KNOWLEDGE-AIDED REDUCED-RANK STAP FOR MIMO RADAR BASED ON JOINT ITERATIVE CONSTRAINED OPTIMIZATION OF ADAPTIVE FILTERS WITH MULTIPLE CONSTRAINTS

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ABSTRACT

In this paper, a reduced-rank knowledge-aided technique for MIMO radar space-time adaptive processing (STAP) design is proposed. We focus on the advantage of MIMO radars in achieving better spatial resolution by employing the colocated antennas. The scheme is based on knowledge-aided constrained joint iterative optimization of adaptive filters (KAC-JIOAF) and takes advantage of the a priori covariance matrix by employing additional linear constraints in the design. A recursive least squares (RLS) implementation is derived to reduce the computational complexity. We evaluate the algorithm in terms of signal-to-interference-plus-noise ratio (SINR) and probability of detection \( P_D \) performance and compare it with the state-of-the-art reduced-rank algorithms. Simulations show that the proposed algorithm outperforms existing reduced-rank algorithms.

Index Terms— Knowledge-aided techniques, MIMO radar, Space-time adaptive processing, Reduced-rank.

1. INTRODUCTION

The concept of multiple-input multiple-output (MIMO) radar has received considerable attention since the pioneer work was reported in [1–3]. Two advantages of MIMO radars, spatial diversity [2, 5] and spatial resolution [1, 4], were discussed respectively. This paper focuses on the advantage of MIMO radars in achieving better spatial resolution by employing the colocated antennas. More details about spatial diversity can be found in [5] and the references therein.

Following the landmark publication by Brennan and Reed [6], space-time adaptive processing (STAP) techniques for the SIMO radar have been well developed. The STAP techniques can be easily adopted to the MIMO radar systems with slight modifications [1, 7]. However, the large computational complexity of STAP often prohibits full-rank processing in real-time. This is further complicated in the MIMO radar case, due to the extra degrees of freedom (DOFs). Reduced-rank adaptive filtering is a key technique to lower the computational complexity and improve convergence performance. The family of Krylov subspace methods, including the multistage Wiener filter (MSWF) [8] and auxiliary-vector filters (AVF) [9], has been well investigated in the recent years. Despite their improved convergence and tracking performance, these methods are very complex to implement in practice and suffer from numerical problems. Recently, reduced-rank filtering algorithms based on joint iterative optimization of filters were proposed [10] and applied to the airborne radar systems [11], outperforming the MSWF and the AVF.

In this paper, we develop a reduced-rank knowledge-aided approach for MIMO radar STAP design, which is an extension of a JOINT scheme in [11]. In [12, 13], knowledge-aided signal processing techniques were reported to offer the promise of significantly improved performance for all radar systems. The term "knowledge" can refer to either clutter predictions based on digital terrain databases study or estimates from a previous scanning. In this case, we assume that a priori covariance matrix \( R_0 \) is known at the receiver. The proposed knowledge-aided constrained joint iterative optimization of adaptive filters (KAC-JIOAF) algorithm takes advantage of prior knowledge by employing additional linear constraints. A recursive least squares (RLS) implementation is derived to reduce the computational complexity. We evaluate the algorithm in terms of SINR and probability of detection \( P_D \) performance, and compare it with the state-of-the-art reduced-rank algorithms.

This paper is organized as follows. Section 2 states the signal model and the problem with which we are concerned. Section 3 presents the proposed knowledge-aided reduced-rank STAP algorithm for MIMO radar. In Section 4, we present the proposed adaptive RLS algorithm. The performance results of the proposed reduced-rank STAP are provided in Section 5 using simulated radar data. Finally, conclusions are given in Section 6.

2. SIGNAL MODEL AND PROBLEM STATEMENT

In this section, we consider the STAP problem in a MIMO radar residing on an airborne platform. The transmitting and receiving antenna arrays are linear and parallel, consisting of \( N_T \) and \( N_R \) elements, respectively. Assuming that the transmitter and receiver, which are moving at the same speed and in the same direction, are close enough, we can simplify the bistatic radar case and consider they share the same inject angle \( \theta \) [7]. The MIMO radar transmitter emits omnidirectional orthogonal or noncoherent waveforms and at each receiving antenna, these orthogonal waveforms are extracted by \( N_T \) matched filters. Considering \( M \) pulses at the range of interest and properly deploying the spacing between the transmitting antennas \( d_T \) and the spacing between the receiving antennas \( d_R \), we can obtain an \( N_T \times N_R \times M \) 3-dimensional space-time observation data cube. Thus, MIMO radars create \( N_T \) times as many DOFs as conventional SIMO radars.

