AN EFFICIENT RANGING METHOD FOR WIRELESS SENSOR NETWORKS

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ABSTRACT
The Radio Interferometric Positioning System (RIPS) provides a localization method with high accuracy and simple hardware configuration for sensor networks by means of measuring the phase difference of the interference signal. However, due to the periodicity of phase, it is hard to determine the actual distance difference only from a single phase measurement. To solve the problem, RIPS makes multiple measurements at different frequencies so as to convert phase difference to distance difference, which is a computationally intensive searching process and not suitable for the energy-constrained wireless sensor nodes. In this paper, we propose an efficient ranging method based on Chinese Remainder Theorem (CRT). Meanwhile, we utilize some properties of the coefficients in the CRT algorithm to avoid the over-sensitivity of the traditional CRT, which increases the robustness of the algorithm. We apply this robust CRT algorithm to the ranging process which calculates the distance difference directly in a closed form equation and therefore reduces the response time and energy consumption of the ranging procedure.

Index Terms—Ranging, Phase Ambiguity, Chinese Remainder Theorem, Measurement noise, Robustness.

1. INTRODUCTION

Position information of node is very important in Wireless Sensor Networks (WSN). By far, the most widely discussed localization techniques in the literature can be classified into two categories, range-free and range-based[7]. Range-free is those positioning methods which do not need to know the actual distance between nodes. It is usually based on connectivity or coverage etc. Of the two the range-based approach which measures the distance between nodes directly usually provides higher localization accuracy than the range-free approach. These distance measurements can be implemented by measuring the received signal strength indicator (RSSI), time of arrival (TOA), or time difference of arrival (TDOA). However, these methods will face many practical challenges when applied to the resource-limited wireless sensor nodes. TDOA usually requires extra acous-

This work is supported by the National Natural Science Foundation of China under Grant 60772095 and the National Hi-Tech Research & Development Program under Grants No. 2006AA01Z220 of China.
phase is always the residue distance which is the actual distance modulo wavelength, and ignores the integer wavelength information.

In order to solve the phase ambiguity problem, RIPS measures the phase offset on different frequencies, namely, measures the same unknown distance with different wavelengths and gets the corresponding residual distance. By searching the possible integer combinations of wavelengths, it finally finds the distance with minimum mean square error. If this computationally intensive searching algorithm is implemented on the energy-restrained wireless sensor nodes, it certainly becomes a heavy burden of the node and the response time of the localization process will be increased. In short, how to obtain the distance from phase quickly and efficiently becomes a key problem in localization system based on phase measurement.

In fact, the Chinese Remainder Theorem (CRT) is an effective tool to solve the problem. It is an analytic procedure for calculating the unambiguous integer dividend from the remainders which are produced from integer division by several prime numbers. By analogy with the phase ambiguity problem here, it is not hard to find out, the dividend in the theorem corresponds to the distance we need to estimate, the prime divisors correspond to different wavelengths, the remainders correspond to the remainder parts of the distance measured by different wavelengths. The problem of applying CRT is it is too sensitive to noise. Even a small amount of noise in phase measurement can result in enormous error. Unfortunately, phase measurement is not perfect and it is inevitably affected by noise. For this reason, some literatures propose robust CRT algorithms which reduce the sensitivity to noise and apply them to the velocity and range estimation in pulse Doppler radar and continuous wave radar system[10][11]. Inspired by those related research work, this paper presents an efficient ranging method based on robust CRT, which avoids the time and energy-consuming search process in RIPS and calculates the distance difference in a closed form equation.

2. RANGING METHOD BASED ON CHINESE REMAINDER THEOREM

2.1. Problem Description

Four nodes are involved in a single RIPS measurement, as illustrated in Fig. 1. Two nodes, say node A and node B, transmit sine wave at slightly different frequencies to obtain an interference signal which is received by the other two nodes (node C and node D). The phase difference between the two received signals on each node is a function of the distances between the four nodes[2]. Let \( \theta \) be this phase difference of the two received signals, we have

\[
\theta = 2\pi \frac{d_{AB} - d_{BD} + d_{BC} - d_{AC}}{\lambda_{\text{carrier}}} \pmod{2\pi} \tag{1}
\]

where \( \lambda_{\text{carrier}} \) is the wavelength of carrier signal.

After figuring out the combination of distances such as \( d_{ABCD} \), \( d_{ACBD} \) etc, RIPS utilizes genetic algorithm to determine the position of each nodes. In addition, if node A, B, and C are anchor nodes whose position is known, we can further calculate distance \( d_{AD} - d_{BD} \) and \( d_{CD} - d_{AD} \) with which two hyperbolas are determined and the position node D is the intersection point of the two hyperbolas.

