ITERATIVE HOS-SOS (IHOSS) BASED SENSOR LOCALIZATION AND DIRECTION-OF-ARRIVAL ESTIMATION

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ABSTRACT

A new method for joint Direction-of-Arrival (DOA) and sensor position estimation is introduced. The proposed method exploits the advantages of both higher-order-statistics (HOS) and second-order-statistics (SOS) with an iterative algorithm. The ambiguity problem in sensor position estimation is solved by observing source signals at least in two different frequencies. These frequencies should satisfy a certain condition which is presented in the paper. A new cost function is defined for the DOA and position estimation. The proposed method works even when the source signals are correlated contrary to the HOS approaches. The performance of the method approaches to the Cramer-Rao bound (CRB) for both DOA and position estimation.

Index Terms— Direction-of-Arrival Estimation, Sensor Localization, Higher-Order-Statistics

1. INTRODUCTION

Previous literature in DOA estimation is mostly based on sensor arrays where there are only small perturbations for the sensor positions in the neighborhood of nominal sensor positions. In this work, the sensors are randomly distributed except that the distance and direction between two reference sensors are assumed to be known. In this setting, both the unknown positions of the sensors and the DOA angles for multiple sources are found.

In [1], it is shown that the combination of HOS and the ESPRIT algorithm allows one to compute DOA estimates for any arbitrary sensor geometry without knowing the sensor positions. Therefore, the requirement for special array geometry for the ESPRIT algorithm is eliminated. The only required information is the distance and direction between two reference sensors. In [1], source signals are assumed to be independent and long observations are used for accurate DOA estimation. The estimation performance degrades significantly in case of multiple sources [2] with small number of observations and correlated signals. To our knowledge there is no previous work which estimates DOA and sensor positions in a joint manner without knowing the nominal sensor positions.

The proposed method uses HOS and SOS approaches in a joint manner to take advantages of both techniques. For this purpose, ESPRIT and MUSIC algorithms are employed in an iterative manner to find the unambiguous positions of the sensors as well as the DOAs of multiple sources. CLEAN algorithm [3] is used to improve the HOS estimations for multiple sources. Unambiguous sensor positions are found by using multiple frequencies [4]. Proposed approach improves the estimation performance significantly even when the number of snapshots is small and there is some correlation between source signals in comparison to HOS approach [1].

2. PROBLEM STATEMENT

It is assumed that the array is composed of randomly deployed M sensors and there are L far-field sources. The two of the sensors are the reference sensors. It is assumed that the distance between the reference sensors can be adjusted to be less than or equal to $\lambda/2$. Furthermore the direction vector between the reference sensors is known. The received signal vector for the sensor array can be written as,

$$x(t) = A(\Theta, P)s(t) + v(t), \quad t = 1, 2, \ldots, N$$

where, $N$ is the number of snapshots, $s(t) = [s_1(t), \ldots, s_L(t)]^T$ is the $L \times 1$ vector of $L$ sources, $v(t)$ is the $M \times 1$ vector of Gaussian noise and $A(\Theta, P)$ is the $M \times L$ array steering matrix. Source DOA vector, $\Theta = [\theta_1, \ldots, \theta_L]$ and sensor positions, $P = [p_{1x}, \ldots, p_{Mx}]^T$ are used to write the steering matrix as,

$$[A(\Theta, P)]_{mi} = \exp \left( j2\pi f_c \frac{p_m \omega \cos(\theta_i) + p_m \omega \sin(\theta_i)}{\omega} \right)$$

where, $\theta_i$ is the direction-of-arrival of $i^{th}$ source in azimuth, $p_m = [p_{mx}, p_{my}]$ is the 2D position of the $m^{th}$ sensor, $f_c$ is the center frequency of the sources and $\omega$ is the speed of propagation. ($\cdot$)$^T$ is the transpose operator.

The goal in this paper is to estimate both DOAs of $L$ sources and the positions of $M - 2$ sensors. Note that the positions of these sensors are found relative to the reference sensors. If the positions of the reference sensors are known, then the relativity for the positions can be eliminated. In order to solve the ambiguity on sensor positions, it is assumed that the array output is available at multiple frequencies for the same sources.

