ADAPTIVE FEEDBACK CANCELLATION IN HEARING AIDS USING THE IPLS ALGORITHM

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ABSTRACT

Hearing aids suffer from the presence of a positive feedback loop between the output transducer and microphone. This feedback reduces both the stable gain achievable in the forward path as well as the sound quality of the output. Existing solutions to this problem perform feedback cancellation using adaptive filtering. The most common methods of adaptive filtering use the Least Mean Square (LMS) or Recursive Least Squares (RLS) algorithms, with the LMS being the most common choice for hearing aid applications. However, these approaches suffer from various drawbacks due to the unique challenges inherent in hearing aids. A new approach employing the Interior Point Least Squares (IPLS) algorithm demonstrates some distinct advantages over these other algorithms for use in the hearing aid schematic. Specifically, it is capable of achieving slightly faster convergence and, more significantly, maintains desired performance trade-offs with fixed parameter values despite changes in input signal power.

Index Terms— IPLS, adaptive filtering, hearing aid

1. INTRODUCTION

One major hindrance to the performance of digital hearing aids is the presence of a feedback loop between the output transducer and microphone. This feedback path consists of both the tissues and bones of the ear as well as the air space close to the ear; thus, its transfer function is both unknown and time-varying. A common approach to removing this feedback is to model the feedback path as an FIR filter and estimate its tap values with an adaptive filter. The hearing aid output is then sent through this estimated filter and subtracted from the hearing aid input signal (Fig. 1). Thus, the feedback signal is eliminated if the estimated filter exactly matches the feedback path. The taps of this adaptive filter can be updated using one of many algorithms such as Least Mean Square (LMS) [1–3], Recursive Least Squares (RLS) [4, 5], Wiener filtering [6], etc. By far the most commonly used algorithm in practice is the LMS due to its computational simplicity.

1.1. Challenges and Motivation

Several factors complicate the estimation process, some of which are unique to feedback cancellation as opposed to other adaptive filtering schemes. First, it is evident that the signal to be canceled is highly correlated with the input signal since it is a delayed and filtered version of the input. This hinders the ability of most algorithms to properly estimate the feedback path taps. Second, a unique challenge in the hearing aid application is limited computational power due the desire to make hearing aids as small and power efficient as possible. Thus, complex algorithms that perform well may not be implementable in practice. Third, the hearing aid input signal is very unpredictable both in terms of type and power (volume) level, and its characteristics can change quickly. Thus, any algorithm used for hearing aid feedback cancellation should be able to adapt well to different input signals automatically. Finally, we again emphasize that the feedback path is time-varying, undergoing changes during typical daily tasks such as sitting next to a car window or talking on the telephone. The algorithm must be able to not only adapt to an unknown path initially, but then track changes in this path as it varies with time.

The LMS algorithm is well-suited to hearing aids in terms of meeting the complexity constraints due to its simplicity, and is therefore the most commonly used algorithm in this scenario. However, the trade-offs of its computational efficiency are limited performance in terms of convergence rate and steady-state error, and sensitivity to parameter values that must be optimized for certain input signal characteristics and amplitudes. The RLS algorithm, on the other hand, demonstrates much superior performance as compared to the LMS both in terms of convergence rate and steady-state error in the feedback cancellation application; the trade-off is increased computational complexity. Whereas the complexity of the LMS algorithm increases with $O(M)$, where $M$ is the length of the adaptive filter, the RLS increases with $O(M^3)$. Additional drawbacks of the RLS algorithm are that it needs to be re-tuned to perform optimally for different input signal powers and can become unstable in finite-precision implementations.
The Interior Point Least Squares (IPLS) algorithm is a newer algorithm that was developed for the noise cancellation (as opposed to feedback cancellation) scenario [7] and demonstrates superior performance to the RLS algorithm in this scheme. Advantages seen include faster convergence and robustness in terms of not needing to be re-tuned for various SNR levels. Due to these initial results, it was analyzed and tested in the feedback cancellation case to determine if it could be a viable option for hearing aid applications.

2. IPLS Algorithm

A brief derivation of the IPLS algorithm will now be presented; details can be found in [7]. Following this, some of the properties of the algorithm such as transient convergence, complexity, and parameter choices will be discussed. Regarding notation, scalar quantities are labeled with normal font (e.g. \(d\)) while bold lower-case characters (e.g. \(w\)) denote vectors and bold upper-case characters (e.g. \(R\)) denote matrices. Bold characters with a subscript \(M\) denotes column vectors of the previous \(M\) samples (i.e. samples \((n - M + 1)\) through \(n\)), where \(M\) is the length of the filter weight vector. \(w\); \(d_n\) is the column vector of length \(n\) of all previous samples of the signal \(d(n)\).

