We present cooperative MIMO for alien noise cancellation (CoMAC): a per-tone, blind, low-complexity, linear, and adaptive noise whitening algorithm for alien crosstalk mitigation in upstream vectored VDSL systems. CoMAC directly acts on the residual errors of the vectored users after self-FEXT cancellation and frequency domain equalization, and thus, leverages the inherent alien-crosstalk-induced spatial correlation across users. CoMAC employs a low-complexity recursion scheme derived from the optimal MMSE noise whitener to non-disruptively initialize, engage, and adapt the noise canceller while the vectored users operate in data mode. Assuming reliable transmit symbol estimation at its input, we show that CoMAC achieves the Cramer-Rao lower bound. Further, the SNR improvements accruing from CoMAC can be translated into substantial rate improvements for upstream vectored VDSL.

Index Terms— MIMO, VDSL, crosstalk, blind noise whitening

1. INTRODUCTION

VDSL promises to be the last mile solution for high speed data communications. The data rate for VDSL is impaired by many sources of disturbance, like self-crosstalk, impulse noise, alien-noise, etc. Dynamic spectrum management level 3 (DSM3), a scheme based on the concept of signal cooperation among users at the central office (CO), is a current initiative undertaken by the VDSL standard body to mitigate the impact of self-far end crosstalk (FEXT), i.e., FEXT originating from co-operating users also called the vectored group [6], in both upstream (US) and the downstream directions. In the DSM3 compliant devices, a self-FEXT canceller operating on a per tone basis is usually employed at the receiver (CO) to mitigate the upstream self-FEXT. However, alien crosstalk mitigation does not fall under the purview of the DSM3 standard.

Alien crosstalk is an additive noise at the receiver of the vectored users in the VDSL system that generally stems from the services outside the vectored system that share the same cable. In the presence of strong alien crosstalk, the SNR gain resulting from self-FEXT cancellation might be marginal. In such situations, it can be possible to further improve the SNR by performing alien crosstalk mitigation. Due to the inception from the same sources, the alien noise experienced at the receivers of the different vectored users exhibits correlation. In the upstream direction, the vectored system can take advantage of the simultaneous access to the received signals of all vectored users at the CO to effectively mitigate alien crosstalk.

Most previous attempts to cancel the alien cross-talk are hardware based [1, 2] and employ common-mode sensor techniques. Software based solutions involve whitening the additive noise at the receiver by factorizing the noise correlation matrix, which requires matrix inversion, and hence, is computationally expensive and perhaps prohibitive, especially when applied simultaneously to hundreds of tones. Other solutions found in the literature require two-sided coordination [3], which is not feasible since the upstream transmitter CPEs do not cooperate. Reference [4] performs noise de-correlation by first factorizing the noise correlation and then using noise-predictive decision feed-back equalization.

In this paper, we derive a software based per-tone blind alien noise cancellation technique called cooperative MIMO for alien cancellation (CoMAC) for upstream vectored discrete-multi-tone (DMT)-based VDSL systems. Since the CO does not have access to signals that cause alien crosstalk, any alien crosstalk mitigation technique must necessarily be blind. The CoMAC algorithm is motivated by the fact that in the presence of alien crosstalk, the residual noise on a tone subsequent to self-FEXT cancellation still exhibits spatial correlation across the vectored users. This correlation across vectored users (channels) can be readily leveraged to construct a noise cancellation technique that predicts the noise on a particular channel based on the noises gathered from all the other channels via cooperation at the CO. Based on this rationale, we derive CoMAC, which directly acts on the residual error signals of the vectored users at the output of the slicer (demapper) subsequent to self-FEXIT cancellation and frequency domain equalization (FEQ). Thus, the CoMAC algorithm can be conveniently inserted into a vectored VDSL system that already employs a self-FEXIT canceller. Utilizing the slicer-error (detection-error) for a given tone is not only convenient, but also accurate, since it represents the residual noise during the learning of the canceller coefficients, thanks to the low symbol detection error probability for nominal DMT-DSL operation.