3. HOS BASED BLIND DOA ESTIMATION

In this section, we present the general expression for the cumulant matrix which is valid when the assumptions in [1] do not hold. More specifically, the presented expression is valid when the source signals are not independent and there is finite number of observations.

HOS can effectively be used in DOA estimation for arbitrary sensor deployment by employing the ESPRIT algorithm. Once the reference sensors are selected, the fourth order cumulant has cross-correlation information between the actual and some virtual sensors located relative to the actual sensors. Therefore, an ESPRIT structure where the two sub-arrays composed of the actual and virtual
sensors is obtained. The relation between the actual and virtual sub-
arrays is obtained by “virtual cross-correlation computation” [1] approach expressed in matrix form as [5],
\[
C = \begin{bmatrix}
(A \otimes a^*_1) C_s (A \otimes a^*_1)^H & (A \otimes a^*_2) C_s (A \otimes a^*_2)^H \\
(A \otimes a^*_2) C_s (A \otimes a^*_1)^H & (A \otimes a^*_1) C_s (A \otimes a^*_2)^H
\end{bmatrix}
\]
where, \(\otimes\) is the Kronecker product, \((.)^H\) is the complex conjugate operation and \((.)^T\) is the conjugate transpose operator, \(a_i\), is the \(i^{th}\) row of the array steering matrix, \(A\), and \(C_s\), is the \(L^2 \times L^2\) source cumulant matrix in the form of,
\[
C_{s}(i,j) = Cum\{s_k(t), s'_l(t), s_m(t), s'_n(t)\}
\]
where, \(i = L(m-1) + l\) and \(j = L(n-1) + k\) for \(1 \leq k, l, m, n \leq L\). Sensors 1 and 2 are selected as the reference sensors and their row vectors are related as,
\[
a_{r2} = a_{r1}D
\]
\[
D = diag\left(e^{-j2\pi f_c \Delta \cos(\theta_{s1})}, \ldots, e^{-j2\pi f_c \Delta \cos(\theta_{sL})}\right)
\]
Note that without loss of generality, we assume that the reference sensors are located at \([0, 0]\) and \([\Delta, 0]\) on the coordinate system where \(\Delta \leq \lambda/2\).

When the source signals are statistically independent, \(C_s\) is a diagonal matrix with only \(L\) non-zero diagonal elements. The non-
diagonal elements are located with the indices \(L(i-1) + i\) for \(1 \leq i \leq L\). When there are multiple sources with limited number of observations, source signals cannot be assumed to be independent. In this case, the cumulant matrix in (3) can be written in a more general form as in equation (6) by substituting (5) into (3). The matrix \(C^d\) is in the same form of the correlation matrix in the ESPRIT algorithm. The only difference is the source correlation matrix defined for SOS is replaced by \(C^d_{HOS}\). \(C^d_{s}\) corresponds to the error term in the source cumulant matrix due to undesired nonzero terms when the independency assumption is not satisfied.

In this paper, the error term due to multiple sources and finite length observations is decreased by converting the problem into a single source case. This is done by using the CLEAN algorithm [3]. It is known that the CLEAN algorithm removes the effect of additional sources sufficiently well even when the source DOA is estimated with an error.

4. IHOSS ALGORITHM

In this section, we introduce a new approach for improving the performance of parameter estimations in the presence of errors in the cumulant matrix without changing the constraints on array structures. The proposed approach is based on improving the accuracy of the cumulant matrix with the joint use of HOS and SOS techniques in an iterative manner. Therefore, the algorithm is called as IHOSS (Iterative Higher-Order Second-Order Statistics). Under the given problem setting, ESPRIT and MUSIC algorithms are selected for the practical implementation for HOS and SOS approaches respectively. The details of IHOSS algorithm are explained in the following subsections.

4.1. Improved Cumulant Matrix Estimation

The cumulant matrix in (6) contains the error term, \(C^e\), due to the undesired nonzero elements in source cumulant matrix. In this paper, we propose to decrease the errors in cumulant matrix by converting the multiple source problem into a single source case since, when there is a single source, \(C^e\) is equal to zero. For this purpose, we use the CLEAN algorithm to estimate the array outputs of each individual source.

