DISTORTION EXPONENTS OF SOURCE TRANSMISSION OVER TWO-WAY RELAYING COOPERATIVE NETWORKS

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ABSTRACT

In this paper, we consider the source transmission in a three-node, half-duplex, and two-way relaying network, where two users communicate with the help of one relay. The relay employs the decode-and-forward (DF) based relaying protocol. We study the distortion exponent that characterizes the high signal-to-noise ratio (SNR) behavior of the end-to-end distortion of the reconstructed signal at each user node. We first provide an upper bound on the achievable distortion exponents of two-way relaying communications, which is tight at large bandwidth ratio. We then investigate the performance of various coding and transmission schemes, including the conventional one-way relaying strategies and source-channel coding in two-way relaying with single-rate coding or limited channel state feedback. We derive the achievable distortion exponents of all these schemes and illustrate the effect of the bandwidth ratio, feedback resolution, and relaying strategies on the optimal distortion exponent.

1. INTRODUCTION

Cooperative communication is a promising technique to combat fading and facilitate robust transmission in wireless communication systems. Recently, there have been growing interests in the field of two-way relaying communications, also known as bidirectional relaying, where two source nodes communicate simultaneously in both directions with the help of one relay. Instead of simply employing the traditional one-way relaying protocols to let the relay take turns to forward the message from each user, intelligent two-way relaying strategies have been proposed in, for example, [1–3], which significantly improve the system throughput. The problem has also been investigated from a physical network coding perspective [4,5], where the network coding is directly performed at the physical layer. Most of these works assume perfect channel state information (CSI) at the transmitters (CSIT). Recently, a two-way relaying scheme based on the limited channel state feedback is proposed in [6].

In this paper, we consider the problem of transmitting discrete-time analog-amplitude sources over two-way relaying cooperative networks. Different from most of the current works, we study the performance of the overall system in terms of the end-to-end distortion between the source signal and its reconstruction at the destination, which is typically used in multimedia communications.

It was first proposed in [7] to use the distortion exponent as a performance measure of the end-to-end distortion, which is defined to be the asymptotic exponential decay rate of the expected end-to-end distortion in the high-SNR regime. Since then, various results have been obtained in the distortion exponent analysis of source transmissions over non-ergodic fading channels, such as parallel fading channels [7], the multiple-input multiple-output (MIMO) fading channels [8,9], and fading relay channels [10–12]. However, to the best of our knowledge, the distortion exponent of two-way relaying cooperative systems has not been studied in the literature yet.

Our goal in this paper is to study the distortion exponent of a half-duplex two-way relaying system. We provide an upper bound on the achievable distortion exponent of any possible relaying and transmission strategy, which is tight at large bandwidth ratio. We also derive the achievable distortion exponents of various coding and transmission schemes, including conventional one-way relaying strategies and source-channel coding in two-way relaying with single-rate coding or limited channel state feedback. The results illustrate the effect of the bandwidth ratio, feedback resolution, and relaying strategies on the overall performance of the two-way relaying system via the distortion exponent. We show that, without CSIT, even with the simple single-rate source-channel coding, an improved performance of the two-way relaying protocol can be observed over sophisticated one-way relaying strategies such as the layered coding with progressive transmission or the broadcast strategy [11,12]. Furthermore, the distortion exponent can be improved with only a few bits of feedback information if perfect CSIT is available at the relay.

2. SYSTEM MODEL

We consider a three-node two-way relaying communication system, where two source nodes T1 and T2 communicate with the help of a relay node T3. We assume there is no direct link between the two source nodes. The system model is shown in Fig. 1. All nodes are equipped with single antenna and operate in half-duplex mode. The relay employs the two-phase DF-based two-way relaying protocol in [3] with perfect synchronization. All nodes are assumed to have full CSI of the incoming links only, if not otherwise mentioned.

The transmission is done in two phases. In Phase 1, both source nodes transmit simultaneously to the relay node, which resembles a multiple-access channel (MAC). The source signals x1 and x2 are assumed to be memoryless, zero-mean, unit-variance complex Gaussian with the same channel coding rate of R bits per channel use. We focus exclusively on the symmetric-rate case in this paper for its tractability, which also allows a better insight into the general problem. The received signal at time instant k at the relay node is then y[k] = h13[k]x1[k] + h23[k]x2[k] + n[k], where n[k] is the additive noise. In Phase 2, the relay jointly decodes the received signal.
The distortion exponent of a half-duplex two-way relaying system as a function of the bandwidth ratio $b$.

