PHASE CORRECTION AND DENOISING FOR ICA OF COMPLEX FMRI DATA

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ABSTRACT
Analysis of functional magnetic resonance imaging (fMRI) data in its native, complex form has been shown to increase the sensitivity of the analysis both for data driven techniques such as independent component analysis (ICA) and for model-driven techniques; however, the noisy nature of the phase poses a challenge for successful study of fMRI data. In addition, for complex ICA, the inherent scaling ambiguity, which has a phase term, introduces additional difficulty for group analysis and visualization of the results. In this paper, we address these issues, which have been among the main reasons phase information has been traditionally discarded and introduce a phase correction scheme that can be either applied subsequent to ICA of fMRI data or can be incorporated into the ICA algorithm in the form of prior information to eliminate the need for further processing for phase correction. In addition, we introduce methods for visualization of the analysis results as well as preprocessing the complex fMRI data to mitigate the effects of noise in the phase which are not limited to ICA algorithms. We demonstrate the successful application of the methods using actual fMRI data.

Index Terms— Phase correction, denoising, complex-valued fMRI, ICA

1. INTRODUCTION
Functional magnetic resonance imaging is a technique that provides the opportunity to study brain function non-invasively and is a powerful tool utilized in both research and clinical arenas since the early 90s[1]. Independent component analysis (ICA) has been shown to be quite effective for the analysis of fMRI data, as by using a simple generative model based on linear mixing, ICA can minimize the constraints imposed on the temporal or the spatial dimension of the fMRI data. ICA of fMRI hence provides valuable new insights, especially when studying paradigms for which reliable models of brain activity are not available.

Using the fMRI data in its native, complex form for ICA of fMRI promises to provide more statistically significant information [2, 3, 4]; however, the phase images are usually discarded since their noisy nature poses a challenge for successful study of fMRI [5]. Additionally, the inherent scaling ambiguity in ICA, which has a phase term in the case of complex ICA, introduces additional difficulty for group analysis and visualization of the results. In this paper, we address these issues, which have been among the main reasons phase information has been traditionally discarded and introduce a phase correction scheme that can be either applied subsequent to ICA of fMRI or can be incorporated into the ICA algorithm in the form of prior information—when available—to eliminate the need for further processing for phase correction. We also introduce a denoising scheme as a preprocessing step and, address thresholding and visualization of the results of complex ICA results.

In Section 2 we describe complex-valued ICA of fMRI data and introduce the proposed phase correction scheme. Section 3 describes the denoising scheme and the visualization scheme, based on the Mahalonobis distance, that uses the complex-valued data of the estimated phase corrected independent components to identify voxels—3D pixels—of interest. Finally, in Section 4 we demonstrate the effectiveness of the phase correction scheme on actual fMRI data.

2. COMPLEX-VALUED ICA OF FMRI DATA

2.1. ICA of fMRI Data
We can form a matrix \( X \in \mathbb{C}^{T \times V} \) using the fMRI data such that the \( t \)th row is formed by flattening the volume image data of \( V \) voxels, at time instant \( l \), into a row, and the rows are indexed as a function of time, \( l = 1, \ldots, T \). In spatial ICA of fMRI data, we assume a simple linear mixing model such that \( X = AS \), and determine both the activation maps and the corresponding waveforms, i.e., both \( S \) and \( A \), typically without constraining either. The additional assumption we impose is that the rows of matrix \( S \) represent observations of statistically independent random variables. Thus, spatial ICA finds systematically nonoverlapping, temporally coherent brain regions without constraining the temporal domain. A principal advantage of this approach is its applicability to cognitive paradigms for which detailed a priori models of brain activity are not available. Following its first application by McKeown et al. [6], ICA has been successfully used in a number of exciting fMRI applications, especially in those that have proven challenging with the standard regression-type approaches. These include identification of various signal types (e.g., task-related, transiently task-related, and physiology-related signals) in the spatial or temporal domain, analysis of multisubject fMRI data, incorporation of a priori information to improve the estimates, clinical applications, and for the analysis of complex-valued fMRI data. A comprehensive review of ICA approaches for fMRI data along with main references in the area is given in [7].

