A KERNEL-APPROACH FOR ESTIMATING THE POSITION OF MOVING OBJECTS

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ABSTRACT

Kernel regression is introduced as a method for solving ill-posed localization problems. To obtain a unique solution the missing data is augmented by the use of a kernel function that comprises the dynamic behavior of the studied system. The proposed approach is based on the minimization of a cost term which combines a least squares estimator and a regularizer in a reproducing kernel Hilbert space. The solution is represented by a finite number of parameters. While the method works for a large class of positive definite kernels we further point out the impact of the kernel design on the quality of the solution. The design of the preferred kernel function is physically motivated. The validity of the method is demonstrated by a real world problem where the available data origins from unsynchronized and singular range measurements to nodes of unknown position.

Index Terms— localization, SLAM, regularization, reproducing kernel Hilbert space, stochastic process

1. INTRODUCTION

Indoor localization is a vibrant field of research and of great interest. While the satellite based Global Positioning System GPS works extremely well with an accuracy that is sufficient for most outdoor applications, the GPS signal is too weak to penetrate most buildings so that GPS based indoor localization persists intractable for a lot of applications with regard to accuracy, availability and reliability. At present a lot of effort is made in developing alternative techniques to expand the range of service from outdoor to indoor. To achieve fast deployment there is a strong tendency towards a self adjusting system that works without a priori knowledge of the environment.

The authors of [1, 2, 3] propose solutions to the problem of geometry estimation of a static cell network when there is sufficient range distance information available between different beacons such that a unique solution can be found. For self localization of a mobile unit, probably the most notable development within this context is a technique called Simultaneous Localization And Mapping (SLAM) where an a priori unknown environment is mapped while keeping track of the own position. Recent research has shown that especially an implementation called FastSLAM mainly based on the work of M. Montemerlo and S. Thrun [4] is a suitable approach with low complexity for localization by fusing information of stereoscopic cameras and inertial navigation sensors. A lot of derivatives of this method for different demands like monocular cameras [5] can be found throughout the literature. For most of the available implementations, control information from inertial sensors or similar are essential for the FastSLAM framework. For the problem of tracking an autonomous underwater vehicle diving on a fixed depth and equipped only with acoustic transceivers measuring ranges to small transponders of unknown position, Newman and Leonard propose a piecewise linear representation of the trajectory on a predefined grid [6]. As the resulting equation system suffers from significant convergence problems, Olson et al. [7] propose a Kalman Filter approach that uses pairwise synchronized range measurements and assumes no movement in between timesteps. A quite similar setup can be found for the LaSLAT algorithm by Taylor et al. [8]. Within this approach synchronized range measurements are used to estimate collections of consecutive positions and the respective transceiver positions via a least-square approach. This is repeated in time for online capability of the method.

In this paper, the main focus will be on solving a special ill-posed range based localization and mapping problem. Unlike former approaches the proposed method does not require synchronized range data sets but incorporates single range measurements into the solution. As a result we obtain a method that works in two steps. An offline step where an initial solution is attained followed by continuous online estimation. The method is generic enough such that it also may find application for other kind of localization problems.

The paper is organized as follows. Section 2 defines the problem of range based localization and mapping and Section 3 proposes kernel regression for such a localization problem. The reduction of the space of possible solutions to a compact set of functions will allow to obtain a unique solution. The kernel design that customizes the approach to our special localization problem will be addressed in Section 4. Simulations within a real world application in section 5 show the applicability of the approach. We conclude in Section 6.
2. PROBLEM STATEMENT

Let $x : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $t \mapsto x(t) = [x(t) \ y(t) \ z(t)]^T$ denote the trajectory of a vehicle moving within a network of beacons positioned at $p_k = [p_{1k} \ p_{2k} \ p_{3k}]^T$. Let further $r^n = \{r_1, \ldots, r_n\}$ be a set of erroneous range observations that can be described by a linear model

$$r_i = ||x(t_i) - p_{ai}|| + \eta_i,$$

and $\alpha^n = \{a_1, \ldots, a_n\}$ the set of given data associations that links the measurement uniquely to a beacon. The additive measurement noise $\eta_i \in \mathbb{R}$ is supposed to be mutually independent and described by a known probability density function $f(\eta_i)$. The Range Only Localization And Mapping (ROLAM) problem is then defined to find the landmark positions $p_k$ while simultaneously tracking the path $x(t)$ of the vehicle. Clearly, the cartesian relationship between two positions $x(t_i)$ and $x(t_{i+1})$ is not observable given only the ranges $r_i$ and $r_{i+1}$ between them. Thus the problem can be considered to be ill-posed as depicted in Figure 1. Moreover, without any additional absolute position information available, there is an increased degree of freedom within this problem definition.

Apparently, the whole scenario can be translated or rotated freely without changing the observations. To resolve this additional multiplicity of the solution we define a local coordinate system through $p_k$.

