REDUCED-RANK MULTIUSER RELAYING (RR-MUR) SCHEME FOR UPLINK CDMA NETWORKS

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ABSTRACT

Cooperative relaying has been studied extensively in the literature to exploit spatial diversity gains by having each source transmit its messages through multiple independently fading relay paths. In multiuser systems where multiple sources may access the same set of relays simultaneously, CDMA spreading techniques along with multiuser detection schemes have been proposed in the literature to eliminate multiple access interference (MAI). In order for each relay to forward messages from all sources, a tremendous increase in dimensions (or spreading gain) is used to accommodate the relay transmissions. To reduce the required bandwidth or dimensions, we propose a reduced-rank multiuser relaying (RR-MUR) scheme where the data received from multiple users are first compressed into lower dimensions before being retransmitted. More specifically, linear compression precoders at the relays and decoder at the destination are found by imposing a recursive joint optimization procedure with the objective of minimizing the mean square error (MMSE) of the estimate at the destination. We show through numerical simulations that the RR-MUR scheme outperforms the often adopted $Q$-selection scheme in terms of increased spectral efficiency.

Index Terms— cooperative communications, relay networks, multiuser detection, CDMA.

1. INTRODUCTION

Multiple antenna systems have been proposed in the literature to combat fading by exploiting spatial diversity gains. However, due to limitations in size and cost, it is becoming increasingly difficult and impractical to equip mobile devices with multiple antennas. Recently, user cooperation has been studied extensively, e.g. in [1, 2], to achieve similar spatial diversity gains by allowing users to cooperate by relaying each others messages to the destination. Specifically, with cooperation, users that are experiencing deep fades may utilize the quality channels provided by the relays to transmit their data to the destination. By doing so, users can effectively reduce their transmission outages and enlarge the system throughput without having to employ multiple antennas on each device.

Many cooperative relaying techniques have been proposed in the literature, among which amplify-and-forward (AF) and decode-and-forward (DF) [1] have been the most popular. In this work, we shall consider the AF scheme where a relay simply amplifies and retransmits the received signal without explicitly decoding the messages. Most previous work on cooperative relay networks consider only the simple case with a single source-relay pair or the case where a single source is served by multiple relays. To enable simultaneous access by multiple sources, code division multiple access (CDMA) techniques along with multiuser detection schemes have been proposed in [3, 4, 5]. Specifically, in [3, 4], multiuser detection schemes have been employed at both the relay and the destination to mitigate the effect of multiple access interference (MAI) in pair-wise cooperation systems where each relay forwards messages for only one source. The case where multiple sources can access the same set of relays simultaneously is considered by the authors in [5] where a relay-assisted-decorrelating multiuser detector (RAD-MUD) scheme is proposed to further reduce the MAI with the help of multiuser precoding at the relays. It has been shown in [5] that the effective mitigation of MAI is crucial to attain diversity gains in the bit-error-rate since it would otherwise be dominated by the MAI.

In this paper, we consider the case where each relay is to forward the messages from all sources simultaneously. To effectively separate the forwarded messages, the number of dimensions (i.e. the spreading gain) required at the relays may increase rapidly with the number of relays or the number of sources. To reduce the required dimensions, the relays may elect to transmit with lower dimensions, i.e. the messages received by each relay must be compressed. An intuitive approach that is often adopted in the literature is the selective relaying scheme (i.e. the $Q$-selection scheme referred to in this paper) where only the message of the best user is being forwarded at each relay. We argue that this class of strategies do not fully utilize the available dimensions in transmitting the most useful data.

The main contribution of this paper is to propose the reduced-rank multiuser relaying (RR-MUR) scheme where we use a set of linear reduced-rank precoders at the relays (to perform the compression) along with a linear multiuser detector at the destination to mitigate the MAI. The linear precoders at the relays and the multiuser detector at the destination are jointly optimized in terms of minimizing the mean square error (MSE). Due to the complexity of the optimization, we propose a suboptimal iterative procedure to solve for the precoders and decoders, which is similar to the approach used in [6] for dimension reduction in the context of sensor networks. This can be implemented in practice by allowing relays and the destination to broadcast the updated precoders or decoders in turn until the MSE converges. We show through numerical simulations that the RR-MUR scheme outperforms the often adopted $Q$-selection scheme in terms of increased spectral efficiency.

2. SYSTEM MODEL

Consider an uplink CDMA network with $K$ sources transmitting to a common destination (i.e. base station) through the help of $L$ relays, as shown in Fig.1. The cooperation scheme takes on two phases of transmission. In phase I, data is first transmitted from the source to
the relays and then, in phase II, the signals received at the relays are amplified and forwarded to the destination in a cooperative manner.