When the array steering matrix estimate, supplied by the HOS approach, is used, CLEAN algorithm estimates the source signals as,
\[
\hat{s}(t) = \hat{A}^\dagger(\Theta, P)x(t)
\]
where, \(\dagger\) corresponds to the Moore-Penrose pseudoinverse. Then, the array output for the \(i^{th}\) source is found as,
\[
\hat{x}^{(i)}(t) = x(t) - \sum_{j \neq i} a_j s_j(t)
\]
where, \(a_j\) is the \(j^{th}\) column of the array steering matrix and \(s_j(t)\) is the \(j^{th}\) source. Substituting (7) into (8), the array output for the \(i^{th}\) source can be rewritten as,
\[
\hat{x}^{(i)}(t) = \left(I - AE; A^\dagger\right)x(t)
\]
\[
\hat{x}^{(i)}(t) = Q_i x(t)
\]
where, \(E_i\) is the \(L \times L\) diagonal matrix whose diagonal elements are one except the \(i^{th}\) element, where it is set to zero. IHOSS finds the cumulative matrix as the sum of cumulative matrices corresponding to the individual sources as given in (10). \(Q_i^c\) is the complex conjugate of the \(i^{th}\) row of the matrix \(Q_i\). \(C_s\) is the \(M^2 \times M^2\) cumulant matrix which contains all the cumulant components including the elements in matrix \(C\) (3) such as,
\[
C_s(k,l) = Cum\{x_{i1}, x_{i2}, x_{s1}, x_{s2}\}
\]
where, \(x_i\) is the output of the \(i^{th}\) sensor, \(k = (k_1 - 1)M + k_2\) and \(l = (l_2 - 1)M + l_1\) for \(1 \leq k_1, k_2, l_1, l_2 \leq M\). Note that if the array output for each individual source is known exactly, it can be shown that the sum of the cumulative matrices of these outputs is equal to the matrix \(C^d\) in (10). In practice, the array outputs for each source signal are estimated as in (9) and hence, the matrix \(C_{s1}^{d1}\) (10) is the estimate of the matrix \(C^d\). Note that while IHOSS uses all sets of cumulants in \(C_s\) to obtain the cumulant matrix, the classical cumulant, \(C\), (3) is composed of the subset of cumulant matrix \(C_s\).

4.2. Unambiguous Sensor Localization

When the DOA estimations are used in (2), sensor locations can be estimated ambiguously. In this paper, the ambiguity problem in sensor positions is solved by using multiple frequencies [4]. In the proposed approach, \(d\) number of frequencies and \(d - 1\) number of sources are sufficient for localizing the sensors in a \(d\)-dimensions. In this paper, we assume that the sensors are deployed in \(2\)-dimensions on a plane.

The elements of the array steering matrix (2) corresponding to \(m^{th}\) sensor and \(i^{th}\) source with center frequency \(f_c\) can be written in the following form,
\[
a_{m}^{i}(f_c) = e^{j2\pi f_c \Delta \cos(\theta_{si})} \left(p_m u(\theta_i) - \frac{\partial}{\partial \theta} k_{fi}\right)
\]
where, \(k_{fi}\) is an integer specified for frequency \(f_c\) and the \(i^{th}\) source. \(u(\theta_i) = [-\cos(\theta_i), -\sin(\theta_i)]^T\) is the unit direction vector of the \(i^{th}\) incoming source. The ambiguity in the position estimate is due to the unknown value of \(k_{fi}\). Let \(f_1^{(i)}\) and \(f_2^{(i)}\) be two different frequencies, where the array output is observed for the same
\[
C = C^d + C^e
\]
\[
C = \begin{bmatrix}
AR_HOS A^H & AR_HOS D A^H \\
AR_HOS D A^H & AR_HOS A^H
\end{bmatrix} + \begin{bmatrix}
(A \otimes a^1) C_s (A \otimes a^1)^H & (A \otimes a^1) C_s (A \otimes a^2)^H \\
(A \otimes a^2) C_s (A \otimes a^1)^H & (A \otimes a^1) C_s (A \otimes a^1)^H
\end{bmatrix}
\]
\[
R_s^{HOS} = diag(\gamma_1, \gamma_2, \ldots, \gamma_L), \quad \gamma_i = Cum(s_i(t), s_i^*(t))
\]
\[
C_{est}^{d} = \sum_{i=1}^{L} \begin{bmatrix}
Q_i (Q_i \otimes q_i^1) C_s (Q_i \otimes q_i^1)^H \\
Q_i (Q_i \otimes q_i^2) C_s (Q_i \otimes q_i^2)^H
\end{bmatrix}
\]