2.1. MIMO Radar Signal Model

As shown in Fig. 1, at the MIMO radar receiver, the sufficient statistics can be extracted by a bank of matched filters. The space-time snapshot is formed by stacking the 3-dimensional data cube to form

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an $K \times 1, K = N_T N_R M$ column-wise vector. Given a space-time snapshot, radar detection is a binary hypothesis problem, where hypothesis $H_0$ corresponds to target absence and hypothesis $H_1$ corresponds to target presence. The $K \times K$ covariance matrix $R$ of the undesired clutter-plus-jammer-plus-noise component can be modelled as
\[
R = \mathbb{E}\{v(i) v^H(i)\} = R_c + R_j + R_n,
\]
where $v(i)$ denotes the input interference-plus-noise vector, $(.)^H$ represents Hermitian transpose and $\mathbb{E}$ denotes expectation. $R_n$ denotes the noise covariance matrix, $R_j$ is the jamming covariance matrix and $R_c$ is the clutter covariance matrix.

The vector $s$, which is the $K \times 1$ normalized space-time steering vector in the space-time look-direction, can be defined as:
\[
s = s_{tx}(\vartheta_i) \otimes s_{rx}(\vartheta_i) \otimes s_t(\varpi_i),
\]
where $s_{tx}(\vartheta_i)$ and $s_{rx}(\vartheta_i)$ are the $N_T \times 1$ normalized transmitting spatial steering vector and $N_R \times 1$ normalized receiving spatial steering vector in the direction provided by the target spatial frequency $\vartheta_i$, respectively and $s_t(\varpi_i)$ is the $M \times 1$ normalized temporal steering vector at the target Doppler frequency $\varpi_i$.

### 2.2. Full-Rank KA-STAP

The full-rank KA-STAP [13] is obtained by considering the optimization problem as follows:
\[
\omega_{opt} = \arg \min_{\omega(i)} \mathbb{E}[\|\omega^H(i)r(i)\|^2]
\]
\[
\begin{aligned}
&\text{s. t.} \\
&\omega^H(i) R_0 \omega(i) = 1 \\
&\omega^H(i) R_0 \omega(i) \leq \delta_d \\
&\omega^H(i) \omega(i) \leq \delta_g
\end{aligned}
\]
where $r(i)$ is the received observation vector at the time $i$, $\delta_d$ is chosen to be a desired gain and the quadratic constraint $\omega^H(i) R_0 \omega(i) \leq \delta_g$

\[
\delta_d \text{ forces the solution to be orthogonal or nearly orthogonal to the a priori clutter covariance matrix } R_0, \text{ which can be derived from the digital terrain database or the data probed by radar in previous scans. The solution to this optimization problem is}
\]
\[
\omega_{opt} = \frac{(R + Q)^{-1}s}{s^H(R + Q)^{-1}s},
\]
where $R = \mathbb{E}[r(i)r^H(i)]$ and $Q = \beta_2 R_0 + \beta_3 I_K$ [13]. If we require the solution strictly orthogonal to the a priori clutter covariance matrix $R_0$, the quadratic constraint in (3) can be substituted by $V^H \omega(i) = 0$ where $V$ is a unit vector set of $R_0$ and $0$ is a zero vector.

### 3. PROPOSED KAC-JIOAF ALGORITHM

In this section, we detail the principles of the proposed knowledge-aided constrained joint iterative optimization of adaptive filters (KAC-JIOAF) scheme, which is depicted in Fig. 2. The proposed KAC-JIOAF algorithm employs a transformation matrix $S_D(i)$ with dimensions $K \times D$ to project the $K \times 1$ observation vector $r(i)$ onto a lower dimensional subspace to form the $D \times 1$ reduced input vector $\tilde{r}(i)$. The reduced-rank filter $\bar{\omega}(i)$ with dimensions $D \times 1$ processes the reduced-rank input vector $\tilde{r}(i)$ and yields a scalar estimate $\bar{y}(i)$, which is sent to detection device to determine if a target exists.

Let us construct the transformation matrix as a bank of $D$ full-rank filters $\{t_d(i)|d = 1, \ldots, D\}$ with the dimensionality of $K$, that is, $S_D(i) = [t_1(i), t_2(i), \ldots, t_D(i)]$. The output estimate $y(i)$ of the proposed reduced-rank STAP can be expressed as a function of the transformation matrix $S_D(i)$, the reduced-rank filter $\bar{\omega}(i)$ and the received observation vector $r(i)$, shown as follows:
\[
y(i) = \bar{\omega}^H(i) S_D^H(i) r(i).
\]