CoMAC employs a low complexity recursion scheme to non-disruptively initialize, engage, and adapt the alien noise canceller coefficients while the vectored users operate in data mode. This proposed alien-crosstalk cancellation technique also does not require any, usually expensive to implement, hardware modifications over and above those already necessitated by DSM3 requirements. Under the assumption of reliable transmit symbol estimation at the input to the alien crosstalk canceller (output of the self-FEXIT canceller), the signals at the output of the resultant alien-crosstalk canceller can be shown to achieve the Cramer-Rao lower bound (CRLB).

The outline of the paper is as follows. Section 2 describes the CoMAC technique in the context of the upstream vectored VDSL system. Section 3 is devoted to the theoretical performance analysis of CoMAC and shows that CoMAC achieves the CRLB. In section 4, we detail a linear and adaptive procedure for the CoMAC algorithm and explain how rate improvements can be achieved using the seamless rate adaptation (SRA) mechanism. Simulation results are given in the section 5, followed by conclusions in the section 6.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION FOR COMAC

2.1. US Receiver Structure and Mathematical Modeling

We consider a system of $N$ vectored VDSL users sharing the bundle with additional $R$ alien users also employing VDSL services. We
focus our attention on transmission in the upstream direction and assume that the DMT symbol boundaries for the vectored and alien users are perfectly aligned since they are deployed from the same cabinet. This assumption is also supported by the VDSL standard [7] and is therefore justified. Hence, the alien canceller we derive in this paper will be per tone and memoryless.

Fig. 1 shows the schematic of the suggested CoMAC algorithm in the VDSL receiver framework. The received time domain signal from each of the N vectored users is first processed by the FFT block. Let \( y[q, t] \) be the corresponding \( N \times 1 \) frequency domain signal vector on a particular tone \( q \) and for DMT symbol indexed by \( t \). These symbols are jointly accessible at the CO and are related to the \( N \times 1 \) transmit constellation vector \( x[q,t] \) as

\[
y[q, t] = H x[q, t] + u[q, t],
\]

where, (a) \( H = H_d(I + C) \) is the \( N \times N \) MIMO channel matrix for the vectored users, (b) \( H_d \) represents the direct-channel component of \( H \) with only diagonal terms, (c) \( C \) denotes the off-diagonal self-FEXT coupling matrix, (d) \( u \) is the \( N \times 1 \) receiver noise vector, which is a combination of alien crosstalk and additive white gaussian noise. The signal \( y[q, t] \) is then processed by the diagonal \( N \times N \) frequency domain equalizer \( F_d[q, t] \) and the self-crosstalk canceller \( M[q,t] \) \(^5\) [5] and the resulting output of these operations is given by

\[
z[q, t] = (MF_d y)[q, t].
\]

Remaining aspects of Fig. 1 will be discussed in Section 4. Meanwhile, discarding indices \( q \) and \( t \) and assuming all diagonal equalization and self-FEXT cancellation (i.e., \( MF_d H = I \)), from (1) and (2), we have \( z = x + MF_d u = x + w \). Let \( \Theta \) denote the \( N \times R \) alien coupling matrix and \( x_a \) denote the \( R \times 1 \) alien channel vector. Also, let the \( N \times 1 \) vector \( \varphi \) represent the receiver AWGN. Then, we have \( u = \Theta x_a + \varphi \), and

\[
w = MF_d u = Ax_a + v,
\]

where \( A = MF_d \Theta \) and \( v = MF_d \varphi \) represent the equivalent crosstalk coupling from the alien disturbers to the vectored users and the equivalent background noise, respectively, after FEQ and self-FEXT cancellation.

The cross-correlation amongst the different elements of \( v \) is negligible, since realistic self-FEXIT cancellers conforming to the model of (2) are diagonally dominant due to the characteristics of the MIMO-DSL channel [5, 7, 8]. In the sequel, we further assume that the covariance matrix of \( v \) is given by \( E[vv^H] = \sigma_v^2 I \). Note that although the noise model represented by (3) is a close approximation to the real system, the CoMAC algorithm that we present later is agnostic of the underlying physics that generates spatial correlation in the noise.

