IMAGE DENOISING BASED ON TRANSLATION INVARIANT DIRECTIONAL LIFTING

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ABSTRACT

Adaptive Directional Lifting (ADL) has been successfully implemented in image compression and denoising due to the feature of simple structure and flexible directional selectivity. However, image denoising by means of ADL introduces many visual artifacts caused by Gibbs phenomena due to the lack of translation invariance. In this paper, we propose a translation invariant directional lifting (TI-DL) by employing the cycle-spinning based technique to reduce artifacts in denoising results. Moreover, the inefficiency and high computational complexity of the orientation estimation technique in ADL strongly influences the performance. In order to achieve better denoising results, in this paper, 2-D Gabor filters are adopted for orientation estimation to achieve better orientation estimation results with lower complexity. Experimental results demonstrate that the proposed method achieves state-of-art denoising performance in terms of both objective (PSNR) and subjective (SSIM) evaluation.

Index Terms—adaptive directional lifting, image denoising, translation invariance, 2-D Gabor filters

1. INTRODUCTION

Adaptive Directional Lifting (ADL) has been successfully applied in image compression due to the characteristic of representing the edges and textures in images efficiently [1, 2]. Recent researches have shown that the application of image denoising can also benefit form this technique [3, 4]. Due to the flexible directional selectivity, ADL can effectively decorrelate the dependencies found over image discontinuities and compact high frequency components induced by image features into the low-pass sub-band. After the transform, the dominant singularities left in high-pass sub-bands are induced by point (point-wise-noise). Therefore, the widely used thresholding technique can efficiently eliminate the noise with the image features been protected. Previous work has shown the potential of ADL. However, in the context of image denoising, this technique still has great improvement space. For example, some problems such as decreasing the artifacts caused by Gibbs phenomenon and increasing the efficiency and precision of the orientation estimation algorithm need to be solved.

To construct an efficient image transform scheme, two criteria are necessary: anisotropy and translation invariance. ADL incorporated local spatial direction prediction into each lifting stage to achieve flexible directional selectivity. Therefore, ADL is an anisotropic transform lacking of translation invariance, which is really important for image denoising. The lack of translation invariance would lead to Gibbs phenomena in the neighborhood of discontinuities which greatly affects the denoising results. Previous researches about applying ADL in image denoising [3, 4] mainly worked on improving the robustness of ADL in the noisy circumstance. The drawback of lacking of translation invariance still exists. In order to compensate for the lack of translation invariance property of ADL, in this paper, we use the technique of cycle spinning [5] for the construction of ADL to achieve translation invariant directional lifting (TI-DL).

Another problem of ADL is the inefficiency of the orientation estimation technique in the noisy circumstance. In [4], it has been proved that the accuracy of the orientation estimation determines the performance of ADL. In previous work of ADL, the local orientation was determined by minimizing the prediction error. This technique has two drawbacks. First, since it takes all coefficients in the high-pass sub-bands into account to determine the local orientation, it is very noise-sensitive. Second, the computational complexity increases rapidly when the number of directions N predefined increases. In order to overcome the drawbacks mentioned above, in this paper, we adopt Gabor filters (GFs) for local orientation estimation. 2-D GFs have been widely used in image texture analysis due to the desirable characteristics of spatial locality and orientation selectivity[6]. It has been proved that GFs are optimally localized in the space and frequency domains. Therefore, they can be used as detecting operators for direction, scale and edge. In this paper, GFs are used to extract the local dominant orientation for the construction of ADL. Such 2-D GFs can obtain the directional information through one time of directional filtering. Therefore, the transform traverse of N directions is avoided and the computational complexity has no connection with N anymore.
2. BACKGROUND INFORMATION

2.1. Adaptive Directional Lifting

The directional selective feature of ADL comes from the directional predict and update operators. It is well known that 2-D conventional lifting is lacking of directional flexibility because it applies horizontal and vertical lifting steps alternately. The predictor and updater of conventional lifting scheme can be represented as follows:

\[
P(x_e[m,n]) = \sum p_i x_e[m+i,n] \\
U(d[m,n]) = \sum_j u_j d[m+j,n]
\]

in which, \(P(\cdot)\) is the predictor, \(U(\cdot)\) is the updater, \(x_e[m,n]\) is the even subset of a 2-D signal, \(d[m,n]\) is the prediction residual, \(p_i\) and \(u_j\) are the filter coefficients.

In ADL, the lifting scheme is performed in the direction of the highest pixel correlation:

\[
P(x^p_{\text{dir}}, \text{dir}) = \sum p_i x^p_{\text{dir}}[m+i,n+\text{sign}(i-1) \ast \text{dir}] \\
U(d^*_{\text{dir}}, \text{dir}) = \sum u_j d^*[m+i,n+\text{sign}(i-1) \ast \text{dir}]
\]

in which, \(\text{dir}\) is the local dominant direction, \(x^p_{\text{dir}}[m,n]\) is the interpolated version of \(x_e[m,n]\) and \(d^*[m,n]\) is the interpolated version of prediction residual \(d[m,n]\).