2.2. Complex ICA
For complex ICA, if we use the notation based on random variables, we use the generative model, \( x = As \), where \( x, s \in \mathbb{C}^N \) and \( A \in \mathbb{C}^{N \times N} \), and achieve demixing by estimating a weight matrix \( W \) such that \( u = Wx = PAx \). Here, \( P \), a permutation matrix, represents the permutation ambiguity and \( A \), a diagonal matrix, represents the scaling ambiguity of ICA, which has a magnitude and phase term in the complex-valued implementation of ICA. The entries of the multivariate vector \( x \) represent the mixture random variables and are replaced by the given observations for the application.
in question, e.g., by the volume image at time \( t \) for fMRI data analysis.

To achieve independence of the sources \( s_k \) that form the source vector \( s \), an appropriate measure of independence has to be selected to compute the demixing matrix \( W \). By the nature of the goal, the measure requires computation of higher-order statistics in the data, either explicitly as in the approaches based on cumulants, or implicitly through the use of nonlinear functions. ICA approaches that rely on nonlinear functions to implicitly generate the higher-order statistics to achieve independence offer practical and effective solutions to the ICA problem. Two such popular approaches are based on maximum likelihood (ML)—which is equivalent to information maximization and nonlinear decorrelations—and maximization of negentropy (MN), which can be shown to be equivalent to ML when the demixing matrix is constrained to be unitary [8]. For both ML and MN, the algorithms are optimal when the form of the nonlinear function used in the cost function matches the form of the probability density functions (pdf) of the sources \( s_k = s_{k, re} + js_{k, im} \), which in the complex case are described by the joint density \( p(s_{re}, s_{im}) \).

However, as discussed in [3], a number of simple functions from the trigonometric family provide robust and effective solutions for the ICA problem in the complex domain as in the real case, or simple adaptive mechanisms can be employed to estimate the independent components in a deflationary mode as discussed in [9]. The developments in both [10] and [9] do not make any assumptions such as the circularity—rotation invariance—of source distributions, an assumption common in complex ICA, making these approaches more suitable to applications such as fMRI data. Since very little is known about the nature of the fMRI data when used in its native complex form, it is desirable to avoid making additional assumptions such as the circularity of source distributions. However, certain information such as an important preprocessing step used in the scanner can be incorporated into the algorithm as we discuss next.

### 2.3. Phase Correction

Here, we first introduce an ICA approach that can effectively take the prior information in the form of preprocessing used in the scanner into account to correct for phase rotation. This approach can be used by any ICA algorithm that relies on non-linear functions to implicitly generate higher order statistics. Next, we introduce an approach that can be used to correct for phase rotation in the absence of such prior information and/or when we are interested in using a flexible ICA algorithm, such as one that adaptively estimates the source distributions as in [9].

#### 2.3.1. Phase Correction through Prior Information

A common pre-processing step that is applied in the fMRI scanner is to re-normalize the data such that most of the power is concentrated in the real part of the complex-valued fMRI signal [11]. Such a processing also makes the visualization easy as all the acquired data then has the same orientation and also can be easily added in the case of group analysis, which is commonly done in fMRI studies.

Hence, the fMRI data with this preprocessing has a noncircular distribution—i.e., is not rotationally invariant—such that most of the power is in the real component. This prior information can be incorporated into the selection of the nonlinear function for the ICA algorithm so that the estimated sources will have the same orientation as the preprocessed data.

In [3], it has been shown that a number of trigonometric functions and their hyperbolic counterparts can be effectively used for achieving ICA, and in [12] it has been noted that by selecting the appropriate function, we can effectively alleviate the phase ambiguity. In Figure 1, we show the approximate density the use of score function atanh implies (given by \( \exp(-u^\ast \text{atanh} u) \)), which exactly corresponds to the data that results from the fMRI preprocessing we have described. In the second row of the same figure, we show estimation results of joint approximate diagonalization of eigenmatrices (JADE) [13] and ML with atanh as the score function. In the estimation results, we note that when the direction of the source matches that of the pdf implied by the score function, the shape of the distribution of the estimated components is preserved, i.e., the phase ambiguity that exists for the complex ICA is alleviated. Thus, even though the correlations of the magnitude with the original sources are close to unity for both JADE and ML-atanh in this example, the correlation of real and imaginary parts are high (close to unity) only for ML-atanh for the first three sources, those for which the direction of the source density matches with that of the nonlinearity. The red source estimate shows a rotated source estimate as its direction of asymmetry does not match with that of the density model given by atanh. These results were consistent over 100 different realizations of the source distributions.