### 2.2. Minimum Mean Square Error Noise-Whitening

Since the noises across the vectored users \( w_m \), \( 1 \leq m \leq N \) are correlated, we can always find a \( N - 1 \) length vector \( \alpha_m \) for each user \( m \), such that the variance of the alien-noise spatial-prediction-error, \( \sigma_m^2 = \text{var}(w_m - w_{\text{opt}}^T \alpha_m) \)

\(^5\)The equalization of the MIMO channel \( H = H_d(I + C) \) is split into the diagonal-matrix component \( F_d \) and the full-matrix component \( M \) to ensure backward compatibility with legacy VDSL systems that do not exploit signal cooperation; such legacy VDSL systems equalize only \( H_d \) using \( F_d \).

is minimized for each user \( m \), \( 1 \leq m \leq N \). Here, \( w_{\text{opt}} \) is an \( N - 1 \) length noise vector comprising all elements of \( w \) but \( w_m \). The optimal \( \alpha_m \) using the orthogonality principle is given by:

\[
\alpha_m = \Gamma_{%,m}^{-1} p_m ,
\]

where \( \Gamma_{%,m} = E(w_m^T w_m^T) \) is the noise covariance matrix of all the users except \( m \) and \( p_m = E(w_m w_m^T) \). The noise cancellation for each user \( m \) can be easily performed as

\[
\hat{z}_m = z_m - \alpha_m (w_m^T)^{-1} w_m .
\]

The resulting alien cancelled symbol for channel \( m \), \( \hat{z}_m \), then passes through the TCM demodulation block for further processing. This procedure can be applied concurrently for all the users. The next logical step therefore is to evaluate the performance of CoMAC and compare it with the maximum achievable performance.

### 3. COMAC ALGORITHM PERFORMANCE EVALUATION

Assuming that all the alien disturbers transmit with the same power, \( E[x,v] = \rho^2 I \), and by equation (3), the total noise for the \( m \)-th channel is expressed as \( w_m = v_m + a_m x_a \), where \( a_m \) is the \( m \)-th row of \( A \). Additionally, the variance of \( v_m \) is given by

\[
\sigma_v^2 = \begin{bmatrix} \sigma_v^2 & \rho^2 \sigma_v^2 & \cdots & \rho^2 \sigma_v^2 \end{bmatrix}.
\]

Inserting (5) into (4) and with some algebra, we get

\[
\sigma_m^2 = \text{var}(w_m - w_{\text{opt}}^T \alpha_m) = \sigma_v^2 - \rho^2 \sigma_v^2 |\alpha_m|_2^2 - \rho^2 |\alpha_m|_2^2 .
\]

From (8), it is clear that the reduction in the noise variance due to the optimal noise canceller is given by the term \( \text{var}(w_m - w_{\text{opt}}) \).

Expressing \( w_{\text{opt}} \) in terms of its components using (3), we have

\[
\Gamma_{%,m} = E[(v_{\text{opt}} + A_{%,m} x_a)(v_{\text{opt}} + A_{%,m} x_a)^T] = \sigma_v^2 I + \rho^2 A_{%,m} A_{%,m}^T ,
\]

where \( A_{%,m} \) is an \( N - 1 \times 1 \) submatrix of \( A \) without the \( m \)-th row. Combining (7), (8), (9), and (10), defining \( \lambda = \rho^2 / \sigma_v^2 \), using the matrix inversion lemma, and with a little bit of algebra, the final noise variance achieved by the optimal alien canceller is given by

\[
\sigma_m^2 = \sigma_v^2 + A_{%,m}^T (I + \lambda A_{%,m} A_{%,m}^T) A_{%,m} \frac{1}{\lambda - \sigma_v^2} .
\]

Equation (11) implies that the CoMAC does not attain the alien noise free variance of \( \sigma_v^2 \); nevertheless, it will be seen next that this optimum alien canceller still achieves the CRLB.

