MULTICHANNEL SAR AUTOFOCUS USING MULTIPLE LOW-RETURN CONSTRAINTS

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ABSTRACT

MultiChannel Autofocus (MCA) and Reversed-step MCA (RMCA) for Synthetic Aperture Radar (SAR) assume a region of low return in the focused data, and work best when the region is known accurately. In practice, the returns from within the side-lobes of the antenna footprint are highly attenuated; thus a region of highly-probable low returns is provided. However, scene-dependent features like strong reflectors in the presumed low-return region may violate the assumptions underlying MCA and RMCA, with resulting performance degradation. We consider such a scenario and propose scene-independent, random selection methods of the low-return constraints. Our simulation results show that a well-focused image can be obtained within a few trials, even when the number of anomalies is moderately large.

Index Terms— Synthetic aperture radar, autofocus

1. INTRODUCTION

Synthetic Aperture Radar (SAR) is widely used to acquire microwave images with both high cross-range and range resolution. In the tomographic formulation of spotlight-mode SAR, the post-processed data are the Fourier transform of the underlying reflectivity, sampled on a polar grid [1]. Thus, in principle, the original image can be reconstructed by applying an inverse Fourier transform. In practice, the Fourier data often are contaminated with unknown phase shifts due to demodulation timing errors, which cause the resulting image to be improperly focused.

In extensive literature on the SAR defocusing problem [2] a number of well-motivated autofocus algorithms have been proposed. However, many of these algorithms are iterative in nature and their theoretical justification is mostly heuristic. The MultiChannel Autofocus (MCA) algorithm, proposed in [3], is a non-iterative algorithm developed under two key assumptions: i) the 1-D defocusing assumption and ii) the low-return assumption. When these assumptions are satisfied, it was shown that MCA recovers a near-perfect solution. However, the 1-D defocusing model does not apply exactly in reality since the phase error is a function of the polar angular variable, rather than the Cartesian Fourier coordinate. It so happens that MCA is very sensitive to the departure from the 1-D assumption. Thus, a modified version of MCA, termed Reversed-step MCA (RMCA), was proposed in [4] to account for this sensitivity.

The low-return assumption refers to the region of low return in the perfectly focused data that results from the attenuation provided by the antenna pattern. The low-return region, along with the defocused data, defines a linear solution space for the phase error, where the performance of both MCA and RMCA depends highly on the choice of the low-return constraints. In practice, the scene-dependent nature of the low-return region may make it difficult or even impossible to accurately identify the low-return region, especially in the presence of strong reflectors in the antenna sidelobes. Analytical or data-dependent estimation of the true low-return region seems difficult, so we consider a rather simple way to solve this problem. Although the presumed low-return region, based on the antenna pattern, may contain some pixels with significant magnitude, it is highly likely the presumed low-return region will have far lower magnitude than elsewhere. Thus, we assume that the true low-return region is a subset of a possibly oversized, presumed low-return region.

In Section 2, we introduce the problem and establish the notation to be used. Section 3 suggests a random subregion approach. Then, we demonstrate this method through simulations in Section 4. We study the performance of RMCA for three different random geometries of the subregion and with different numbers of anomalies in the presumed low-return region. Lastly, we conclude and discuss future work in Section 5.

2. BACKGROUND

2.1. SAR defocusing problem

Consider spotlight-mode SAR, where we acquire data for vantage angles \( \theta_l, l = 0, 1, \ldots, M - 1 \). We denote \( g_o(x, y) \) to be the complex-valued SAR image, where \( x \) and \( y \) are the range and cross-range coordinates, respectively. After demodulation and sampling, we obtain the discrete SAR data, \( \{ G_p[l, k], l = 0, \ldots, M - 1, k = 0, \ldots, N - 1 \} \), which essentially are a set of Fourier transformations of \( g_o \) sampled on an offset polar-grid [1]. The demodulated data often are contaminated with unknown phase shifts, which are constant for a fixed look angle \( \theta_l \). That is,

\[
\hat{G}_p[l, k] = G_p[l, k] e^{j\phi_l}, \quad \forall l, k.
\]

Due to the phase errors, \( \phi_l \), the image reconstructed from the contaminated data \( \{ G_p[l, k] \} \) becomes improperly focused. The aim of SAR autofocus is to restore the perfectly focused image.