3. SYSTEM MODEL

The basic idea is to restrict the space of solutions in such a way that the problem becomes well posed. To this end, we apply the framework of kernel regression to the problem. Any element of the velocity function $\dot{x}(t)$ is assumed to be an element of a Reproducing Kernel Hilbert Space (RKHS),

$$\mathcal{H} = \text{span} \{ k(t, \cdot) \mid t \in T \},$$

spanned by the kernel function $k(t_i, t_j)$, i.e. $\dot{x}(t) \in \mathcal{H}$, $\dot{y}(t) \in \mathcal{H}$, and $\dot{z}(t) \in \mathcal{H}$. A quadratic loss function

$$c(t_i, r_i, x_i, p_{ai}) = \frac{1}{\sigma_i^2} (||x(t_i) - p_{ai}|| - r_i)^2$$

with $x_i = x(t_i)$ describes the quality of the fit with respect to our measurements $r_i$. The sum defines the empirical risk

$$R_{\text{emp}}[x, p^k] = \frac{1}{n} \sum_{i=1}^{n} c(t_i, r_i, x_i, p_{ai}).$$

The regularizer is chosen to be

$$\Omega[x] = ||\dot{x}||_{\mathcal{H}} + ||\dot{y}||_{\mathcal{H}} + ||\dot{z}||_{\mathcal{H}}$$

such that the regularized risk functional becomes

$$R_{\text{reg}}[x, p^k] = R_{\text{emp}}[x, p^k] + \mu \Omega[x],$$

with the regularization parameter $\mu \in \mathbb{R}$. Following the mathematical framework of kernel regression [9], the minimum of (6) can be expressed as

$$x(t) = x_0 + \sum_{i=1}^{n} \alpha_i \kappa(t, t_i)$$

with an unknown starting position $x_0 \in \mathbb{R}^3$ at $t = 0$, $\alpha_i = [\alpha_{ix} \ \alpha_{iy} \ \alpha_{iz}]^T$, and the ensemble $\alpha^n = \{\alpha_1, \ldots, \alpha_n\}$. With equation (7) the regularizer can be rewritten as

$$\Omega[x] = \alpha_x^T K \alpha_x + \alpha_y^T K \alpha_y + \alpha_z^T K \alpha_z$$

with the definitions $\alpha_x = [\alpha_{ix} \ \ldots \ \alpha_{nx}]^T$, analogously for $\alpha_y, \alpha_z$ and the $n \times n$ Gram matrix defined as $K_{ij} = k(t_i, t_j)$. The optimization problem finally becomes

$$[\alpha^n, p^k] = \arg\min_{\alpha^n, p^k} \left( R_{\text{emp}}[x, p^k] + \mu \Omega[x] \right).$$

Note: For solving (10) we assume that every unknown quantity is bounded.

4. KERNEL DESIGN

The kernel function is defining the RKHS $\mathcal{H}$ and has a major impact on the regularizer. We choose the kernel mainly based on two assumptions. At first we assume that the trajectory is subject to the following simple equation of motion

$$m \ddot{v} = -\beta v + f(t), \quad v = \dot{x},$$
that is modeled by the parameters $m$ describing the mass, friction parameterized by $\beta$, and an unknown force vector $f(t)$. Second, we assume that the force function is bounded in power. To this end, the power spectral density of the force function is modelled by the two real valued parameters $a > 0$ and $b > 0$

$$S_f(\omega) = a^2 e^{-\frac{b^2}{\omega^2}}. \quad (12)$$

We now construct a stochastic process representing $v$ that exhibits realizations with the upper postulated properties (11) and (12). It’s worth to note that for $S_f(\omega) \triangleq 1$ this process becomes the well known Ornstein-Uhlenbeck process [10]. It emerges that the process representing the trajectory $x$ would become a nonstationary process that can not be modelled by a RKHS spanned by a symmetric kernel function. This is rather clear as a vehicle may start and stop at different locations. Therefore we choose to model the stochastic process that is representing $v$ and covers all relevant realizations of this process by a RKHS that is spanned by the autocovariance function

$$k(t_i, t_j) \triangleq c_0 \left( e^{\frac{\beta \Delta t}{m}} f_1(\Delta t) + e^{\frac{-\beta \Delta t}{m}} f_2(\Delta t) \right) \quad (13)$$

with $\Delta t \triangleq t_j - t_i$, $c_0 \triangleq \frac{a^2}{\beta^2 m} e^{\frac{b^2}{\beta^2 m}}$ and $f_1(\Delta t) \triangleq 1 - \text{erf} \left( \frac{\Delta t}{2} + \frac{\beta}{2 m} \right)$, $f_2(\Delta t) \triangleq 1 + \text{erf} \left( \frac{\Delta t}{2} - \frac{\beta}{2 m} \right)$. 