2.1. Derivation

The IPLS algorithm is based on the minimization of the mean squared error, yielding a cost function:

\[
F(w(n)) = E[(d(n) - \hat{d}(n))^2] = \frac{1}{n} \sum_{i=1}^{n} (d(i) - x_M(i)^T \hat{w}(i))^2,
\]

where the signals \(d(n)\) and \(\hat{d}(n)\) are the desired and estimated signals, respectively, as seen in Fig. 1. Since this is a convex function, the minimum can be found by setting the gradient equal to zero and solving for \(w\), which gives us the well-known least squares, or Wiener, solution

\[
w^{ls}(n) = R_{xx}^{-1}(n)p_{xd}(n), \tag{2}
\]

where \(p_{xd}(n)\) denotes the cross-correlation vector and \(R_{xx}(n)\) the auto-correlation matrix. The IPLS algorithm estimates this solution using barrier/interior point methods. First, a convex feasibility region is defined as

\[
\Omega(n) = \left\{ w(n) \in \mathbb{R}^M | F(w(n)) \leq \tau(n), \|w(n)\|_2 \leq R \right\}. \tag{3}
\]

This definition restricts feasible weight vectors at time \(n\) to those yielding a cost less than the value of \(\tau(n)\), which is a positive, time-varying parameter that will be updated each iteration, and with overall squared magnitude less than \(R^2\), where \(R\) is a positive, fixed parameter chosen at initialization. This inequality constrained minimization problem can be transformed to an unconstrained minimization problem with infinite cost incurred at the boundary by introducing the logarithmic barrier function

\[
\phi(w(n)) = -\ln\{\tau(n) - F(w(n))\} - \ln\{R^2 - \|w(n)\|_2^2\}. \tag{4}
\]

The minimizer of this barrier function, \(\tilde{w}(n)\), is a good approximation to the minimizer of the original cost function if \(\tau(n)\) and \(R\) are chosen such that the first constraint of (3) is emphasized. We observe that this barrier function is convex since both of our constraints are convex, and thus the global minimum will exist at the point where the gradient is equal to zero. The resulting IPLS weight vector is

\[
\tilde{w}(n) = [\alpha(n, \tilde{w}(n))I + R_{xx}(n)]^{-1} p_{xd}(n), \tag{5}
\]

where \(I\) is the \(M \times M\) identity matrix and \(\alpha(n, \tilde{w}(n)) := \frac{\tau(n) - F(w(n))}{R^2 - \|w(n)\|_2^2}\) is a time-varying regularization term. The estimate \(R_{xx}(n)\) will be inaccurate when \(n\) is small and there are fewer data samples to base it on, and thus it is common to add a regularization term such as this to ensure invertibility. As \(R_{xx}(n)\) becomes more accurate, we desire the regularization to dissipate to zero so as to not introduce significant bias.

The last remaining term, \(\tau(n)\), is defined as

\[
\tau(n) := F(w(n - 1)) + \beta \frac{R}{\sqrt{2}} \|\nabla F(w(n - 1))\|_2,
\]

which ensures that \(\alpha(n, w(n - 1))\) is proportional to \(\|\nabla F(w(n - 1))\|_2\) so that it will dissipate at a rate proportional to how close the most recent estimate is to the optimal estimate. It is also noted that this definition for \(\tau(n)\) ensures that \(w(n - 1)\) remains a feasible vector (i.e. \(F(w(n - 1)) \leq \tau(n)\)) when the feasible region is updated. The slack between the updated boundary of the feasible region, \(\Omega(n)\), and the weight vector is controlled by the parameter \(\beta\), which is a fixed constant chosen by the user; a larger value will result in a rather conservative update (large slack) whereas a small value will result in a more aggressive shrinking of the feasible region (small slack).

To actually implement the IPLS algorithm, we cannot simply calculate the weight vector at each iteration from (5) because \(\alpha\) is dependent on \(w(n)\) rather than \(w(n - 1)\). Therefore, we generate \(w(n)\) by finding an approximate minimum of the barrier function using a single Newton iteration each step:

\[
w(n) = w(n - 1) - \left[\nabla^2 \phi(w(n - 1))\right]^{-1} \nabla \phi(w(n - 1))
\]

where \(\nabla \phi(w(n - 1))\) is the gradient of the barrier function and \(\nabla^2 \phi(w(n - 1))\) is its Hessian.