Specifically, in phase I, each source, say source $k$, transmits a block of symbols $\{x_k[m]\}_{m=1}^M$ to the relays with transmission power $E[x_k[m]^2] = 1$ for all $m$. The signal transmitted by the $k$-th source ($k = 1, 2, \ldots, K$) is given by

$$x_k(t) = \sum_{m=1}^M \sqrt{P_{sk}} x_k[m] s_k(t - mT_s),$$  

where $s_k(t)$ is the spreading waveform of source $k$ and $T_s$ is the symbol duration. The spreading waveform $s_k(t)$ is given by

$$s_k(t) = \frac{1}{\sqrt{N_s}} \sum_{n=1}^{N_s} c_k[n] \varphi(t - nT_s),$$

where $c_k = [c_k[1], c_k[2], \ldots, c_k[N_s]]$ is the $\pm 1$ spreading sequence assigned to source $k$, $N_s$ is the spreading gain, and $\varphi(t)$ is the normalized chip waveform with chip period $T_c = T_s/N_s$. We assume that the spreading sequences $\{c_k[n]\}_{k=1}^K$ are linearly independent. To simplify our discussions and focus on the effectiveness of dimension reduction, we consider a synchronous CDMA system where all transmitted symbols arrive at the receivers simultaneously. Thus, the signal received at the relay $l$ is given by

$$y_l(t) = \sum_{m=1}^M \sum_{k=1}^K h_{skR_l} \sqrt{P_{sk}} x_k[m] s_k(t - mT_s) + n_l(t),$$

where $h_{skR_l}$ is the channel coefficient between source $k$ and relay $l$, and $n_l(t)$ is the additive white Gaussian noise at the $l$-th relay. Let us consider a quasi-static Rayleigh fading channel where we assume that $h_{skR_l}$ is complex Gaussian with mean 0 and variance $\sigma_{skR_l}^2$, i.e. $h_{skR_l} \sim \mathcal{CN}(0, \sigma_{skR_l}^2)$, and that it remains constant over the transmission of each data block. Moreover, we assume that the channel coefficients are known at all relay nodes and the destination.

At each relay, the received signal is first passed through a match filter bank (MFB) corresponding to the spreading waveforms $s_1(t), s_2(t), \ldots, s_K(t)$. In the $m$-th symbol period, the output of the MFB relay $l$ can be expressed as

$$y_l[m] = RH_0 x[m] + n_l[m],$$

where $x[m] = [x_1[m], x_2[m], \ldots, x_K[m]]^T$, $R$ is the correlation matrix with the $(j,k)$-th element $R_{jk} = \int_{-\infty}^{\infty} s_j(t) s_k(t) dt$, $H_0 = \text{diag}(\sqrt{P_{sk}} h_{s1R_l}, \sqrt{P_{sk}} h_{s2R_l}, \ldots, \sqrt{P_{sk}} h_{sKR_l})$, and $n_l[m]$ is the noise vector with distribution $\mathcal{CN}(0, \sigma^2 n_l R)$. When the spreading waveforms are not perfectly orthogonal, the off-diagonal terms of $R$ may be non-zero, resulting in MAI. In this case, linear multiuser pre-coders at the relays as well as multiuser detectors at the destination can be employed to reduce MAI and to further exploit cooperative diversity gains, as shown in [5].

### 3. Reduced-Rank Multiuser Relaying (RR-MUR)

By employing the AF scheme at the relays, the noisy signal received at the relays, e.g. $y_1[m]$, is simply multiplied by a linear precoding matrix and retransmitted to the destination without computing the hard decisions. Suppose that each relay, say relay $l$, is assigned $Q$ ($Q \leq K$) linearly independent spreading waveforms to retransmit the information embedded in the received vector $y_l[m]$ (which has $K$ dimensions). That is, a total of $L \cdot Q$ dimensions are required in order to fully separate all signals transmitted by the relays.

Specifically, in the $m$-th symbol period at relay $l$, we first multiply the vector $y_l[m]$ with a $Q$-by-$K$ precoding matrix $B_l$ to yield the vector of precoded symbols

$$t_l[m] = [t_{l,1}[m], t_{l,2}[m], \ldots, t_{l,Q}[m]]^T = B_l y_l[m].$$

The role of the precoding matrices $B_1, B_2, \ldots, B_L$ is to compress the size of received symbols without significantly increasing the MAI at the destination. The precoded symbols at relay $l$, i.e. $t_l$, are then forwarded to the destination in phase II with the spreading waveforms $s_{1,l}(t), s_{2,l}(t), \ldots, s_{Q,l}(t)$, which also has duration $T_s$. The spreading waveform $s_{i,l}(t)$ is given

$$s_{i,l}(t) = \frac{1}{\sqrt{N_c}} \sum_{n=1}^{N_c} c_{i,l,q}[n] \varphi(t - nT_c),$$