Fig. 1 shows the a generic ADL scheme.

![ADL transform scheme](image)

(b) Forward transform  (c) Inverse transform

2.2. Orientation Estimation

As mentioned before, in ADL, the orientation estimation was carried out by minimizing the prediction error. This method is very noise sensitive and for each image sub-block it has to transform traverse \(N\) directions to obtain \(\text{dir}\). The computational complexity increases rapidly with the increasing of \(N\). Moreover, in the case of lifting along row, the estimated orientations \(\text{dir}_x\) are limited from 45° to 135°. This means one has to perform the orientation estimation operation again in the column lifting when the orientation information \(\text{dir}_y\) ranged from -45° to 45° is needed. Fig. 2 shows the flow chart of one level 2-D ADL transform with orientation estimation.

In order to overcome the drawbacks mentioned above, in this paper, we extract the local orientation information by 2-D GFs. 2-D Gabor function can be represented as follows:

\[
g(x,y) = \left( \frac{1}{2\pi \sigma_x \sigma_y} \right)^{\frac{1}{2}} \exp \left[ -\frac{1}{2} \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right) + 2\pi j \omega x \right]
\]

\[g(x,y)\] can be regarded as the mother wavelet. Through proper scaling and rotation of \(g(x,y)\) as formula (4), a set of self-similar filters can be obtained.

\[g_{mn}(x,y) = a^{-m} g(x',y') a > 1, m, n \in \mathbb{Z},\]

in which, \(x' = a^{-m}(x \cos \theta + y \sin \theta), y' = a^{-m}(-x \cos \theta + y \sin \theta)\). Here, \(\theta = n \pi / K, K\) is the number of the directions, \(a^{-m}\) is the scale factor. Through proper adjusting of \(m\) and \(n\), one can obtain a set of GFs with different scales and directions. Therefore, we can design our own 2-D GFs which are suitable for the construction of ADL.

3. TRANSLATION INVARIANT DIRECTIONAL LIFTING

3.1. ADL with Cycle Spinning

ADL is constructed based on lifting scheme, on one hand, it inherits lots of features of conventional lifting scheme such as simple structure, in-place operation, suitable for hardware implementation and easily inverse transform. On the other hand, like the conventional lifting, it is lack of translation invariance. Once implemented in image denoising, Gibbs phenomena - alternating undershoot and overshoot of a specific target level - appear in the neighborhood of discontinuities.

In this paper, cycle spinning[5] is used to compensate for the lack of translation invariance of ADL. According to [5], an important observation about the Gibbs phenomena artifacts is that the location has strong connection with the actual location of the discontinuity. Therefore, cycle spinning can effectively reduce the artifacts by averaging all the denoising results obtained from various time shifts of the noisy signal.

In case of 2-D signal denoising by thresholding in lifting scheme, cycle spinning can be expressed as:

\[
\hat{s} = \frac{1}{(M+1)^2} \sum_{i,j=0}^{M} S_{-i,-j}(L^{-1}(T(L(S_{i,j}(x))))),
\]
in which, \( x \) is the noised image, \( \hat{s} \) is the denoised image, \( M \) is the number of shifts along row and column, \( L \) is the lifting scheme, \( T \) is the thresholding operator and \( S_{i,j} \) is the time shift operator whose inverse operation is \( S_{-i,-j} \). The number of shifts \( M \) is determined by the period of the periodic invariance. Taking \( 5/3 \) and \( 9/7 \) lifting as an example, in each lifting step of these two schemes, there are only two different phases, odd and even. In addition, all the predict and update operations only take the adjacent samples as the input of the calculation. Therefore, the transform outputs repeat after only one shift. In other words, the period of the periodic invariance is 2 and thus \( M = 1 \).

As mentioned before, ADL is adaptive directional lifting which is constructed based on lifting scheme. Hence one can construct translation invariant directional lifting by means of cycle spinning and expect better denoising result from TI-DL.

### 3.2. Orientation Estimation using 2-D Gabor Filters

This section introduces the orientation estimation based on 2-D GFs. Considering the diversity of image features, the input image is first segmented into blocks with textures of clear directional bias. For each given block, we first apply a set of GFs to it, and then calculate the energy of each sub-band. The direction in which the sub-band having the maximum energy is the dominant direction of the block.

According to the structure of ADL, we set \( K = 16 \) and \( m = 5 \) in formula (4) for the designation of the GFs. Fig. 3 shows the magnitude of the Gabor filters at five different scales in 16 different directions.

![Fig. 3. Magnitude of the GFs at 5 scales in 16 directions](image)

The frequency support of the designed GFs is shown in Fig. 4. The flow chart of 2-D ADL using the orientation estimation result from the GFs is also shown in Fig. 4. In this case, it only needs to perform the orientation estimation operation once before the transform to get both \( \text{dir}_v \) and \( \text{dir}_h \).