2.2. Characteristics

The convergence analysis of [7] assumes bounded autocorrelation matrix as well as bounded inputs and outputs. While this is a valid assumption for the general noise cancellation scheme, it is not necessarily true of the feedback cancellation case due to the presence of the positive feedback loop. Thus, while it is shown that the IPLS algorithm obtains the same asymptotic convergence rate of \(O(1/n)\) as the RLS algorithm for the noise cancellation case, new convergence analysis needs to be performed for the feedback case.

However, an intuitive comparison between the IPLS algorithm and the RLS algorithm is helpful in understanding the convergence characteristics of the two algorithms and the advantage of the IPLS algorithm. The RLS algorithm estimates the Wiener solution using recursive computations and includes a fixed regularization term, \(\delta\), which decays at a rate of \(\lambda^n\), where \(\lambda\) is the exponential weighting of the auto-correlation estimate [8]. Thus, the weight vector found by the RLS can be represented by

\[
w(n) = [\lambda^n sI + R_{xx}(n)]^{-1} p_{xd}(n).
\]

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Comparing this with equation (5), we see that the IPLS differs from the RLS in that the fixed term, $\delta I$, has been replaced by the time-varying term $\alpha(n, \hat{\beta}(n))$. The two main advantages of this time-varying term is that the amount of regularization will automatically adapt to signal input level and also decay in proportion to how quickly the algorithm converges.

Nothing comes free, and this time-varying regularization which can provide improved performance in the IPLS algorithm over the RLS algorithm also increases its computational complexity. The RLS algorithm’s complexity increases with $O(M^2)$ computations per iteration. However, the overall complexity of the ICLS algorithm as presented here is $O(M^3)$. It is possible to employ a recursive update for the Newton direction [7], but this only reduces the complexity to $O(M^2 \cdot 3)$: a significant improvement, but still higher than the RLS.

As discussed in the derivation, the IPLS algorithm requires the choice of two constants prior to implementation: $R$ and $\beta$. The former determines the maximum magnitude of a feasible weight vector. Referring to equation (4), we see that if $R$ is large, then the first constraint on the feasibility region is emphasized and the minimum of the function will tend to be centrally located, meaning that the minimizer of $\phi(w(n))$ will be a good approximation to the minimizer of $f(w(n))$. Thus, larger values of $R$ will result in lower steady-state error. However, they also allow for larger initial divergence of the tap values when the estimate of the input autocorrelation matrix is not very accurate.

As mentioned, $\beta$ controls the slack allowed when adjusting the feasibility region after each weight update. A smaller value corresponds to shrinking the feasibility region more aggressively while larger values result in more conservative updates. Simulations demonstrate that similar (but inverse) effects occur with various values of $\beta$ as with various values of $R$: larger $\beta$’s result in smaller initial divergence but larger steady-state error, and vice versa. Note that to emphasize the first constraint in (4), $\beta$ is typically chosen much smaller than $R$.

3. SUBBAND IPLS

As noted earlier, one particular challenge of adaptive feedback cancelation in hearing aids is limited computational power due to size and power constraints. While offering superior performance to both the LMS and RLS, the IPLS algorithm is likely too complex to be implemented directly in a practical hearing aid. One common approach used in digital hearing aids to reduce algorithm complexity is subband filtering, which separates the adaptive filtering task into several simpler tasks, each of which operates on only a certain band of frequencies. The hearing aid input signal can be efficiently separated into various subbands using a weighted-overlap-add (WOLA) implementation of a DFT filter bank; this was accomplished in this study using the WOLA Toolbox for Matlab developed by DSPFactory [9]. Each subband has its own adaptive filter and update algorithm which all operate in parallel and the outputs are then recombined to form the hearing aid output.

The computational savings of subband filtering arise from two main characteristics. First, each adaptive filter is significantly shorter than the fullband adaptive filter. Since the complexity of the ICLS algorithm increases exponentially with filter length, several implementations of shorter filters are more efficient than one implementation of a longer filter. Second, to obtain the $N$ subband samples each iteration, several hearing aid input samples can be operated on each iteration of the subband implementation. That is, input samples are taken in blocks of length $B$ and then the adaptive filters are updated once. This is the equivalent of downsampling the subband data streams, and thus the subband filters are updated less frequently than the sample rate of the hearing aid, as is done in the fullband case.

The exact amount of downsampling chosen is an important parameter due to several reasons. First, the downsampling controls the completeness or redundancy of the subband signals, and therefore the complexity of the computations. Second, more downsampling results in more space between subbands, reducing noise due to aliasing. Third, critical sampling (no redundancy) results in maximally white subband signals, while less downsampling increases the coloration due to the fact that the subband signals only possess a limited bandwidth of frequencies. Colored inputs result in a bias in adaptive algorithms, preventing them from converging to the correct weight vector. Thus, we see a trade-off between algorithm performance and complexity with regard to choice of downsampling rate.