sources. When the equation (12) is considered for \( f_s^{(1)} \) and \( f_s^{(2)} \), the sensor positions found for each frequency should be equal to each other. This relation can be written in terms of a cost function to be minimized to find the optimum values for \( k_{f_s} \). This expression is given in (13), where \( \Xi_m(f_c) \) and \( k_{f_c} \) are as follows,
\[
\Xi_m(f_c) = \begin{bmatrix}
\xi_m(f_c) \\
\xi_m^2(f_c) \\
\ldots \\
\xi_m^L(f_c)
\end{bmatrix}
\]
\[
k_{f_c} = \begin{bmatrix}
k_{f_c,1} \\
k_{f_c,2} \\
\ldots \\
k_{f_c,L}
\end{bmatrix}
\]
\( \xi_m(f_c) \) is the phase term of the array steering matrix element in (12). The two frequencies \( f_s^{(1)} \) and \( f_s^{(2)} \) should be selected properly to resolve the ambiguity. The constraint on the frequencies is given in Lemma-1.

**Lemma-1:** Let the coordinate of the most distant sensor with respect to the reference sensor positioned at \([0, 0]\) is given as \([h_x, h_y]\). The ambiguity in sensor positions is resolved if the following constraint on the two frequencies is satisfied.
\[
\frac{f_s^{(2)}}{f_s^{(1)}} \neq \frac{g_2}{g_1}
\]
\( g_1 \) and \( g_2 \) are the integers that have no common divisors and they are bounded by
\[
\{g_1, g_2\} \leq 2 \left[ \frac{\text{max}(f_s^{(1)}, f_s^{(2)})}{\vartheta_s} \sqrt{h_x^2 + h_y^2} \right] + 1
\]

The proof of Lemma-1 is not given due to space limitations. If the two frequencies satisfy the Lemma-1 then, the unambiguous position of the \( m^{th} \) sensor is found as,
\[
P_m = \left[ \frac{\vartheta_s}{2\pi f_s^{(1)}} \Xi_m(f_s^{(1)}) + \frac{\vartheta_s}{f_s^{(1)} f_s^{(2)}} k_{f_s} \right] U^T(\Theta)
\]
where, \( U(\Theta) = [u(\theta_1) \ldots u(\theta_L)] \).

While two frequencies constrained as in Lemma-1 are sufficient for unambiguous sensor position estimation, additional frequencies can be used to improve the performance especially at low SNR. In this case, the unambiguous position is selected by using the majority vote.

### 4.3. Cost Function and Algorithmic Steps

IHOSS algorithm is composed of a number of steps. It is an iterative algorithm where a new set of sensor positions and DOAs are estimated at each iteration. The main problem in such an iterative approach is that it is not possible to guarantee the convergence of the algorithm due to the number of unknowns and the limited information available. For this reason, we have chosen an alternative approach where the results for each iteration are tested by a cost function in order to select the best possible outcome. In this paper, we propose a cost function, \( \Psi \), which is based on the MUSIC spectra as given below,
\[
\Psi = \frac{1}{\frac{1}{2} L} \int \left( \Gamma(\theta) - \sum_{i=1}^{L} \Gamma(\theta_i) \delta(\theta - \theta_i) \right) d\theta
\]

where, \( \delta(.) \) is the dirac delta function and \( G \) is the \( M \times (M - L) \) matrix whose columns are composed of eigenvectors corresponding to \( M - L \) smallest eigenvalues of correlation matrix obtained in SOS approach. The proposed cost function tries to maximize the peaks in the MUSIC pseudo spectra. In addition, it tries to minimize the residuals in the same spectra.