2.2. Reversed-step MultiChannel Autofocus (RMCA)

The warped-domain data \( g_p \) corresponding to the polar-grid Fourier data \( \{ G_p[l, k] \} \) is defined as

\[
g_p[m, n] = \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} G_p[l, k] e^{j2\pi ml/M} e^{j2\pi nk/N},
\]

where \( m \) and \( n \) are the indices corresponding to the y-coordinate and the x-coordinate of the warped domain, respectively. In Reversed-step MCA (RMCA) [4], the phase errors are estimated by applying

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the MCA algorithm to the warped-domain data instead of the spatial-domain data, since the defocusing effect remains one-dimensional in the warped domain.

From now on, we use bold letters for matrix and vector representations of 2D and 1D data, respectively; for example, \( \mathbf{g}_p \) is the \( M \times N \) matrix representation of \( \{ g_p[m, n] \} \). Then, the defocusing relationship in the warped domain can be expressed as

\[
\tilde{\mathbf{g}}_p = \mathbf{F}^H \mathbf{D}(e^{i\phi}) \mathbf{g}_p = \mathbf{C}(b) \mathbf{g}_p,
\]

where \( \mathbf{g}_p \in \mathbb{C}^{M \times N} \) and \( \tilde{\mathbf{g}}_p \in \mathbb{C}^{M \times N} \) are the focused and defocused warped-domain data, respectively, \( \mathbf{F} \in \mathbb{C}^{M \times M} \) is the 1-D DFT operator, \( e^{i\phi} \in \mathbb{C}^{M \times 1} \) is a vector with entries \( e^{i\phi_l} \), \( l = 0, \ldots, M-1 \), \( \mathbf{D}(e^{i\phi}) \in \mathbb{C}^{M \times M} \) is a diagonal matrix formed with \( e^{i\phi} \), and \( \mathbf{C}(b) \) is a circulant matrix formed with the blurring kernel \( b = \mathbf{F}^H e^{i\phi} \). The solution space is defined as the set of all warped-domain data formed from \( \tilde{\mathbf{g}}_p \) with different phase corrections \( \phi \), or equivalently with different all-pass correction filters \( \mathbf{f}_A = \mathbf{F}^H e^{i\phi} \). By relaxing the all-pass condition [3], any warped-domain data, \( \tilde{\mathbf{g}}_p \), in the solution space can be expressed in terms of the basis expansion

\[
\text{vec}(\tilde{\mathbf{g}}_p) = \Phi \mathbf{f},
\]

where \( \text{vec}(\tilde{\mathbf{g}}_p) \in \mathbb{C}^{MN \times 1} \) is the vector composed of the concatenated columns of \( \tilde{\mathbf{g}}_p \), \( \mathbf{f} \in \mathbb{C}^{M \times 1} \) is the correction filter using a basis \( \{ e_{m} \}_{m=0}^{M-1} \) for \( \mathbb{C}^{M \times 1} \), and the \( m \)-th column of the \( NM \times M \) matrix \( \Phi \) is \( \text{vec}(\mathbf{C}(e_{m})\tilde{\mathbf{g}}_p) \).

Now, suppose \( g_p[m, n] \) has low value for \( (m, n) \in \Omega \). Then, the correction filter is obtained by solving

\[
\min_{\mathbf{f}} ||\Phi_\Omega \mathbf{f}||_2,
\]

where \( \Phi_\Omega \) is composed of rows of \( \Phi \), corresponding to the low-return indices \( \Omega \). Note that the solution is the right singular vector corresponding to the smallest singular value of \( \Phi_\Omega \).

### 3. RANDOM SUBREGION APPROACH

Since the actual low-return region depends somewhat on the scene, the presumed low-return region, \( \Omega \), provides only a superset of the most probable low-return pixels. Without further complicating the estimation process, we may apply RMCA to the same defocused data using several different random subsets of \( \Omega \) as the low-return constraints. Then, given that the number and strength of anomalies in the presumed low-return region are small, it is possible to obtain a reasonably focused image within several trials, with high probability. Image focus quality can be evaluated by computing a sharpness measure, such as entropy, and we may choose the image with the highest sharpness measure.