![Fig. 4. Flow chart of 2-D ADL with GFs.](image)

Fig. 5 shows the first orientation estimation results obtained from different methods in different circumstances. Fig. 5(a) and (b) are the orientation estimation results of ADL [1] in clear image and noised image (\( \sigma = 20 \)) respectively. Apparently, the accuracy of the orientation estimation of ADL decreases rapidly with the effect of noise. Fig. 5(c) is the estimation result of RADL [4] for noised image. Although RADL achieved more accurate estimation than ADL in the noised circumstance, the estimated orientations are limited in the range of \( 45^\circ \) to \( 135^\circ \). Another orientation estimation needs to be done to obtain the directional information belonging to \( \text{dir}_h \). Fig. 5(d) is the estimation result of the GFs. All the directional information including \( \text{dir}_v \) and \( \text{dir}_h \) has been obtained with higher accuracy.

![Fig. 5. Orientation estimation results.](image)

### 4. EXPERIMENTAL RESULTS

In this section we report initial efforts at image denoising based on this new approach. In order to test our method, we choose four standard 512 x 512 gray level images “Barbara”, “Lena”, “Pepper” and “Boat” as test images which are contaminated by simulated zero-mean additive white Gaussian noise with standard variance \( \sigma = 20 \). In our experiment, the biorthogonal Debauches 9/7 filter banks are used for all the wavelet related methods, and Bivariate-Shrinkage [7] is used to deal with the high-pass coefficients.

Table 1 shows the PSNR comparison for different techniques including those of the state-of-art techniques non-local mean (NLM) [8] and translation-invariant contourlet transform (TI-CT) [9]. Moreover, since TI-DL is inspired by the work of translation invariant wavelet (TI-WT) [5] and robust adaptive directional lifting (RADL) [4], the denoising results...
of these two techniques are used for comparison too. It can be seen that except for Barbara, TI-DL outperforms the other methods. We adopt the perceptual quality index structural similarity (SSIM) [10] as another quality evaluation method to compare the denoising results more subjectively as shown in Table 2. In this case, TI-DL outperforms NLM even in the case of Barbara. The denoising results of Barbara obtained from different techniques are shown in Fig. 6. We visualize the denoising performance by zooming into Barbara’s trousers.

![Original image](image1) ![Noised image](image2) ![TI-WT](image3) ![RADL](image4) ![NLM](image5) ![TI-DL](image6)

Fig. 6. Denoising results for Barbara.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Barbara</th>
<th>Lena</th>
<th>Boat</th>
<th>Pepper</th>
</tr>
</thead>
<tbody>
<tr>
<td>noised image</td>
<td>22.15</td>
<td>22.13</td>
<td>22.32</td>
<td>22.18</td>
</tr>
<tr>
<td>RADL</td>
<td>28.84</td>
<td>31.24</td>
<td>29.65</td>
<td>30.75</td>
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<tr>
<td>TI-WT</td>
<td>28.93</td>
<td>31.74</td>
<td>30.21</td>
<td>31.33</td>
</tr>
<tr>
<td>TI-ADL</td>
<td>29.80</td>
<td>32.21</td>
<td>30.70</td>
<td>31.71</td>
</tr>
<tr>
<td>TI-CT[9]</td>
<td>29.53</td>
<td>31.75</td>
<td>30.03</td>
<td>31.15</td>
</tr>
<tr>
<td>NLM[8]</td>
<td><strong>30.31</strong></td>
<td>31.78</td>
<td>29.34</td>
<td>29.62</td>
</tr>
</tbody>
</table>

Table 1. Comparison of PSNR (dB)

<table>
<thead>
<tr>
<th>Methods</th>
<th>Barbara</th>
<th>Lena</th>
<th>Boat</th>
<th>Pepper</th>
</tr>
</thead>
<tbody>
<tr>
<td>noised image</td>
<td>0.4792</td>
<td>0.3455</td>
<td>0.4012</td>
<td>0.3439</td>
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<tr>
<td>RADL</td>
<td>0.8350</td>
<td>0.8250</td>
<td>0.8234</td>
<td>0.8045</td>
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<tr>
<td>TI-WT</td>
<td>0.8449</td>
<td>0.8533</td>
<td>0.8447</td>
<td>0.8053</td>
</tr>
<tr>
<td>TI-ADL</td>
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<td>0.8584</td>
<td>0.8567</td>
<td>0.8298</td>
</tr>
<tr>
<td>NLM</td>
<td>0.8533</td>
<td>0.8380</td>
<td>0.8119</td>
<td>0.8047</td>
</tr>
</tbody>
</table>

Table 2. Comparison of SSIM

5. CONCLUSION

In this paper, translation invariant directional lifting is proposed for image denoising. Cycle spinning is adopted to reduce artifacts caused by lacking of translation invariance. 2-D GFs are used for orientation to achieve higher computational complexity and estimation accuracy. Experimental results show that this algorithm can achieve both good visual quality and high objective evaluation for the image denoising.

6. REFERENCES