Hence, we can use atanh when such a preprocessing—a desirable form for complex fMRI data—is utilized. The only correction that will then have to be carried out is the sign correction to make sure that most background voxels are around 0, a postprocessing step common to real-valued ICA of fMRI [7]. In Figure 3, we show an example of the application of this scheme by calculating the mean phase image of a motor component identified across subjects.

The strength of this scheme lays on identifying prior information that can help in the selection of the non-linear function used in the ICA cost functions. Similar prior information can be used to identify the optimal non-linearities in fMRI data with different preprocessing techniques.

#### 2.3.2. Post-ICA Phase Correction

Prior information on fMRI data preprocessing and expected source noncircularity might not be always available. Also, when one uses an adaptive ICA algorithm that estimates the source distributions adaptively, it might not be desirable to use a fixed nonlinearity as discussed in section 2.3.1. In these cases, the estimated distribution of these sources will have a unknown rotation across subjects that
creates problems for group analysis. The method introduced here is not limited to ICA algorithms that rely on nonlinear function like in the previous section.

In this case, we can use a phase correction scheme based on principal component analysis (PCA) that adjusts the phase values of the voxels of interest to make the identification of task related functional changes in the fMRI data and group analysis possible. The steps in Algorithm 1 change the orientation of the pdf, $p(\hat{s}_{k, re}, \hat{s}_{k, im})$, of the estimated sources so that it is mostly concentrated on the positive side of the real part of the complex domain. This algorithm can be applied to estimated components from all the subjects, therefore obtaining phase images with similar phase values in the voxels of interest.

Algorithm 1 – Post-ICA Phase Correction

1: Find rotation angle $\theta$ that maximizes: $\arg \max_\theta E \left[ (\text{M} \hat{s}_k)^2 \right]

2: Resolve $180^\circ$ phase ambiguity: $\arg \max_\theta E \left[ (\text{M} \hat{s}_k)^3 \right]

where $\text{M} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ and $\hat{s}_k = [\hat{s}_{k, re}, \hat{s}_{k, im}]^T$. The first step of Algorithm 1 is easily achieved by applying PCA to the complex-valued data $\hat{s}_k$ and aligning the first principal component—component with the highest variance—with the real axis. The second step of the algorithm resolves the sign ambiguity of the first step, since the aligned data may be rotated $180^\circ$ degrees so that the majority of values are concentrated on the negative part of the real axis. Figure 2 shows the scatter plot of the real and imaginary data of an estimated source before and after the PCA based phase correction. This method does not change the magnitude of the estimated source, but it makes sure that the phase value of the voxels of interest—those with high magnitude—are close to 0 radian in the obtained wrapped ($[-\pi, \pi]$ radians) phase image.

3. DE NOISING, THRESHOLDING, AND VISUALIZATION OF ICA RESULTS

3.1. Multi-Subject fMRI Denoising

In this section, we introduce a physiologically based Quality Map Phase Denoising (QMPD) method that can be applied to the fMRI data prior to processing by ICA or any other analysis algorithm for group data. In [14], we showed the effectiveness and benefits of this method when applied as a pre-processing step in complex-valued ICA of fMRI data for single subjects. The output of the QMPD method is a binary quality mask that identifies the good quality voxels in each 2D slice for every fMRI data volume obtained at every time point. The quality masks are multiplied with the complex-valued data, prior to smoothing. In fMRI group studies, direct comparison between subjects is difficult since the voxels with good quality across the subjects are usually different, probably due to scanner motion registration errors.

We define a procedure in Algorithm 2 that can be used to obtain the QMPD quality mask for the denoising of all subjects in a study. In this paper, we also implement an automatic histogram-based method to define the threshold $t$ in step three, such that we determine the minimum in the histogram of the multi-subject map.

Algorithm 2 – Multi-Subject QMPD

1: Obtain QMPD binary mask ($Q_n$) as in [14] for all $n$ subjects;

2: Create a new map: $V = \sum_{j=1}^{n} Q_j$;

3: Threshold all voxels in $V$ by $t$ to obtain binary multi-subject mask: $Q$

4: Smooth complex data of all subjects with the new mask $Q$.