The geometry of the low-return region is another factor that affects the performance of MCA. Extending the low-return region along the range direction introduces redundancies in the defocusing operator, while extending the region along the direction of blur forces the algorithm to reduce the mainlobe width of the blurring kernel. Thus, adding constraints that extend in both the range and cross-range directions may have a positive effect on the performance of the algorithm. Analytically solving for the best possible geometry of the low-return region is seemingly intractable. In order to get an idea of how different geometries affect RMCA performance, the next section provides simulation results for three different types of subregions: i) subregion consisting of randomly chosen pixels, ii) subregion consisting of randomly chosen vertical columns, and iii) subregion consisting of randomly chosen horizontal rows. Here, the horizontal coordinate represents the range dimension and the vertical coordinate represents cross-range.

Figure 1 shows an example of a presumed low-return region and three subregions with different geometries. The presumed low-return region depicted in Fig. 1(a) corresponds to an antenna pattern with uniform power and rectangular shape. The range of data-collection angles \( \Theta \) is narrow, and the presumed low-return region in the warped domain resembles the ideal antenna footprint. Figures 1 (b), (c), and (d) are randomly chosen subregions of i.i.d. random, random vertical, and random horizontal geometries, respectively.

### 4. SIMULATION RESULTS

In this section, we present simulation results under a specific scenario, and then discuss generalization of the results. A SAR image from Sandia National Laboratories (http://www.sandia.gov) was used to model the magnitude of the reflectivity, and the spatial phases were driven from an i.i.d. uniform distribution ranging from \(-\pi\) to \(\pi\). The reflectivity of the outermost 20 rows of the 128-by-128 image were set to zero, except for few anomalies. The magnitude of the anomalies were chosen to be three times larger than the maximum magnitude in the middle 108 rows, where the relative magnitude of the anomalies is much lower in most real situations. The Fourier data was synthesized on a polar-grid with a range of angles \( \Theta = 3^\circ \), and a white phase-error function was applied to model the worst-case defocusing. An example of a focused image is shown in Fig. 2(a), where three anomalies are located in the presumed low-return region. Figure 2(b) shows the corresponding defocused image.

A presumed low-return region of a large size (1792 pixels) was identified, based on the antenna pattern and the Fourier data grid, and was used as the superset of the low-return constraints throughout the set of simulations. For each simulation of RMCA, we varied four...
Fig. 2. Simulated SAR images for $\Theta = 3^\circ$: (a) perfectly focused image with three anomalies in the zero-return rows, (b) defocused image, where a white phase-error function was applied, (c) RMCA restoration using the presumed low-return constraints, and (d) RMCA restoration using a randomly chosen subset of a larger set of presumed low-return constraints.

parameters: i) the number of anomalies, ii) the location of anomalies, iii) the type of geometry of the subregion, and iv) the random constraints. For each number of anomalies, ranging from 0 to 10, we chose 50 different locations of the anomalies, except for the case where no anomaly existed. Then for each of these 501 corrupted data sets, we applied RMCA with 50 different subregions (for each type of geometry discussed in Section 3) as the low-return constraints. Here, the size of the low-return constraints was chosen to be as close as possible to 640 pixels.

Figures 2(c) and (d) both show images restored by RMCA from the same defocused data, Fig. 2(b), but with different low-return constraints. Figure 2(c) was obtained by using a presumed low-return region of size 640 as the constraints, and Fig. 2(d) was obtained by using a randomly chosen subset (of size 640) of a presumed low-return region of size 1792. Figure 2(d) had the fifth smallest entropy among the 50 restorations using different subregions of i.i.d. random geometry; it is neither the best nor the worst restoration obtainable by using the random-subregion approach.

Figure 3 shows three plots summarizing our simulation results. The plots show the average performance of RMCA as the number of anomalies get larger, where the performance is measured by the image entropy of the restored data. The connected circles in Figures 3 (a), (b), and (c) represent the averaged entropies of the restored images for i.i.d. random, random horizontal, and random vertical subregions, respectively. While all three types tend to show degradation in performance as the number of anomalies gets larger, several details should be noted.

First, close comparison of the three plots reveals that the average performance of i.i.d. random subregion is always between the aver-
Fig. 4. Rate of restoring a well-focused image.

