I/Q Imbalance Mitigation for Time-Reversal STBC Systems Over Frequency-Selective Fading Channels

Mingzheng Cao and Hongya Ge
Center for Wireless Communications and Signal Processing Research
Dept. of ECE, New Jersey Institute of Technology
Newark, New Jersey 07102–1982
Email: mc229@njit.edu, ge@njit.edu

Abstract—This work studies the effect of in-phase/quadrature (I/Q) imbalance on time-reversal space-time block coded (TR-STBC) communication systems operating over frequency-selective fading channels. The transceiver I/Q imbalance in a 2 × 1 TR orthogonal STBC (TR-OSTBC) system is studied in detail, and low-complexity mitigation solutions are proposed by exploiting the special structure of the received data. Our results show that the proposed solutions both in time domain and in frequency domain can effectively mitigate the I/Q distortion in such a system.

Index Terms—I/Q imbalance, time reversal, space-time block coding

I. INTRODUCTION

STBC communication systems provide reliable data transmission by exploiting spatial diversity in flat fading channels [1]. To further exploit the multipath diversity embedded in the frequency-selective fading channels, TR-STBC system, both orthogonal and quasi-orthogonal, have been proposed and studied extensively [2]–[5].

In practical implementation, the I/Q imbalance (the non-ideal matching between the relative amplitudes and phases of I and Q branches of a transceiver) exists in many RF systems due to analogue imperfections [6]. This commonly results in a small complex conjugate term in time domain (see eqs.(1-2)), hence an equivalent mirror-image distortion term in frequency domain in the data structure of communication systems [7]–[8]. Therefore, the I/Q imbalance increases symbol error rate (SER) drastically, especially in STBC systems utilizing both symbols and their complex-conjugates.

Although there exist various I/Q mitigation methods in literature, the methods in [8]–[9] are preferred when the transmitter I/Q imbalance is taken into account. The receiver I/Q imbalance compensation is studied for an OSTBC-OFDM systems over frequency-selective fading channels in [8]. Taking into account the transmitter I/Q imbalance, the transceiver I/Q imbalance compensation for STBC OFDM systems in studied in [9]. Specifically, the method in [9] can mitigate the transceiver I/Q imbalance at the receiver with only the estimated effective channel state information (ECII) (\( \{ u_t, v_t \} \) in eq.(4)). It combines the tasks of estimating the transmitter I/Q imbalance parameters \( \{ \alpha_T, \beta_T \} \), the channel \( h_t \), and receiver I/Q imbalance \( \{ \alpha_R, \beta_R \} \). In this work, we develop a new transmission scheme that enables simple yet effective solutions, both in time domain and in frequency domain, to mitigate transceiver I/Q imbalance for TR-OSTBC systems operating over frequency-selective fading channels. Simulation results demonstrate that the transceiver I/Q imbalance can be effectively compensated by employing the proposed solutions with either known or estimated ECSII.

Notation: \( (\cdot)^* \), \( (\cdot)^T \) and \( (\cdot)^H \) denote complex-conjugate, transpose and complex-conjugate transpose operations, respectively; \( \mathbb{R}\{\cdot\} \) and \( \mathbb{I}\{\cdot\} \) denote real and imaginary parts, respectively; \( \odot \) and \( \otimes \) denote convolution and Kronecker product, respectively; \( (\cdot)_K \) denotes mod-\( K \); \( \text{circ}(x) \) denotes a circulant matrix with \( x \) being its first column.

II. SIGNAL AND SYSTEM MODELS

A. Transceiver I/Q Imbalance Model

In this work, we adopt a two-parameter frequency-independent I/Q imbalance model, and assume the same transmitter/receiver I/Q imbalance parameters for all transmitting/receiving antennas sharing the same local oscillator (LO).

When I/Q imbalance exists, the LO output at the transmitter can be expressed as \( c_T(t) = \alpha_T e^{j\varphi_T} + \beta_T e^{-j\varphi_T} \), where \( \alpha_T = \frac{1}{2} \left[ 1 + (1 + \epsilon_T) e^{j\varphi_T} \right] \) and \( \beta_T = \frac{1}{2} \left[ 1 - (1 + \epsilon_T) e^{j\varphi_T} \right] \), with \( \epsilon_T \) and \( \varphi_T \) representing amplitude and phase imbalance parameters of the transmitting antenna. Consequently, the up-converted band-pass signal for the intended transmission \( s(t) \) becomes \( \mathbb{R}\{ s(t)c_T(t) \} = \mathbb{R}\{ s(t)e^{j\varphi_T} \} \), resulting in the equivalent baseband signal

\[
\begin{equation}
   s_T(t) = \alpha_T s(t) + \beta_T s^*(t),
\end{equation}
\]

containing both the intended signal and its complex-conjugate.