3.2. Visualization of ICA Results

Estimated sources in ICA are usually presented using z-score thresholded magnitude images to highlight the voxels of interest, therefore ignoring the phase information. The visualization method we introduce here takes into account the phase by using a two dimensional Mahalanobis distance metric in the real and imaginary data of the estimated sources given by

$$d_{k,i} = \sqrt{[\hat{s}_{k,i} - \mu_k]^T C_k^{-1} [\hat{s}_{k,i} - \mu_k]}$$

where $\hat{s}_{k,i} = [\hat{s}_{k,i, re}, \hat{s}_{k,i, im}]^T$; and $\mu_k$ and $C_k$ are the corresponding mean and covariance of the estimated sources. Voxels of interest are identified if they have a value higher than a previously specified threshold for the values obtained using (1).

An optional step is to eliminate low quality—noisy—voxels from the estimated sources, which are usually low magnitude voxels. In our application we use the phase derivative variance (PDV) quality map [15], since as desired, it assigns bad quality values to noisy areas in the complex images, i.e., volumes in the data where the voxels phase values and their gradients exhibit high variation.

4. RESULTS

4.1. fMRI Data

The dataset used in this paper is from sixteen subjects performing a finger-tapping motor task while receiving auditory instructions. The paradigm involves a block design with alternating periods of 30 s ON (finger tapping) and 30 s OFF (rest). The experiments were performed on a 3T Siemens TRIO TIM system with a 12-channel radio frequency (RF) coil. The fMRI experiment used a standard Siemens gradient-echo EPI sequence modified to store real and imaginary data separately. The data was pre-processed for motion correction and spatial normalization into standard Montreal Neurological Institute space using the MATLAB Toolbox for Statistical Parametric Mapping (SPM)\(^1\).

\(^1\) SPM, URL: http://www.fil.ion.ucl.ac.uk/spm/software/spm5
4.2. ICA using Nonlinear Decorrelations with atanh Nonlinearity

We use the ICA algorithm using atanh as the nonlinearity as described in section 2.3.1, hence no phase correction is needed for the estimates. Figure 3 shows the magnitude and phase image—of selected slices—for a motor task related component identified averaged over all the subjects. The motor component was identified by regressing its estimated time-courses with the ideal hemodynamic time response. The identified voxels in the mean images are consistent with the right motor neuronal areas that are expected to be activated during the task, e.g., Brodmann area 4.

![Mean magnitude and phase maps for the estimated motor component using atanh nonlinearity (Mahalonobis z-score of 3.5).](image)

**Fig. 3.** Mean magnitude and phase maps for the estimated motor component using atanh nonlinearity (Mahalonobis z-score of 3.5).

4.3. ICA using Infomax with a Circular Nonlinearity

Next, we estimate the components from fMRI data using Infomax algorithm [16] with a circular nonlinear function proposed in [17], which does not impose any particular orientation to the distribution of the estimated sources. The obtained ICA results at this point identify the motor task component in each subject, but group studies are limited to the magnitude image. Applying the post-ICA phase correction scheme described in section 2.3.2 to the estimated components in each subject, makes group analysis possible by eliminating the phase ambiguity across subjects.

The identified task related voxels in the mean magnitude and phase images in Figure 4 correlate with the areas identified and showed in Fig. 3. Therefore, once again, we obtain group results that are meaningful for the phase.

![Mean magnitude and phase maps of Infomax with a circular nonlinearity for estimated motor component (Mahalonobis z-score of 3.5).](image)

**Fig. 4.** Mean magnitude and phase maps of Infomax with a circular nonlinearity for estimated motor component (Mahalonobis z-score of 3.5).

5. DISCUSSION

We addressed the main issues that enable successful analysis of complex fMRI data from multiple subjects, and thus enables group fMRI analysis using the phase information. We implemented a simple, but effective, method to robustly denoise fMRI complex-valued data for group analysis. The multi-subject QMPD method accurately identifies and eliminates areas of the fMRI images that are corrupted by noise, measurement errors, and aliasing. QMPD can be used as a pre-processing step by any fMRI complex group analysis algorithm, including but not limited to ICA of fMRI. We also presented two effective phase correction schemes to eliminate the inherent scaling ambiguity of complex-valued ICA to allow for group analysis, using prior information or subsequent to ICA analysis. Additionally, we presented a visualization method that uses the phase in the estimated fMRI sources to identify task—functional—related voxels. Hence, we provide a complete framework that allows the utilization of the phase in the analysis of complex fMRI data, in particular using ICA that has already shown to hold much promise for the task.

6. REFERENCES