Note that the full-rank filter can be viewed as a special case of the proposed scheme when $D = 1$. For $D > 1$, the signal processing tasks are changed and the full-rank filters compute a subspace decomposition and the reduced-rank filter estimates the desired signal. According to the linearly constrained minimum variance (LCMV) criterion, the filters are designed to solve the following optimization problem
\[
[S_{D, opt}, \tilde{\omega}_{opt}] = \arg \min_{S_{D(i)}, \omega(i)} \mathbb{E}\left[\|\omega^H(i) S_D^H(i) r(i)\|^2\right]
\]
\[
\begin{aligned}
&\text{s. t.} \\
&\omega^H(i) S_D^H(i) U = f^H,
\end{aligned}
\]
where $U$ is the $K \times L$ designed constraint matrix and $f$ is a $K \times 1$ vector. As is well known, the rank of clutter subspace is low. It motivates us to incorporate a set of eigen vectors of $R_0$ corresponding to the dominant eigen values as the nulling constraint together with
the directional constraint $\omega^H(i)S_D^H(i)s = 1$. Thus, the constraint is expressed by

$$U = \begin{bmatrix} s \ V_{L-1} \end{bmatrix}$$

$$f = \begin{bmatrix} 1\ 0 \cdots \ 0 \end{bmatrix}^T_{L-1}$$

where $V_{L-1}$ denotes the eigen vector set containing the first $L - 1$ columns of $V$.

The constrained optimization problem in (6) can be transformed into an unconstrained optimization problem by the method of Lagrange multipliers. The cost function can be defined as

$$\mathcal{L}(S_D(i), \omega(i)) = \mathbb{E} \left[ |\omega^H(i)S_D^H(i)r(i)|^2 \right] + 2\Re \left\{ (\omega^H(i)S_D^H(i)U - f^H)\lambda \right\}$$

where $\lambda$ is a $L \times 1$ vector of Lagrange multipliers and the operator $\Re\{\cdot\}$ denotes the real part of the argument. By fixing $\omega(i)$, we calculate the gradient of (8) with respect to $S_D(i)$ as

$$\nabla_{S_D(i)} \mathcal{L} = RS_D(i)\omega(i)\omega^H(i) + UL\lambda\omega^H(i). \tag{9}$$

By equating (9) to zero and solving for $\lambda$, we get

$$S_D(i) = R^{-1}U\left[U^HRU\right]^{-1}f\omega^H(i) \tag{10}$$

where the operator $(\cdot)^+$ denotes the pseudo-inverse. The expression of $S_D(i)$ can be further simplified to

$$S_D(i) = R^{-1}U\left[U^HRU\right]^{-1}fs^H(i). \tag{11}$$

By fixing $S_D(i)$, the gradient of (8) with respect to $\omega(i)$ can be written as

$$\nabla_{\omega(i)} \mathcal{L} = S_D^H(i)\mathbf{R}_S(i)\omega(i) + S_D^H(i)U\lambda. \tag{12}$$

By equating (12) to zero and solving for $\lambda$, we obtain

$$\omega(i) = \hat{\mathbf{R}}^{-1}S_D^H(i)U\left[U^H\mathbf{R}_S(i)\hat{\mathbf{R}}^{-1}S_D^H(i)U\right]^{-1}f \tag{13}$$

where $\hat{\mathbf{R}} = S_D^H(i)\mathbf{R}_S(i)$ denotes the reduced-rank covariance matrix.

### 4. ADAPTIVE ALGORITHM

In this section, we develop an RLS algorithm that adaptively adjusts the coefficients of the transformation matrix $S_D(i)$ and the reduced-rank filter $\omega(i)$ based on the least squares (LS) cost function shown as

$$L_{LS}(S_D(i), \omega(i)) = \sum_{n=1}^{i} \alpha^{i-n} \left| \omega^H(i)S_D^H(i)r(i) \right|^2 + 2\Re \left\{ \omega^H(i)S_D^H(i)U - f^H \right\} \lambda,$$

where $\alpha$ is the forgetting factor. By computing the gradients of (14) with respect to $S_D(i)$ and $\omega(i)$, respectively, and equating them to zero, we obtain

$$S_D(i) = \hat{\mathbf{R}}^{-1}(i)U\left[U^H\hat{\mathbf{R}}(i)U\right]^{-1}fs^H(i) \tag{15}$$

$$\omega(i) = \hat{\mathbf{R}}^{-1}(i)S_D^H(i)U\left[U^H\hat{\mathbf{R}}(i)\hat{\mathbf{R}}^{-1}(i)S_D^H(i)U\right]^{-1}f.$$

### 5. SIMULATIONS

In this section, we compare the performance of our proposed KAC-JOAF algorithm and other existing algorithms, say full rank RLS, MSWF with the RLS [8], AVF [9] and JOINT with the RLS [11] algorithms. In the simulations, the clutter is generated by using the model in [7] and parameters are shown in Table 1. We assume that there are two jammers at $45^\circ$ and $60^\circ$ with jammer-to-noise ratio (JNR) equal to 40 dB. All presented results are averages over 1000 independent Monte-Carlo runs.