The algorithmic steps of IHOSS algorithm is given below,

**S1** - Estimate the cumulant matrix, \( C \), from the output array as in (3) and [2]. Then find \( \hat{\Theta}^{(1)} \) and \( \hat{A}^{(1)} \) using estimated cumulant matrix with the ESPRIT algorithm.

**for** \( n = 1: \text{Iteration Number} \)

**S2** - Calculate \( C_{est}^{d} \) (10) using (11) and \( \hat{A}^{(n)} \) in (9).

**S3** - Find \( \hat{\Theta}^{(n)} \) and \( \hat{A}^{(n)} \) using \( C_{est}^{d} \) with the ESPRIT algorithm.

**S4** - Find \( \hat{P}^{(n)} \) as in (17) using (13) and (14).

**S5** - Update \( \hat{\Theta}^{(n)} \) using \( \hat{P}^{(n)} \) with the MUSIC algorithm.

Update \( \hat{A}^{(n)} \) using \( \hat{P}^{(n)} \) and updated \( \hat{\Theta}^{(n)} \) using (2).

**S6** - Find the cost function \( \Psi^{(n)} \) as given in (18)

**end**

**S7** - \( \text{Update} \) the final estimate of DOA, \( \hat{\Theta} = \hat{\Theta}^{(n_{opt})} \)

**S8** - Find the final estimate of DOA, \( \hat{\Theta} = \hat{\Theta}^{(n_{opt})} \)

**S9** - Find the final estimate of sensor positions, \( \hat{P} = \hat{P}^{(n_{opt})} \)

### 5. PERFORMANCE RESULTS

IHOSS algorithm is compared with the VESPA algorithm [1] for DOA estimation, since VESPA cannot estimate the sensor positions. The position estimation with single frequency and single iteration is denoted by IHOSS-Single. Note that IHOSS-Single has ambiguous results for position estimation. CRB is also evaluated for both DOA and sensor position estimation.

There are \( M = 10 \) sensors and the sensors are randomly deployed to an area of 50x50 meters. Two reference sensors are assumed to be located at \([0, 0]\) and \([15, 0]\) meters. There are two far field sources with the corresponding DOA angles of 80 and 120 degrees respectively. It is assumed that the narrowband data for the
source signals is collected at three different frequencies, \( f_1 = 9.85 \), \( f_2 = 9.925 \) and \( f_3 = 10.0 \) MHz. The results are the average of 100 trials. At each trial, source and noise signals as well as the sensor positions except the reference sensors are changed. The source signals have uniform distribution and the noise sequence is additive white Gaussian.

The performance results for DOA and sensor position estimations at different SNR values and iteration numbers are illustrated in Fig. 1. The number of snapshots is selected as \( N = 500 \). In Fig. 1-(a), it is seen that VESPA has a flooring effect. This is due to errors in cumulant matrix at finite length data and multiple sources. IHOSS performs well and closely follows CRB. This is due to the fact that IHOSS converts multiple source problem into a single source one and uses HOS and SOS techniques effectively. The position estimation accuracy in Fig. 1-(b) is especially good at high SNR where the position ambiguity is solved effectively. It is also seen that the required number of iterations for the best result in DOA and sensor position estimation is SNR dependent. As the SNR increases, IHOSS can follow the CRB as the number of iterations is increased.

In Fig. 2 the performance of the algorithm is reported when the number of snapshots is changed. SNR is set to 20 dB. Fig. 2-(a) shows that IHOSS algorithm performs significantly better than VESPA and achieves the CRB even for the small number of snapshots. The number of iterations to find the best result is also small. In Fig. 2-(b), it is seen that IHOSS finds the sensor positions effectively and closely follows the CRB.

6. CONCLUSION

A new method for joint DOA and sensor position estimation is presented when the sensors are deployed to unknown random positions. It is assumed that the distance and the direction between two reference sensors are known. Both HOS and SOS approaches are employed in an iterative manner. The error in cumulant matrix estimation is decreased by using the CLEAN algorithm. The ambiguity problem in sensor positions is solved by using multiple frequencies. Several simulations are done and it is shown that the proposed method improves the performance of DOA and sensor position estimation significantly and approaches to the CRB.

7. REFERENCES