Similarly, the LO output at the receiver can be expressed as \( c_R(t) = \alpha_R^* e^{-j\varphi_R} + \beta_R e^{j\varphi_R} \), where \( \alpha_R = \frac{1}{2} \left[ 1 + (1 + \epsilon_R) e^{j\varphi_R} \right] \) and \( \beta_R = \frac{1}{2} \left[ 1 - (1 + \epsilon_R) e^{j\varphi_R} \right] \), with \( \epsilon_R \) and \( \varphi_R \) representing amplitude and phase imbalance parameters of the receiving antenna. Taking into account the effect of channel \( h(t) \) and additive noise \( n(t) \), the received data down-converted by \( c_R(t) \) takes the equivalent baseband form,

\[
\begin{equation}
   r(t) = \alpha_R^* [h(t) \odot s_T(t)] + \beta_R [h(t) \odot s_T(t)]^* + n(t).
\end{equation}
\]

B. Data Structure With CP Aided Transmission

When two transmit-antennas are used to simultaneously transmit \( K \times 1 \) data blocks \( s_t \) over frequency-selective fading
channels $h_i = [h_i[0] \cdots h_i[L]]^T$, $i = 1, 2, L \ll K$, the received $K \times 1$ data vector containing I/Q imbalance has the following discrete-time baseband form,

$$r = \sum_{i=1}^{2} (U_i s_i + V_i s_i^*) + n. \quad (3)$$

Here the square matrices $U_i$ and $V_i$ in (3) are circulant due to the use of a cyclic-prefix (CP) preamble copying from $L$ trailing symbols of the data vector $s_i$ at the transmitter. Specifically, matrices $U_i = \text{circ}(u_i)$ and $V_i = \text{circ}(v_i)$ are parameterized by the channel and transceiver I/Q imbalance parameters as follows,

$$u_i = \alpha_T \alpha_R^* h_i + \beta_T \beta_R^* h_i^*, \quad i = 1, 2. \quad (4)$$

$$v_i = \beta_T \alpha_R^* h_i + \alpha_T \beta_R^* h_i^*, \quad i = 1, 2.$$

Here the $K \times 1$ vector $h_i = [h_i^T \ 0^T_{K-L+1}]^T$ is a zero-padded channel. Compared to the effect of I/Q imbalance on the signal, such effect on noise is relatively small at reasonably high SNRs, hence a proper $n \sim \mathcal{CN}(0, \sigma_n^2 I_K)$ is adopted.

The matrices $U_i$ and $V_i$ can be diagonalized by the unitary discrete Fourier transform (DFT) matrix $W$ as $U_i = W^H \Lambda_u W$, and $V_i = W^H \Lambda_v W$, with $\Lambda_u = \text{diag}(W u_i)$, and $\Lambda_v = \text{diag}(W v_i)$.

### C. Data Model for the Proposed TR-OSTBC System With CP

In a $2 \times 1$ TR-OSTBC system, symbol vectors in the $2K \times 2$ codeword matrix

$C(s_1, s_2) = \begin{bmatrix} s_1 & s_2 \\ -J_C(s_2) & J_C(s_1) \end{bmatrix}$ → space

are transmitted over two consecutive time slots through two transmit-antennas. Here $J_C(-)$ is a time-reversal conjugate (TRC) operator such that $J_C(s) = Js^*$ with $J$ being an exchange matrix. For any circulant matrix $A$, we have $JA^T = A^H$. Using the results in (3), the received data block in a TR-OSTBC system over two consecutive time slots can be formulated as $r = Us + Vs^* + n$, or, specifically,

$$\begin{bmatrix} r_1 \\ Jr_2 \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ U_2^H & -U_1^H \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} V_1 & V_2 \\ -V_2^H & V_1^H \end{bmatrix} \begin{bmatrix} s_1^* \\ s_2^* \end{bmatrix} + \begin{bmatrix} n_1 \\ Jn_2 \end{bmatrix}. \quad (5)$$