In the first experiment, the probability of detection $P_D$ versus SNR performance is presented for all schemes using 200 snapshots as the training data as shown in Fig. 3. The false alarm rate $P_{FA}$ is set to $10^{-6}$ and we suppose the target injected in the boresight (0°) with Doppler frequency 100Hz. The figure illustrates that the proposed algorithm provides sub-optimal detection performance using

<table>
<thead>
<tr>
<th>Table 1. MIMO Radar System Parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>------------------------------------</td>
</tr>
<tr>
<td>Antenna array</td>
</tr>
<tr>
<td>Carrier frequency ($f_c$)</td>
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<tr>
<td>Transmit pattern</td>
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<tr>
<td>PRF ($f_r$)</td>
</tr>
<tr>
<td>Platform velocity ($v$)</td>
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<tr>
<td>Platform height ($h$)</td>
</tr>
<tr>
<td>Clutter-to-Noise ratio (CNR)</td>
</tr>
<tr>
<td>Jammer-to-Noise ratio (JNR)</td>
</tr>
<tr>
<td>Elements of transmitting antennas</td>
</tr>
<tr>
<td>Elements of receiving antennas</td>
</tr>
<tr>
<td>Ratio of $d_T$ and $d_R$</td>
</tr>
<tr>
<td>Number of Pulses ($M$)</td>
</tr>
</tbody>
</table>

where $\hat{\mathbf{R}} = \sum_{n=1}^{i} \alpha^{i-n}r(i)r^H(i)$ denotes the time average covariance matrix estimate and $\hat{\mathbf{R}} = \sum_{n=1}^{i} \alpha^{i-n}r(i)r^H(i)$ denotes the time average reduced-rank covariance matrix estimate. By employing the matrix inversion lemma, $\hat{\mathbf{R}}^{-1}(i)$ and $\hat{\mathbf{R}}^{-1}(i)$ can be substituted by $\Theta(i)$ and $\hat{\Theta}(i)$ as follows

$$S_D(i) = \Theta(i)U\left[U^H\Theta(i)U\right]^{-1}fs^H(i)$$

where $\Theta(i)$ can be recursively estimated by

$$\Psi(i) = \frac{\alpha^{-1}\Theta(i)r(i)}{1 + \alpha^{-1}r^H(i)\Theta(i)r(i)}$$

$$\Theta(i+1) = \alpha^{-1}\Theta(i) - \alpha^{-1}\Psi(i)r^H(i)\Theta(i)$$

$\Theta(i)$ can be recursively estimated by

$$\hat{\Psi}(i) = \frac{\alpha^{-1}\hat{\Theta}(i)\hat{r}(i)}{1 + \alpha^{-1}\hat{r}^H(i)\hat{\Theta}(i)\hat{r}(i)}$$

$$\Theta(i+1) = \alpha^{-1}\Theta(i) - \alpha^{-1}\hat{\Psi}(i)r^H(i)\Theta(i)$$

where $\Theta(i)$ and $\hat{\Theta}(i)$ are initialized to scaled identity matrices $\delta I$ with dimensions $K$ and $D$, respectively. $\delta$ is a small constant and $I$ is the identity matrix. It is worth noting that the transformation matrix $S_D(i)$ and the reduced-rank filter $\omega(i)$ can be iteratively updated.
short support data. Note that at the detection rate of 90 percent, the proposed scheme obtains 1 dB SNR gain compared with our prior scheme [11] and at least 2 dB SNR gain compared with the AVF.

In the second experiment, we evaluate the SINR performance against the number of snapshots of the proposed KAC-JIOAF-RLS algorithm and compare it with the existing algorithms, as shown in Fig. 4. The schemes are simulated over 700 snapshots and the SNR is set at 10 dB. The numerical results show that the proposed KAC-JIOAF-RLS algorithm outperforms other schemes, achieving faster convergence and better SINR performance.

6. CONCLUSIONS

In this paper, we focused on the advantage of MIMO radars in achieving better spatial resolution by employing the colocated antennas. A reduced-rank knowledge-aided approach for MIMO radar STAP design was developed. The proposed knowledge-aided constrained joint iterative optimization of adaptive filters (KAC-JIOAF) took advantage of prior environmental knowledge by employing additional linear constraints. We considered null constraints together with the directional constraint by using prior covariance matrix knowledge. The RLS implementation was derived to reduce the computational complexity. We evaluated the algorithm in terms of SINR and probability of detection $P_D$ performance, in comparison with the state-of-the-art reduced-rank algorithms. Simulations showed that the proposed algorithm outperforms existing reduced-rank algorithms.

7. REFERENCES