4. SIMULATIONS

Simulation results will now be presented demonstrating the performance of the ICLS algorithm for the task of feedback cancellation. Comparison will be made against the RLS algorithm, since it is the closest to the performance of the ICLS (LMS typically performs worse). The feedback path used is a 40-tap FIR filter whose tap values approximate an example human feedback path and were obtained from tests by Chris Gao and Bob Fretz at Intricon Inc. Both fullband and subband implementations of the ICLS algorithm will be shown, demonstrating the performance sacrifices made when using the less complex, subband structure.

4.1. Fullband ICLS

First, the tap misalignment, $\zeta(n) = \|w(n) - w^*\|^2$, vs. iteration will be compared for the two algorithms for the cases of white noise and speech inputs. The algorithms’ parameters ($\delta$ for RLS and $R$ and $\beta$ for ICLS) have been tuned so as to provide as similar steady-state performance as possible: $\lambda = 1$ and $\delta = 2 \times 10^{-5}$ for RLS, and $R = 1000$ and $\beta = 9 \times 10^{-3}$ for ICLS.

![Fig. 2. Tap misalignment for RLS and IPLS algorithms for white noise input.](image)

For white noise input of variance $4 \times 10^{-3}$, Fig. 2(a) shows that when both algorithms are tuned to provide a steady-state error of -60 dB, the IPLS converges slightly faster than the RLS due to its time-varying regularization, $\alpha(n)$. Similar results were seen for other steady-state error values. Note that both algorithms converge very quickly, within about 100 iterations.

For the case of speech input (Fig. 2(b)), both algorithms perform very similarly. A forward path delay of 300 samples was inserted in both cases to combat bias incurred due to signal correlation. While improving the steady-state error, the insertion of the delay also results in larger initial divergence for both algorithms. In addition to
increased steady-state error, the correlated input signal also increases the number of iterations necessary to reach steady-state.

Figure 3 demonstrates the most significant advantage of the IPLS over the RLS algorithm. We have performed the same simulation as in Fig. 2(a), but have changed the input signal variance by +/- 100. We observe that the IPLS performs almost identically in all 3 cases, while the RLS demonstrates significantly different performance. The value of $\delta$ could be re-tuned in each scenario such that the RLS algorithm performs similarly for differing power levels, but the advantage of the IPLS is that it maintains a constant trade-off between initial divergence and steady-state error without having to modify its parameters.

4.2. Subband IPLS

For the subband implementation we cannot calculate tap misalignment as was done for the fullband case because the taps of each subband filter do not directly correspond to specific taps of the single feedback path filter. Therefore, we show the input and output signals to examine the distortion present in the output for both the fullband (Fig. 4(a)) and subband (Fig. 4(b)) implementations. We see close agreement in both plots, but careful observation yields more deviations of the output from the input curve in the subband implementation than occurred in the fullband due to the issues of aliasing, increased coloration, etc. incurred in the subband scheme.

5. CONCLUSION

We have presented a new algorithm, the Interior Point Least Squares, which was initially developed for the noise cancellation schematic, and have tested its potential for use in feedback cancellation in hearing aids. From the derivation of the algorithm, we see that it solves for a similar weight vector as the RLS algorithm, but uses a method based on interior point/barrier function minimization techniques rather than recursive computations of the Wiener solution. The benefit of the IPLS method is that it allows for a time-varying regularization parameter, $\alpha(n)$, in contrast to the fixed parameter, $\delta$, of the RLS algorithm.

Simulation results demonstrate that this time-varying regularization allows for faster convergence in some cases, but more significantly, allows the algorithm to maintain a constant trade-off of initial tap divergence and steady-state error regardless of input signal power level. For the case of correlated input (speech), we observe increased steady-state error due to the bias that results from the correlation. Inserting forward path delay to compensate reduces the performance of both algorithms and they are seen to perform very similarly.

A subband implementation was developed for the IPLS algorithm to reduce its complexity and make it more implementable in a practical hearing aid. While significantly reducing complexity, the subband implementation sacrifices performance due to aliasing and increased coloration. However, the performance seen in the simulations was still very acceptable. Given the positive results of this initial study, further analysis should be performed on the IPLS algorithm as applied to hearing aid feedback cancellation to further understand its characteristics and capabilities.

6. REFERENCES