where $c_{i,l,q} = [c_{i,l,q}[1], c_{i,l,q}[2], \ldots, c_{i,l,q}[N_c]]$ is the $\pm 1$ spreading sequence with length $N_c$, $\varphi(t)$ is normalized chip waveform with chip period ($T_c = T_s/N_c$). The spreading gain $N_c$ is assumed to be no less than $LQ$. The signal transmitted by the $l$-th relay is

$$u_l(t) = \sum_{m=1}^M \sum_{q=1}^Q t_{l,q}[m] s_{i,l,q}(t - mT_s).$$

Thus, the signal received at the destination in phase II is given by

$$y_D(t) = \sum_{m=1}^M \sum_{l=1}^L \sum_{q=1}^Q h_{Dl} d_{l,q}[m] s_{i,l,q}(t - mT_s) + n_D(t).$$

Fig. 1. Cooperative CDMA uplink with $K$ sources and $L$ relays.

Fig. 2. The block diagram of reduced-rank multiuser relaying (RR-MUR) scheme in CDMA cooperative networks.
where \( h_{R_lD} \sim \mathcal{CN}(0, \sigma^2_{R_lD}) \) is the channel coefficient between relay \( l \) and the destination, and \( n_{D}(t) \) is the AWGN at the destination. The received signal \( y_D(t) \) is passed through an MFB corresponding to the \( LQ \) spreading waveforms \( \{s_1(t), \ldots, s_L(t)\} \). The MFB output at the destination is given by

\[
y_D[m] = h_{R_lD}^{RL} \begin{bmatrix} R_{1,l} \vdots R_{L,l} \end{bmatrix} t[m] + \cdots + h_{R_lD}^{RL} \begin{bmatrix} R_{1,l} \vdots R_{L,l} \end{bmatrix} t_l[m] + n_{D}(m),
\]

where \( t[m] = [t_1[m] \cdots t_L[m]]^T \), \( H_D = \text{diag}(h_{R_1D}, \ldots, h_{R_LD}) \), \( R_D \) is a \( LQ \times LQ \) correlation matrix with \( (l_1, l_2) \)-th block-element being \( R_{l_1,l_2} \) with \( [R_{l_1,l_2}]_{q_1,q_2} = \int_0^T s_{l_1,q_1}(t)s_{l_2,q_2}(t)dt \) for \( 1 \leq l_1, l_2 \leq L, 1 \leq q_1, q_2 \leq Q \). The noise is \( n_D \sim \mathcal{CN}(0, \sigma^2_{nD}) \).

Finally, the MFB output \( y_D[m] \) is multiplied by the matrix \( C \) of size \( K \times LQ \) to obtain an estimate of the transmitted symbol vector \( \hat{x}[m] \), i.e. the source symbols are estimated as

\[
\hat{x}[m] = Cy_D[m].
\]

Detection is then performed by taking the hard decision on \( \hat{x}[m] \).

The performance of this scheme is affected by the choice of matrices \( B_1, B_2, \cdots, B_L \) and \( C \) which will be found in the following section.

4. JOINT OPTIMIZATION OF LINEAR PRECODER AND DECODER

To optimize the performance of the RR-MUR scheme, we propose a scheme where the set of linear precoders \( B_1, B_2, \cdots, B_L \) and the detector \( C \) are chosen jointly to minimize the mean-squared error (MSE) of the estimate \( \hat{x}[m] \). That is, the optimal set of precoders and the optimal detector can be expressed as

\[
\{B_1^*, \ldots, B_L^*, C^*\} = \arg \min_{\{B_1, \ldots, B_L, C\}} \mathbb{E}[\|x[m] - \hat{x}[m]\|^2],
\]

s.t. \( \mathbb{E} \left[ \int_{m=1}^{mT} \|u_l(t)\|^2dt \right] \leq P_{R_l}, \quad l = 1, 2, \ldots, L \)

where \( P_{R_l} \) is the average power constraint at the \( l \)-th relay node. The power constraint above can be expressed alternatively as

\[
\mathbb{E}[t_l^H[m]R_l[t_l[m]]] = \text{tr}(R_lB_l[R_lH_lH_l^HR_l + \sigma^2_{n}B_l^H]) \leq P_{R_l}.
\]

From (4), (7), and (8), the MSE of the estimated symbols in \( \hat{x}[m] \) can be expressed more clearly as

\[
\mathbb{E}[\|x[m] - \hat{x}[m]\|^2] = \mathbb{E}[\|x[m] - CRDH_Dt[m] + n_{D}(m)\|^2] = \text{tr}\left\{I_K + CRDH_DH_D^H + \sigma^2_{n}B_l^H \right\}
\]

\[
-\text{tr} \left\{ R_l^{H}B_l^{H}R_l^{H}D_l^{H}C_{l}^{H} - CRDH_DH_D^H + \sigma^2_{n} \left( CRDH_DB_l^H + CRD_lC_{l}^H \right) \right\}.
\]

Subproblem II, we search for the optimum precoders \( \{B_l^*\}_{l=1}^L \), given a fixed detector \( C \), subject to the power constraint given in (10). The two subproblems are performed in turn until the MSE converges to a constant value.