In the absence of I/Q imbalance, $V = 0$, symbol blocks $s_1$ and $s_2$ can be decoded separately due to the circulant nature of matrices $\{U_i\}$. However, the presence of I/Q imbalance forces us to take into consideration a widely linear relation containing both $s$ and its complex-conjugate $s^*$ in (5) to effectively decode symbols $\{s_i\}$. $i = 1, 2$. Under white Gaussian noise (WGN) assumption, the optimal maximum-likelihood sequence estimate (MLSE) of $\{s_i\}$ can be obtained from

$$\hat{s}_{ML} = \arg \min_{\hat{s} \in S} ||r - Hs||^2,$$

where the set $S$ contains all possible $4K \times 1$ signal vectors $\hat{s}$ from a given symbol constellation. For a TR-OSTBC system with large data block size $K$ (typically $K \gg L$), to reduce the computational complexity of the MLSE, a low-complexity sub-optimal solution built upon the filtering idea can be found for practical implementation. Introducing a linear filter $F$, we can minimize $E(||\hat{s} - Fs||^2)$ to obtain the linear minimum-mean-square-error (LMMSE) estimates of $\{s_i\}$,

$$\hat{s}_{LMMSE} = F_{LMMSE} \hat{r} = H^T (HH^T + \sigma_n^2 / \sigma_s^2 I_K)^{-1} \hat{r},$$

where $\sigma_s^2$ is signal power per sample. Its zero-forcing (ZF) counterpart is simply the result of dropping the diagonal loading factor,

$$\hat{s}_{ZF} = F_{ZF} \hat{r} = H^T (HH^T)^{-1} \hat{r}.$$
decision feedback (MMSE-DFE) and/or ZF-DFE, to remove the remaining inter-symbol interference (ISI). Let $F_f = L^T F_{f,MMSE}$ and $F_\theta = L^T - I_{4K}$ denote the feedforward filter and feedback filter of MMSE-DFE, respectively, the $4K \times 4K$ lower-triangular matrix $L$ can be explicitly obtained by the factorization, $H^H T + \sigma_r^2 / \sigma_n^2 I_{4K} = L L^T$. Here $L_h$ is a $4K \times 4K$ diagonal matrix used to normalize the diagonal entries of $L$. The block symmetric matrix $\hat{G} = H^H T \hat{H}$, with the similar structure as $G$, can be decomposed into

$$\hat{G} = \begin{bmatrix} I_{2K} & 0_{2K} \\ G_{12}^T & G_{11} & 0_{2K} \\ I_{2K} & 0_{2K} & G_s \\ 0_{2K} & G_s & G_{11} \end{bmatrix}(I_{2K} \hat{G}_{12})^T,$$

where $\hat{G}_{11} = I_2 \otimes \hat{A}$, and $\hat{G}_s = \hat{G}_{22} - \hat{G}_{12}^T \hat{G}_{11}^{-1} \hat{G}_{12} = L_h \otimes \hat{E}$. Hence, obtaining matrix $L$, only involves the inversion of $K \times K$ circulant matrix, whose computation load again can be dramatically reduced by DFT operations.

Dropping the diagonal loading factor, the implementation of ZF-DFE is straightforward.

B. Equivalent Frequency Domain Processing

By exploiting the structure of (5), a frequency domain processing can be applied to mitigate the I/Q imbalance. The equivalent frequency domain data model with white noise vector can be formulated as

$$[W_r] = \begin{bmatrix} \Lambda u_1 & \Lambda u_2 \\ \Lambda^* u_1 & -\Lambda^* u_2 \\ W_s \end{bmatrix} [S_1] + \begin{bmatrix} \Lambda v_1 & \Lambda v_2 \\ \Lambda^* v_1 & -\Lambda^* v_2 \\ W_n \end{bmatrix} [N].$$

For $K$-point DFT, we have

$$\mathcal{F}[\hat{s}_i[k]] = \hat{s}_i[k], \quad \mathcal{F}[s_i[k]] = s_i[k], \quad k = 0, \ldots, K-1,$

where $\hat{s}$ is DFT of $s$, and $k' = (k-K)$. Assume $K$ is even, let $r_1 = W_r 1, r_2 = W_r J^2, n_1 = W_n 1$, and $n_2 = W_n J^2$.