...age performance of the other two types. However, we believe that this behavior may be dependent on the scene and the geometry of the presumed low-return region. Second, except for the case where no anomaly exists, using the subregion approach with the random horizontal geometry shows the best average performance. Lastly, the random horizontal subregion shows poor performance when there is no anomaly at all. This may seem implausible at first sight, however, we offer a possible explanation at the end of this section.

The x’s in the three plots represent the average entropies for fixed locations of anomalies, where only the minimum, maximum, the 33rd percentile, and the 67th percentile of the 50 locations are shown. The performance seems to be sensitive to the location of anomalies when there are only few of them, but is less affected by the location as the number of anomalies grows, as expected. Similarly, the dots represent the minimum, maximum, the 33rd percentile, and the 67th percentile of the average entropies corresponding to 50 different random subregions.

Lastly, the horizontal dotted line in the three plots represents the average entropy corresponding to phase errors driven from a uniform distribution of zero mean and standard deviation equal to \( \pi/6 \). We consider restored images with entropy less than this value to be well focused. Using this as a criterion, Fig. 4 shows the percentage of image restorations that were well focused as a function of the number of anomalies. The circles, stars, and triangles correspond to i.i.d. random, random vertical, and random horizontal geometries, respectively. Notice that a modest number of restorations will, with high probability, produce one or more properly focused images. Furthermore, for a scenario where the anomalies are not so strong as those assumed in these simulations, the curves in Fig. 4 fall off far more slowly.

We would like to make some further comments on the results. Suppose RMCA is applied with the low-return constraints \( \Omega_0 \), where \( |\Omega_0| \geq M \). Let \( \Omega_s \) denote the set of indices corresponding to the indices of \( \Omega_0 \), shifted according to the amount \( s \), i.e.,

\[
\Omega_s = \{(m,n) \in [0, \ldots, M-1] \times [0,\ldots, N-1]; (m-s) \text{mod} M, n \in \Omega_0,\}
\]

for \( s \in [1, \ldots, M-1] \), and let \( S = \{s \in [0, \ldots, M-1]; g_0[m,n] = 0, \forall (m,n) \in \Omega_s\} \). Then, when \( |S| = 1 \), RMCA restores the perfectly focused image, whether or not \( 0 \in S \); that is, RMCA restores the perfectly focused image even when the low-return constraints are incorrect if there exists exactly one circularly-shifted version of the constraints that corresponds to zero returns. If \( |S| > 1 \), then

\[
|\Phi_{c_0}f|_2 = 0, \quad \forall f \in \{F^HAc: c \in C\},
\]

where \( A \) is an \( M \times |S| \) matrix, composed of columns \( e^{-j\phi_0}_s \in \mathbb{C}^{M\times1} \), where \( \phi_0[l] = \phi_0 - 2\pi s l / M \), \( \forall s \in S \), and \( C = \{c \in \mathbb{C}^{|S|\times1}; |F^HAc||_2 = 1\} \). In this case, any filter \( f = F^HAc \) with \( c \in C \) is a possible solution to the minimization problem (1), and it could be very different from the true solution.

Therefore, the behavior in the performance of random horizontal geometry may be explained as follows. When no anomaly exists in the presumed low-return region, it is very probable that \( |S| > 1 \) for random horizontal subregions; thus, RMCA might show poor performance. On the other hand, as the number of anomalies increases, random horizontal geometry may show better performance compared to the other two types of random subregion since it is more probable that there is a nonzero \( s \) in \( S \) for the random horizontal geometry than for the other two.

5. CONCLUSION

We have presented a promising demonstration of the random subregion approach for selecting the low-return constraints of RMCA in the case of a few very strong anomalies in the presumed low-return region. With this simple approach, we can restore a well-focused image within a small number of trials, with high probability. Furthermore, even when the restoration is not perfect, one may use the information from the well-focused image to refine the low-return constraints and iterate. However, several key questions remain areas of active research. These include an analysis of the optimal size of the presumed low-return region and its subregion, how the performances differ as the location of anomalies or the range of look angles changes, and the possibility of a different type of subregion with a more robust performance. In wide-angle SAR, choosing the low-return constraints becomes more critical since the focused warped-domain data do not exhibit a clear transition from high to low return in general; thus, when selecting the subregion, we must account for the fact that neighboring pixels of an anomaly in the warped domain may have significant magnitude.

6. REFERENCES