Note: By this construction the minimizer of (10) can be interpreted as a maximum a posteriori estimator. The kernel function was drawn for $a = 1, b = 2, \beta = 6, m = 10$. Modifications are shown in Figure 2. Note: The so constructed stochastic process for the velocity does not fulfill the Markov property for any order. Nevertheless, the impact of velocities from the past becomes negligible over time which suggests that the process may be accurately approximated by a Markov process of higher order.

5. SIMULATION RESULTS

The tests were performed using a real rover that was steered within an area of about $4 \times 4$ meters. The trajectory forms a double loop with a starting point at $x = 0.0074, y = 0.8346, z = 0$. Range measurements were acquired by 8 simulated UWB-Nodes at positions shown in the following tabular.

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>-3.415</td>
<td>-7.018</td>
<td>-4.229</td>
<td>-0.704</td>
</tr>
<tr>
<td>y</td>
<td>3.680</td>
<td>0.530</td>
<td>-2.027</td>
<td>3.472</td>
</tr>
<tr>
<td>z</td>
<td>0.730</td>
<td>0.736</td>
<td>0.736</td>
<td>4.071</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>-0.607</td>
<td>-6.272</td>
<td>-7.090</td>
<td>-0.704</td>
</tr>
<tr>
<td>y</td>
<td>-0.003</td>
<td>2.769</td>
<td>-1.472</td>
<td>-3.430</td>
</tr>
<tr>
<td>z</td>
<td>0.732</td>
<td>4.056</td>
<td>4.065</td>
<td>4.051</td>
</tr>
</tbody>
</table>

The error model of the range measurements was chosen to be exponentially distributed in accordance to experimental measurements using a prototype of an Ultra Wide Band based positioning system. For $\eta \geq 0$,

$$f(\eta) = (a_0 + a_1 r)^{-1} e^{\frac{-\eta}{a_0 + a_1 r}}$$

with $a_0 = 0.119$ and $a_1 = 0.0138$. Here, $r$ denotes the true range and $\eta$ the error as defined in (1). Note, the probability density function is dependent on the range. For reference positioning, the rover was further equipped by a reference navigation system. As the trajectory was generated on a plane, the additional knowledge was used that none of the static nodes has a $z$-coordinate below this surface. Nevertheless, the position estimation of the trajectory and the nodes was performed in three dimensions. The dataset used for testing consists of 125 unsynchronized range measurements from $0 \leq t \leq 159$s. For comparison, the unsynchronized data was temporally aligned to create a synchronized data set on a temporal grid with a resolution of 7s that complies with the LaSLAT approach from [8]. The resulting collection of 23 positions was interpreted as a single batch. Figure 3 shows a two dimensional projection of the original trajectory on the $xy$-plane and the obtained solution by the presented kernel based method as well as the LaSLAT method. The overall Root Mean Squared (RMS) error for the kernel based approach in this example was less than $0.25$ for both, the trajectory and the node positions. The differences in accuracy for the two methods is mainly caused by the error due to the inevitable temporal alignment of the data.

Although, the minimization of the regularized risk (10) yields reliable estimates of the trajectory $x(t)$ and the beacon positions $p^k$, this approach only solves the problem for a set of previously collected range observations. For real-time applications it is preferable to incorporate measurements in a sequential manner. As already indicated before, for this purpose we approximate the constructed Gaussian stochastic process by a Gauss-Markov process of higher order. Thus, it is
straightforward to apply an Extended Kalman Filter (EKF) for online estimation. The initial state and covariance matrix of the EKF was obtained by an offline solution. Fig. 4 shows the result for this scenario. The data for $0 \leq t \leq 80$ was used to obtain the initial estimate. The rest of the samples served for a realtime estimate of the actual position while improving the estimate for the node positions $p_k$. The RMS error for the trajectory within this example was 0.43. We have intentionally chosen a simulation setup of small sample size and rather course temporal resolution to emphasize the potential of the proposed method.

6. CONCLUSIONS

In this paper we presented a novel solution to a range only localization an mapping problem by modelling the solution space with a RKHS. Using this representation it is possible to work in unsynchronized range measurements without loosing information due to any preprocessing for temporal alignment of samples. Moreover, the approximation of the proposed model by a Gauss-Markov process allows self localization while tracking the position of the static beacons in realtime after a short initialization period. This was shown using simulated data. The method is generic enough for appliance on similar localization problems using other kind of incomplete sensor information. For more accuracy, other sensor error models can be incorporated by modifying the cost function. Nevertheless, for online estimation using an EKF, still an approximation of this quantity is required. Of course, the physical model presented here is rather simple as it implicitly assumes a holomorphic platform that may be accelerated in any direction. More sophisticated approaches that incorporate steering models may greatly improve the performance of the regularization approach and widen the field of application.

7. REFERENCES