![Figure 1. Instead of directly measuring the distance, RIPS obtains the distance difference \( d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC} \) from phase measurement.](image)

Obviously, it is not possible to determine the actual distance difference \( d_{ABCD} \) from the measured phase difference merely by a single RIPS measurement because the phase is always modulo \( 2\pi \). The process of converting phase difference to distance difference is what we mainly consider and focus in this paper. In order to solve phase ambiguity, we can measure the same distance with multiple frequencies so that there are more constrains to the equation. To be more specific, If we measure the same distance difference \( d \) by \( k \) different wavelengths \( \lambda_i \) \( (i = 1 \cdots k) \) (i.e. the frequency of the signal is \( f_i = c/\lambda_i \)), we can get the corresponding phase \( \theta_i \in [0,2\pi] \) which can be further converted to the fraction part of the distance \( \gamma_i = \lambda_i \cdot \theta_i / (2\pi) \). This process can be expressed as,

\[
d = n_i \lambda_i + \gamma_i \tag{2}
\]

where \( n_i \) is the unknown integer part of the i-th wavelength. Ranging is the process of estimating \( d \) from a set of \( \gamma_i \).

The simplest but the most inefficient approach is to search every possible value of the \( k \) integer \( n_i \). Each value will calculate \( k \) distances. When the \( k \) distances consistent with each other or mean square error of these distances is small enough, the actual distance is obtained. However, the searching process needs enormous calculation, which is a heavy burden to the wireless sensor nodes who have very limited computational capacity and energy.

2.2. Ranging by CRT and its extension

CRT is an analytical process of calculating dividend from remainders. Assume the unknown integer dividend is \( x \) which is divided by \( k \) prime numbers \( P_i > 1 \), \( (i = 1, 2, \cdots k) \) and get the \( k \) corresponding remainders \( m_i \). Namely,
If \( x \equiv m_i \pmod{P_i} \), \( i = 1 \cdots k \)

(3)

If \( x < M = \prod_{i=1}^{k} P_i \) is satisfied, \( x \) can be determined uniquely by the following equation,

\[
x \equiv (\sum_{i=1}^{k} \delta m_i) \pmod{M}
\]

(4)

where
\[
\delta_i = Q_i q_i
\]

(5)
\[
Q_i = M / P_i
\]

(6)

and \( q_i \) is the modular inverse of \( Q_i \), i.e. \( Q_i q_i \equiv 1 \pmod{P_i} \). Modular inverse can be calculated by extended Euclidean algorithm or other improved algorithms [4].

Usually, the value of \( \delta_i \) is far more than 1, so (4) is very sensitive. If there is some error in measurement, say the error of \( m_i \) is \( \Delta m_i \), the final estimation error of \( x \) is

\[
\Delta x = \sum_{i=1}^{k} \delta_i \Delta m_i \pmod{M}
\]

(7)

Because \( \delta_i \) is far more than 1, from (7) we can see a small amount of error in phase measurement will be amplified greatly by \( \delta_i \), resulting in very large error of distance estimation. However, the following property tells us the sum of \( \delta_i \) is very small modulo \( M \) [11].

Property:
\[
\sum_{i=1}^{k} \delta_i \equiv 1 \pmod{M}
\]

(8)

If the errors on each wavelength are the same, \( \Delta m_i = \Delta m \). Utilizing the property, we have,

\[
\Delta x \equiv \sum_{i=1}^{k} \delta_i \Delta m \pmod{M}
\]

\[
\equiv \Delta m \pmod{M}
\]

(9)

Thus the sensitivity to noise is reduced significantly. We have the extended CRT: If the divisors \( L_i \) are not relative-prime numbers, and let they have the greatest common divisor \( C \ge 1 \). Then \( L_i \) can be expressed as \( L_i = C P_i \), \( (P_i, P_j) = 1 \), \( i \neq j \), where \( (\bullet) \) denotes the greatest common divisor. \( \{L_i\} \) have least common multiple of \( N = CM = C \prod_{i=1}^{k} P_i \). The extended CRT on non-relative-prime divisors condition is stated as follows:

Given that \( x \equiv m_i \pmod{L_i} \), \( i = 1,2,\ldots,k \).

If \( x < N \), \( x \) has a unique solution:

\[
x \equiv C x_0 + r \pmod{N}
\]

(10)

where

\[
x_0 \equiv \sum_{i=1}^{k} \delta_i b_i \pmod{M}
\]

(11)
\[
b_i = \lceil m_i / C \rceil
\]

(12)

\( [\bullet] \) denotes rounding down to the nearest integer. And \( r \) is the common remainder of \( m_i \) modulo \( C \) [11].

\[
r = m_i - b_i \cdot C
\]

(13)

If we examine the extended CRT, it is not hard to find out since the coefficients \( \delta_i \) of \( b_i \) are far more than 1, the theorem has a high sensitivity to the error of \( b_i \) and a low sensitivity to that of \( r \). If each \( b_i \) has the same error, utilizing property can reduce its sensitivity to error. Therefore, the problem is further transformed into how to make the error on each \( b_i \) have the same value.