### Cramer-Rao Lower Bound Computation

The CRLB for an estimator of the transmit symbol of user \( m \) denoted by \( \sigma_m^2 \) serves as a lower bound on \( \sigma_v^2 \) and is given by

\[
\sigma_m^2 \geq \frac{1}{\text{CRLB}_m} = \frac{1}{\ln(p(z|x))}. \quad (12)
\]

Since, \( w \) is circularly symmetric Gaussian with covariance matrix \( \Gamma \), we get the following:

\[
\frac{\partial^2}{\partial x_m \partial x_m^T} \ln(p(z|x)) = -\frac{\partial^2}{\partial x_m \partial x_m^T} \left( \frac{1}{\sigma_m^2} (z - x) \Gamma^{-1} (z - x) \right) , \quad (13)
\]

\[
\sigma_m^2 \geq \frac{1}{\text{CRLB}_m} = \frac{1}{\Gamma_{%,m}} = \left( \frac{1}{\Gamma_{%,m}} - \frac{1}{\sigma_m^2} \right) , \quad (14)
\]

where \( \Gamma_{%,m} \) the \( (m,m)^{\text{th}} \) component of the inverse of covariance matrix \( \Gamma \). Indeed, from (13) and (8), it is clear that the CoMAC algorithm achieves the CRLB.
4. LOW COMPLEXITY ADAPTIVE COMAC FOR PRACTICAL IMPLEMENTATION

The alien crosstalk canceller scheme specified by (5) and (6) assumes that the noise vector \( \mathbf{w} \) is exactly known. In a practical receiver, \( \mathbf{w} \) needs to be estimated. Let \( \hat{\mathbf{x}} \) be the demapped symbol, \( \mathbf{z} \) be the received signal, and \( \mathbf{m} \) be the estimate of the crosstalk canceller coefficients. To avoid the need to learn these statistics before the alien-canceller is in active operation (engaged), the adaptation phase by replacing the true error \( \mathbf{e} \) with \( \hat{\mathbf{e}} \) that depends on the detection-error of the post-alien canceller signal for user \( m \) defined as

\[
\hat{\mathbf{e}}_m[t] = \mathbf{e}_m[t] - \text{demapped} \left( \mathbf{z}_m'[t] \right) .
\]

Thus, the alien-canceller may be iteratively computed as

\[
\mathbf{\alpha}_m[t_n] = \mathbf{\alpha}_m[t_{n-1}] + \mu \mathbf{\hat{w}}_m^T[t_n] \mathbf{\hat{e}}_m[t_n], \quad -L < n \leq 0 \quad (17)
\]

with starting value \( \mathbf{\alpha}_m[t_{-L}] = \mathbf{0} \) and where \( \mu \) is the step size. At DMT symbol time index \( t_0 \), the learned alien-canceller coefficients are inserted into operation and the system transitions from the initialization phase to the adaptations phase.

4.2. Adaptation

Since the alien-canceller is engaged in the adaptation phase, at any time index \( t_n \), \( n \geq 1 \), the existing estimate of the canceller coefficients for user \( m \) given by \( \mathbf{\alpha}_m[t_{n-1}] \) is applied to \( \mathbf{\hat{w}}_m[t_{n}] \) to generate the post-alien canceller signal \( \hat{\mathbf{z}}_m[t_{n}] \) as

\[
\hat{\mathbf{z}}_m[t_n] = \mathbf{z}_m[t_n] - \mathbf{\alpha}_m^T[t_{n-1}] \mathbf{\hat{w}}_m[t_n] .
\]

In the adaptation phase, we could continue to update the alien-canceller coefficients using the LMS recursion of (17), which utilizes the instantaneous prediction-error \( \hat{\mathbf{e}}_m[t_n] \) that depends on the detection-error at the input to the alien-canceller. However, it is also possible to improve the reliability of the updates during the adaptation phase by replacing \( \hat{\mathbf{e}}_m[t_n] \) with the instantaneous detection-error of the post-alien canceller signal for user \( m \) defined as

\[
e_m[t_n] = \mathbf{z}_m'[t_n] - \text{demapped} \left( \mathbf{z}_m'[t_n] \right) .
\]