**Theorem 3.1.** The distortion exponent of a half-duplex two-way relaying system can be upper-bounded by

$$\Delta_{UB} = \min \left\{ \frac{b}{2}, 1 \right\}.$$  

**Proof.** Due to symmetry, it suffices to consider only the source transmission from $T_1$ to $T_2$. The achievable distortion exponent of the transmitted signal from $T_2$ to $T_1$ is then the same. To obtain an upper bound, we let $T_1$ and $T_3$ fully cooperate and assume that perfect CSI is available at all nodes. The original system then reduces to a two-node half-duplex two-way communication system without any relay, as shown in Fig. 2. The same technique is also used to derive a DMT upper bound of the two-way relaying system in [6].

Assume the cooperation node $C$ transmits for a fraction $t$ of the time, and $T_2$ transmits for the remaining fraction $1 - t$ of the time, $t \in (0, 1)$. Let $|h_1|^2 = \gamma^{-\theta_1}$ and $|h_2|^2 = \gamma^{-\theta_2}$. Let the code rate $R = r \log \gamma$, where $r$ is the multiplexing gain. By standard large-deviation arguments, the outage probability is characterized by [6]

$$P_{out} = \Pr \left\{ t(1-\theta_1)^+ + (1-t)(1-\theta_2)^+ < r \right\},$$

where $x^+ \triangleq \max\{x, 0\}$.

By (4), both $C$ and $T_2$ need to transmit at a multiplexing gain of $r \leq r^* \triangleq \min\{t(1-\theta_1)^+, (1-t)(1-\theta_2)^+\}$, otherwise $P_{out} \triangleq \gamma^0$, i.e., the outage probability does not decay with the SNR. As a result, the minimum expected end-to-end distortion is given by

$$D = \int_{\mathbb{R}^2} 2^{-br^* \log \gamma} f(\theta_1, \theta_2) d\theta_1 d\theta_2$$

$$\leq \int_{\mathbb{R}^2} e^{-r \min\{t(1-\theta_1)^+ + \theta_1 + \theta_2\} + \theta_1 + \theta_2} d\theta_1 d\theta_2 \leq e^{-\Delta} \gamma^2$$

where for the first exponential equality, we use the following result from [11]: $f(\theta_t) \triangleq \gamma^{-\theta_t}$ for $\theta_t \geq 0$, and zero otherwise. The second exponential equality follows from Laplace’s method [13].

Optimizing over $t \in (0, 1)$, the distortion exponent upper bound is then

$$\Delta_{UB} = \sup_{t \in (0,1)} \Delta = \min \left\{ \frac{b}{2}, 1 \right\},$$

where the maximum is achieved at $t = t_0$.

We now try to approach the upper bound using traditional one-way DF-relaying based strategies, for which the relay takes turns to forward each user’s information. The communication consists of two independent one-way relay-assisted communications in each direction, i.e., $T_1 \rightarrow T_3 \rightarrow T_2$ and $T_2 \rightarrow T_3 \rightarrow T_1$. And overall four phases of transmissions are needed. The corresponding DMT is given by [6]

$$d(r) = (1 - 4r)^+.$$ (7)

We further combine the one-way DF relaying protocol with two transmission techniques, namely the layered coding with progressive transmission (LS) and the broadcast strategy (BS) [11, 12], which have been shown to effectively improve the distortion exponent for communication over one-way fading relay channels.

**Theorem 3.2.** The maximum distortion exponents achieved by one-way DF relaying protocol with LS and BS strategies for a half-duplex two-way relaying system are given by

$$\Delta_{LS} = 1 - e^{-\frac{b}{2}}, \quad \Delta_{BS} = \min \left\{ \frac{b}{4}, 1 \right\},$$

where both LS and BS have infinite coding layers.

**Proof.** The results in (8) can be directly obtained from the DMT in (7) by using the distortion exponent results for one-way relaying [11]. The proof is straightforward and hence omitted.
A quick examination reveals that conventional one-way relaying schemes can still achieve the distortion exponent upper bound when bandwidth ratio is large, for example, $\Delta_{BS} = 1$ for $b \geq 4$. However, they are in general not optimal as will be shown later in the paper.

4. SINGLE-RATE SOURCE AND CHANNEL CODING

In this section, we consider single-rate source-channel coding for source signals. The coded symbols are sent by the two-way DF relaying protocol. The corresponding achievable DMT is given by [6]
\[
d_{SR}(r) = \begin{cases} 1 - 2r, & 0 \leq r < \frac{1}{6} \\ 2 - \frac{4r}{\beta^*}, & \frac{1}{6} \leq r \leq \frac{1}{2} \end{cases}
\]
where $\beta^* = \frac{5r + 1}{\sqrt{\frac{(3r - 2)^2}{2} - 16}}$.