We first consider the situation without noise. \( b_i \) can not only be calculated by (12), but also can be figured out by (13):

\[
r = m_i - b_i C = m_i - b_i C
\]

(14)
\[
b_i = (m_i - m_0) / C + b_i
\]

(15)

When there are errors in phase measurement, assuming that \( m_i = m_0 + \Delta m_i \) where \( m_0 \) is its ideal value, \( \Delta m_i \) is its error. If we utilize (12) to calculate \( b_i \), we have \( b_i = \lceil m_i / C \rceil = b_0 + \Delta b_i \).

For the rest of \( b_i, i = 2,3,\ldots,k \), we use (15) to calculate \( b_i \):

\[
b_i = (m_i - m_0) / C + b_i + (\Delta m_i - \Delta m_i) / C
\]

(16)

If the term \( (\Delta m_i - \Delta m_i) / C \) does not introduce errors, \( b_i \) and \( \Delta b_i \) will have the same error. Besides, the noise makes \( b_i \) not integers in (16) while the CRT persists to solve in the field of integer. A simple approach is to round \( b_i \) to the nearest integer. That is,

\[
b_i = \text{round}((m_i - m_0) / C) + b_i
\]

\[
= (m_0 - m_0) / C + b_i + \text{round}((\Delta m_i - \Delta m_i) / C)
\]

(17)

When the following condition is satisfied,

\[
| (\Delta m_i - \Delta m_i) / C | < 0.5
\]

(18)

We have \( \text{round}((\Delta m_i - \Delta m_i) / C) = 0 \), then

\[
b_i = (m_0 - m_0) / C + b_i
\]

\[
= b_0 + \Delta b_i
\]

(19)
From (19) we can see that every \( b_i \) have the same error as \( b_i \). According to property 2, if the condition in (18) is satisfied, the final error on \( x \) will be \( \Delta b_i \), rather than \( \sum_{i=1}^{k} b_i \Delta b_i \).

For the remainder part \( r \) in (10), because the measured value \( m_i \) is contaminated by noise, (13) no longer holds. In other words, the remainder of each \( m_i \) is not the same modulo \( C \). We average the remainders of \( m_i \) modulo \( C \) to reduce the impact of noise. That is

\[
r = \sum_{i=1}^{k} r_i / k
\]

where

\[
r_i = m_i - b_i C
\]

2.3. Error Analysis

From the above discussion, the final estimation result would have a normal error which means it does not suffer from over-sensitivity if all \( b_i \) have the same error value. The possibility of having the same error value among \( b_i \) and \( b_i \) is \( P \left| \left| \Delta m_i - \Delta m_j \right| / C \right| < 0.5 = p \). Supposing \( \Delta m_i \) is i.i.d. the possibility of normal error of the final estimation is \( P = p^k \), where \( k \) is the number of wavelengths used in measurement. Here we assume \( \Delta m_i \approx N(0, \sigma^2) \), and then we have \( (\Delta m_i - \Delta m_j) \sim N(0, 2\sigma^2) \). The possibility of the same error among \( b_i \) is

\[
p = \frac{1}{2\sigma \sqrt{\pi}} \int_{-c/2}^{c/2} \exp \left( -\frac{x^2}{4\sigma^2} \right) dx = 2\Phi \left( \frac{C}{2\sqrt{2}\sigma} \right) - 1
\]

where \( \Phi(x) \) is the distribution function of the standard normal distribution. \( p \) increases with \( C / \sigma \). If \( k = 10 \), \( C = 50 \), \( \sigma = 6 \), \( p = 0.9968 \), the possibility with normal error is \( P = 0.9716 \).

3. SIMULATION

The following simulation demonstrates the distribution of estimation error under Gaussian noise. We choose wavelengths to be \( \lambda = \{0.1150, 0.1200, 0.1250\} \) m, quantization step to be 0.1mm, so that \( C = 50 \), \( P = \{23, 24, 25\} \), and the maximum unambiguous range is 69m. The unknown distance difference \( d \) is uniformly distributed in \([0, 69]\) m. Assuming \( \Delta m_i \) obeys normal distribution \( N(0, \sigma^2) \), we have normal estimation error if condition (18) is satisfied. For example, we choose \( \sigma = 10 \), and perform 100000 times of simulation and obtain the histogram of the estimation error \( \Delta x = \hat{x} - x \), as shown in Fig.2 (a).

If condition (18) is not satisfied, over-sensitive error occurs, as the side lobes shown in Fig. 2 (a). If \( \Delta m_i \) is constrained to satisfy (18), we have the distribution of error shown in Fig. 2 (b). The error has zero-mean value, so the estimation is unbiased. If oversensitivity occurs which can be detected by the maximum communication range, we can have another measurement until it satisfy (18).

4. REFERENCES


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