Thus, the alien canceller for user \( m \) may be computed in the adaptation phase by the following decision directed recursion:

\[
\mathbf{\alpha}_m[t_n] = \mathbf{\alpha}_m[t_{n-1}] + \mu \mathbf{\hat{w}}_m^T[t_n] \mathbf{e}_m[t_n], \quad n > 0 \quad .
\]

The recursion of (20) can be initialized with \( \mathbf{\alpha}_m[t_0] \) computed via 17, or alternatively with a null canceller, \( \mathbf{\alpha}_m[t_0] = \mathbf{0} \). The updates described in (20) operate in data mode and allows us to adapt the canceller in a changing noise environment. Further, just like the recursion during the initialization phase, the iterative estimation of (20) also does not require any matrix inversion or factorization.
4.3. Rate improvements via seamless rate adaptation (SRA)

Engaging the adaptive alien-crosstalk canceller results in an improvement in the SNR at the output of the alien-crosstalk canceller (see Fig. 3). In a practical system, this SNR improvement can be translated to an increased data-rate by relying on the SRA mechanism which is a standard feature of VDSL systems [7]. The SRA mechanism triggers a new bit-loading computation when a substantial and sustained change in the SNR is observed. E.g., if a vectored user with 8 bits (256 QAM) on a given tone before CoMAC experiences a 6 dB gain in SNR post-CoMAC, then the SRA will increase the bit-loading to 10 bits (1024 QAM). Once the bit-loading is updated by the SRA, the bit-error rate based on decisioning the received signal on a tone at the input to the alien-crosstalk canceller is no longer guaranteed to be always below $10^{-7}$. This implies that the residual error, $\hat{w}_m$, at the input to the alien-crosstalk canceller is no longer guaranteed to be reliable. This phenomenon may limit the realistic improvement to the bit-loading table; an analysis of its impact on the performance of the alien-crosstalk canceller is a topic of ongoing research.

5. SIMULATION RESULTS

Fig. 3 illustrates the SNR evolution on the 10 MHz tone for three different vectored users in a system of $N = 10$ vectored and $R = 5$ alien users. All users are assumed to have a loop length of 1 kft and the alien canceller is updated at every iteration according to (20) starting with a null initial value. The three chosen users correspond to three different scenarios of low, moderate, and high SNR gain, and the FEXT couplings are selected from [8]. It is clear from Fig. 3 that for all users, the steady-state SNR is very close to the CRLB computed using (14). Further, the user with the worst SNR before CoMAC (user 3) experiences a gain of 23dB after the alien noise is mitigated via CoMAC. This would suggest an increase in bit-loading by 7 bits. However, the realistic gain in bit-loading that can be accommodated without compromising reliability is limited to 2 or 3 bits per tone, as reasoned in Section 4.3. Despite this limited realistic bit-loading gain, the large number of tones used in upstream VDSL allows for the possibility of substantial rate improvements.

6. SUMMARY AND CONCLUSIONS

In upstream vectored VDSL systems impaired by alien crosstalk, the SNR gain stemming from self-FEXT cancellation may be marginal, and may be improved further by performing alien crosstalk mitigation. In this paper, we presented CoMAC (acronym for cooperative MIMO for alien noise cancellation): a per-tone, blind, low-complexity, linear, and adaptive alien noise whitening algorithm for upstream vectored VDSL systems. CoMAC directly acts on the residual error signals of the vectored users subsequent to self-FEXT cancellation and FEQ, and thus, leverages the inherent spatial correlation across users due to the presence of alien crosstalk. The CoMAC algorithm employs a low-complexity adaptive recursion scheme derived from the optimal MMSE noise whitener to non-disruptively initialize, engage, and adapt the alien noise canceller coefficients while the vectored users operate in data mode. CoMAC can be applied concurrently to all vectored users and allows us to adapt the canceller in a changing noise environment without requiring any matrix inversion or factorization. We showed via both theory and simulations that CoMAC achieves the CRLB under the assumption of reliable transmit symbol estimation at its input. Finally, we conclude that the CoMAC algorithm can provide substantial rate improvements for upstream vectored VDSL systems by exploiting the VDSL-standard based SRA mechanism to translate the SNR improvements to improvements in bit-loading.

7. REFERENCES