For single-rate source-channel coding with channel coding rate $R = r \log \gamma$, the expected end-to-end distortion can be written as
\[
D = (1 - P_{out}) \cdot 2^{-bR} + P_{out} \geq \gamma^{-br} + \gamma^{-d_{SR}(r)} \geq \gamma^{-\min\{br, d_{SR}(r)\}},
\]
where $P_{out} = \gamma^{-d_{SR}(r)}$ is the outage probability. The optimal distortion exponent is then given by the following theorem.

**Theorem 4.1.** The maximum distortion exponent of a half-duplex two-way DF relaying system with single-rate source-channel coding is
\[
\Delta_{SR} = \begin{cases} \frac{3(b+1) - \sqrt{(b+1)^2 - 32}}{2(b+3)}, & 0 \leq b < 4 \\ \frac{6}{b+2}, & b \geq 4 \end{cases}
\]

**Proof.** By (9) and (10), the maximum distortion exponent is then
\[
\Delta_{SR} = \max_{r \in [0, \frac{1}{4}]} \min\{br, d_{SR}(r)\}
\]
\[
= \max_{r \in [0, \frac{1}{4}]} \left\{ \min_{r \in [0, \frac{1}{2}]} \left\{ br, 1 - 2r \right\}, \min_{r \in [\frac{1}{2}, \frac{1}{4}]} \left\{ br, 2 - \frac{4r}{\beta^*} \right\} \right\},
\]
which can then be solved to be (11).

5. SOURCE-CHANNEL CODING WITH FEEDBACK

It is natural to consider utilizing the feedback to facilitate the source transmission in two-way communications. In this section, we study two schemes that employ limited channel state feedback from the relay, and analyze their distortion exponent performance.

5.1. No CSIT

We first study a feedback-based scheme (FB-N) proposed in [6], where no CSIT is initially available at any node. In this scheme, all nodes are equipped with a library of $K$ codebooks and a set of fractions $0 < t_1 < \cdots < t_K < 1$. The relay feeds back an index $l \in \{1, 2, \cdots, K\}$ to both source nodes based on $(h_{13}, h_{23})$. Upon receiving index $l$, both sources encode the signals at a rate of $R = r \log \gamma$ bits per channel use, where $r$ is the multiplexing gain, and transmit for a fraction $t_l$ of total channel uses. The relay transmits for $1 - t_l$ of total channel uses with two-way DF relaying. Please refer to [6] for the details of the feedback index mapping rule.

In the limit of $K \to \infty$, the DMT of the feedback scheme is given by $d_{FB-N}(r)$ in Thm. 3 of [6]. From this we can derive the corresponding distortion exponent as follows, which is also the maximum distortion exponent of the feedback scheme.

**Theorem 5.1.** The maximum distortion exponent of FB-N with half-duplex two-way DF relaying in the limit of $K \to \infty$ is given by
\[
\Delta_{FB-N} = \frac{b + 3 - \sqrt{(b+3)^2 - 8b}}{4}.
\]

**Proof.** As in (10), when $K \to \infty$, the expected end-to-end distortion of the feedback scheme is found to be $D = \gamma^{-\min\{br, d_{FB-N}(r)\}}$.

The maximum distortion exponent is then
\[
\Delta_{FB-N} = \max_{r \in [0, \frac{1}{4}]} \min\{br, d_{FB-N}(r)\} = \max_{r \in [0, \frac{1}{4}]} \min\left\{ br, 1 - \frac{3r}{1 - 2r} \right\},
\]
which can then be solved to be (13).

5.2. Perfect CSIT at the relay

It can be quickly verified that the distortion exponent achieved by FB-N in (13) fails to approach the upper bound in (3), which is partly due to the lack of knowledge of CSIT. We now propose a feedback scheme (FB-R) by assuming that perfect CSIT is available at the relay, i.e., we assume $h_{31}$ and $h_{32}$ are known by the relay. It is an extension of the feedback scheme for one-way relaying in [10].

In this scheme, all nodes are equipped with a library of $K$ pairs of source-channel encoder and decoder, each has a coding rate of $R_i = r_i \log \gamma$ bits per channel use, where $0 < r_1 < \cdots < r_K < 1$ are the multiplexing gains. The relay feeds back an index $l \in \{1, 2, \cdots, K\}$ to both sources. The feedback is assumed to be noiseless and zero-delay. Upon receiving index $l$, both source nodes encode the source symbols at a code rate of $R = R_l$. The transmission then follows from the non-feedback case.