For $k = 0, K/2$, it can be obtained that

$$\begin{bmatrix} r_1 \bar{r}_1 \\ r_2 \bar{r}_2 \\ \bar{s}_{12} \bar{s}_{12} \end{bmatrix} = \begin{bmatrix} \Lambda u_1 \Lambda u_2 \\ \Lambda^* u_1 & -\Lambda^* u_2 \\ \Lambda v_1 \Lambda v_2 \\ \Lambda^* v_1 & -\Lambda^* v_2 \\ \Lambda s \end{bmatrix} \begin{bmatrix} s_1 \bar{s}_1 \\ s_2 \bar{s}_2 \\ s_3 \bar{s}_3 \\ s_4 \bar{s}_4 \\ s_5 \bar{s}_5 \end{bmatrix} + \begin{bmatrix} \Lambda n_1 \Lambda n_2 \\ \Lambda^* n_1 & -\Lambda^* n_2 \\ \Lambda \bar{n} \end{bmatrix} \begin{bmatrix} n_1 \bar{n}_1 \\ n_2 \bar{n}_2 \\ n_3 \bar{n}_3 \\ n_4 \bar{n}_4 \\ n_5 \bar{n}_5 \end{bmatrix}(8)$$

where $\Lambda u_n[k]$ and $\Lambda v_n[k]$ are the $k$-th diagonal entries of $\Lambda_{u,n}$ and $\Lambda_{v,n}$, respectively. Reformatting (8) as

$$\begin{bmatrix} [\bar{r}_1, \bar{r}_2] \\ [\bar{s}_{12} \bar{s}_{12}] \end{bmatrix} = \begin{bmatrix} \Lambda \bar{u} \Lambda \bar{u} \\ \Lambda^* \bar{u} & -\Lambda^* \bar{u} \\ \Lambda \bar{s} \end{bmatrix} \begin{bmatrix} \bar{s}_1 \bar{s}_2 \\ \bar{s}_3 \bar{s}_4 \\ \bar{s}_5 \end{bmatrix} + \begin{bmatrix} \Lambda \bar{n} \Lambda \bar{n} \\ \Lambda^* \bar{n} & -\Lambda^* \bar{n} \\ \Lambda \bar{n} \end{bmatrix} \begin{bmatrix} \bar{n}_1 \bar{n}_2 \\ \bar{n}_3 \bar{n}_4 \\ \bar{n}_5 \end{bmatrix}(9)$$

then the least-squares (LS) estimate of $\hat{s}_n$ can be obtained as

$$\hat{s}_n = (H^H_n H_n)^{-1} H^H_n r_n.$$

For other values of $k$, it can be obtained that

$$\begin{bmatrix} [\bar{r}_1, \bar{r}_2] \\ [\bar{s}_{12} \bar{s}_{12}] \end{bmatrix} = \begin{bmatrix} \Lambda u_1 \Lambda u_2 \\ \Lambda^* u_1 & -\Lambda^* u_2 \\ \Lambda v_1 \Lambda v_2 \\ \Lambda^* v_1 & -\Lambda^* v_2 \\ \Lambda \bar{s} \end{bmatrix} \begin{bmatrix} \bar{s}_1 \bar{s}_2 \\ \bar{s}_3 \bar{s}_4 \\ \bar{s}_5 \end{bmatrix} + \begin{bmatrix} \Lambda \bar{n} \Lambda \bar{n} \\ \Lambda^* \bar{n} & -\Lambda^* \bar{n} \\ \Lambda \bar{n} \end{bmatrix} \begin{bmatrix} \bar{n}_1 \bar{n}_2 \\ \bar{n}_3 \bar{n}_4 \\ \bar{n}_5 \end{bmatrix}(9)$$

Then the LS estimate of $\hat{s}_b$ can be obtained as

$$\hat{s}_b = (H^H_b H_b)^{-1} H^H_b F_b.$$
the performance compared to the above two. Moreover, further performance improvement can be achieved by introducing non-linear ZF-DFE and MMSE-DFE, compared to their linear counterparts. Furthermore, Fig.4 shows the proposed solutions with estimated ECSI can mitigate the I/Q imbalance as well.

V. CONCLUSIONS

This work develops the solutions to symbol detection, both in time domain and in frequency domain, for TR-STBC systems with I/Q imbalance operating over frequency-selective fading channels. Results demonstrate the effectiveness of the proposed approaches in mitigating I/Q imbalance and improving the SER performance.

REFERENCES