range error versus SNR is shown in fig.5, where the sweep bandwidth and number of periods are 1.0MHz and 10, respectively. The three curves correspond to different SNR condition at anchor $C$, and the curve identified by green line represents that ideal receiver is adopted at anchor $C$. It is shown that the SNR increase at sensor node and anchor $C$ will both improve the estimation performance. Since the anchor servers for all network, the increase of its receiver performance will introduce the estimation accuracy improvement of whole network. As a note, when SNR at anchor reaches 25dB, it is shown that the estimation performance is basically close to the performance when ideal receiver is equipped for anchor.

Fig.6 shows the average range error versus SNR with different sweep bandwidth, where SNR of anchor $C$ is 20dB and the number of sweep periods is 10. Explicitly, the accuracy of TDOA estimation is improved greatly when the bandwidth or SNR increases. The
range error reduces to around 1m when the sweep bandwidth reaches 2MHz required in 802.15.4 standard. Moreover, for a comparison, consider that the waves emitted by anchors are none-perfect LFM, and in simulation, a Gaussian phase noise with variance \((\pi/10)^2\) is added to each sample of the ideal LFM waves. It is shown that the estimation performance will be deteriorated by approximately 0.5m in this situation.

### 4.2. TDoA Estimation Performance for Mobile Sensor

In the mobile scenario, anchor A and B are located at (0m,0m) and (500m,0m), respectively. A mobile sensor moves from (-100m,100m) to (600m,100m) at a constant speed of 1m/s. During the movement, the TDoA estimation is conducted once every 10s. Fig. 7 shows the range estimation results obtained by TDoA, where SNR of sensor and C are 10dB and 20dB, respectively. It is shown that, when 10 sweep periods are adopted, the estimations are close to the true values and the mobile sensor can obtain the valid TDoA estimations through our method.

![Fig. 5](image.png)  
**Fig. 5.** Average range error obtained by TDoA versus SNR with different SNR of anchor C.

![Fig. 6](image.png)  
**Fig. 6.** Average range error obtained by TDoA versus SNR with different sweep bandwidth.

5. CONCLUSIONS

This paper presents a novel distributed TDOA estimation method which only relies on radio transceivers without other auxiliary measurement equipments. Through frequency measurement of the low frequency RSSI signal, each sensor estimates its TDOA independently. Furthermore, an extended method applied to mobile sensors is also presented. Analysis and simulation results demonstrate the effectiveness of our proposed method.

6. REFERENCES