Recall that the two transmission phases resemble the MAC channel and the BC channel, respectively. Denote $h_1 = (h_{13}, h_{23})$ and $h_2 = (h_{31}, h_{32})$. Define the following regions of $h_1$ and $h_2$

\[
\mathcal{I}_{MAC}(r, t) = \left\{ h_1 : r \log \gamma \leq t \log(1 + r^2 |h_{13}|^2), i = 1, 2, 2r \log \gamma \leq t \log(1 + r^2 |h_{13}|^2 + |h_{23}|^2) \right\},
\]
\[
\mathcal{I}_{BC}(r, t) = \left\{ h_2 : r \log \gamma \leq t \log(1 + r^2 |h_{23}|^2), i = 1, 2 \right\}.
\]

The feedback index $l^*$ sent by the relay node is then
\[
l^* = \max_{1 \leq l \leq K} l, \text{ s.t. } h_1 \in \mathcal{I}_{MAC}(r, t_l) \text{ and } h_2 \in \mathcal{I}_{BC}(r, 1 - t_l),
\]

(16)

That is, the relay informs the source node to use the lowest code rate possible for which the transmission will not be in outage. If $l^* \neq 0$, i.e., an outage occurs even the lowest code rate is used, an arbitrary index will be sent since no reliable communication is possible.

We now present the following theorem that characterizes the optimal distortion exponent of the proposed feedback scheme.

**Theorem 5.2.** The optimal distortion exponent of FB-R with half-duplex two-way DF relaying is
\[
\Delta_{FB-R} = \sup_{0 < r_1 < \cdots < r_K < 1, 0 \geq t_l \leq K} \min_{0 \leq t_l \leq K} \{ d_{SR}(r_{l+1}) + br_l \},
\]
where $d_{SR}(r)$ is the DMT of single-rate coding given by (9), and we define $r_0 = 0$ and $r_{K+1} = 1$.

**Proof.** The proof uses the same technique as that of Proposition 2 in [10], and is omitted due to the space limitation.
Fig. 3. Comparison of various source-channel transmission schemes in a two-way relaying cooperative system.

\[ \Delta_{FB-R}^K \] does not appear to be analytically tractable in general. However, it can be readily verified that the set of functions \( \{ d_{SR}(r_{i+1}) + br \} \) are concave in \( r_i \), and so is their point-wise minimum. Hence, in (17) we have a convex optimization problem that can be efficiently solved by standard tools.

In the limit of infinite feedback resolution \( (K \to \infty) \), we have a continuum of multiplexing gains. The distortion exponent is thus

\[
\Delta_{FB-R}^\infty = \min_{r \in [0, \frac{1}{2}]} d_{SR}(r) + br
\]

\[
= \min \left\{ \min_{r \in [0, \frac{1}{2}]} \left\{ 1 - 2r + br \right\}, \min_{r \in [\frac{1}{2}, 1]} \left\{ 2 - \frac{4r}{3} + br \right\} \right\}
\]

\[
= \min \left\{ \frac{b}{3}, 1 \right\}.
\]

(18)

6. RESULTS AND DISCUSSIONS

The achievable distortion exponents of the single-rate source-channel coding (SR) and the two limited feedback based schemes (FB-N and FB-R) for a two-way DF relaying system are plotted against the upper bound in Fig. 3, respectively. The results of the one-way relaying counterpart with LS and BS strategies are also included for comparison purpose. We see that even with the relatively simple single-rate coding, the two-way relaying can outperform the traditional one-way relaying with sophisticated strategies such as LS and BS at low bandwidth ratio. The distortion exponent can be further improved by using the feedback-based schemes. However, it can be seen that the performance of the FB-N scheme is still limited even with infinite feedback resolution, which is partly due to the lack of CSIT. With perfect CSIT at the relay node, the FB-R scheme approaches the distortion exponent upper bound for a large range of bandwidth ratios. It also gets very close to the limiting case \( (K \to \infty) \) with as few as 2 bits of feedback information \( (K = 4) \).

Compared with the upper bound, it can be seen that although most studied schemes can achieve or approach the upper bound when \( b \) is large, they fail to approach the upper bound for small bandwidth ratios. For example, even as the feedback resolution \( K \) goes to infinity, the distortion exponent of the feedback scheme still does not converge to the upper bound for \( b \leq 3 \). This is actually not unexpected for the following reasons: First, as shown in [6], the two-way DF relaying is DMT-optimal only at low multiplexing gains, which hence limits the overall distortion exponent performance. Second, since the upper bound assumes full CSI at all nodes as well as perfect cooperation, it is still not clear whether such optimality can be achieved or not. Therefore, further investigation on sophisticated two-way relaying strategies that improve the distortion exponent in the low bandwidth ratio regime remains a topic for future study.

7. CONCLUSIONS

In this paper, we study the source transmission in two-way relaying cooperative systems. Different from most of the current work, we focus on the analysis of the end-to-end distortion via the distortion exponent. We provide an upper bound on the achievable distortion exponents of two-way relaying communications, which is tight at large bandwidth ratio. We also derive the distortion exponents of various coding and transmission strategies that involve joint source-channel coding and limited channel state feedback.

8. REFERENCES