Let \( B^{(1)}_1, B^{(1)}_2, \ldots, B^{(1)}_L \) be the set of precoders obtained in the \( j \)-th iteration. The MMSE estimator \( C^{(j)} \) can be obtained by the Wiener-Hopf equation, i.e.,

\[
C^{(j)} = \arg \min_{C} \mathbb{E}[\|x[m] - Cy_D[m]\|^2] = \mathbb{E}[x[m]y_D[m]^H] \mathbb{E}[y_D[m]y_D[m]^H]^{-1}
\]

\[
= \left( R_DH_DB_l^{(j)}H_l \right)^H \left( R_DH_DB_l^{(j)}H_l \right)^{-1} + \sigma^2_{n} \left( R_DH_DB_l^{(j)}H_l \right)^{-1}.
\]

On the other hand, suppose that the detector \( C^{(j)} \) is obtained from the \( j \)-th iteration, then, in the \( j+1 \)-st iteration, we compute for \( \{B_l^{(j+1)}\}_{l=1}^L \) as follows. Let us write the Lagrangian function as

\[
L(\{B_l^{(j)}\}_{l=1}^L) = \text{tr} \left\{ [K + C^{(j)}] R_DB_l^{(j)}H_lH_l^{(j)}R_l^{(j)} + \mu_l R_l^{(j)} \right\} - \text{tr} \left( [K + C^{(j)}] R_DB_l^{(j)}H_lH_l^{(j)} - \sigma^2_{n} \left( R_DB_l^{(j)}H_l^{(j)} \right)^H \right)
\]

\[
- \sum_{l=1}^L \mu_l \left( \text{tr}(P_{R_l} - R_lB_l^{(j)}H_lH_l^{(j)} + \sigma^2_{n}R_l^H) \right)
\]

where \( \mu_l \geq 0 \) (\( l = 1, 2, \ldots, L \)) is the Lagrange multiplier corresponding to the power constraint at relay \( l \). Differentiating (13) with respect to \( B_l^{(j+1)} \), we then have, for \( l = 1, 2, \ldots, L \),

\[
B_l^{(j+1)} = \left( \int_{m=1}^{mT} t_l[m]R_l^{H}C_{l}^{H}R_l^{(j)} + \mu_l R_l^{(j)} \right)^{-1}
\]

\[
\times \left( h_l^{(j)}H_l^{H} + \sigma^2_{n}R_l^H \right)
\]

\[
- \sum_{m \neq t} h_l^{(j)}H_l^{H}H_l^{H}R_l^{(j)} + \mu_l R_l^{(j)} \right)^{-1}
\]

5. PERFORMANCE COMPARISONS

In this section, we compare through numerical simulations the RR-MUR scheme with direct transmission and Q-selection. Specifically, in the Q-selection scheme, each relay performs an MMSE estimate of the source symbols and then selects \( Q \) out of the \( K \) estimates to be forwarded to the destination. The optimal selection is performed here where the \( Q \) symbols selected at the relays are determined jointly through exhaustive search (over \( \binom{K}{Q} \) choices) to minimize the MSE at the destination.
In all cases, the total transmit power at all sources will equal the total transmit power at all relays, and the transmit power are evenly distributed to all sources and relays, i.e. we have $P_{S_k} = P/K, \forall k$, and $P_{R_l} = P/L, \forall l$. For direct transmission, we set $P_{S_k} = 2P/K$. The channel coefficients of all source-relay links and all relay-destination links are i.i.d. with distribution $CN(0, 1)$. We assume that the relay nodes are located in the middle of the source users and destination. Thus, the channel coefficients of all source-destination links are assumed i.i.d. with distribution $CN(0, 1/2^L)$. The variances of AWGN at all receivers are set as $\sigma_n^2 = 1$. We set the initial values $B_0^{(0)} = \lambda_l 1_{Q \times K}$, where $1_{Q \times K}$ is a $Q \times K$ matrix with all elements equal to 1 and $\lambda_l$ is a constant set to meet the power constraint.

In Figs. 3-5, we compare the MSE of the symbol estimates at the destination for direct transmission (dotted line), $Q$-selection (dashed line) and the proposed RR-MUR scheme (solid line). In Fig. 3, we have $K = 2$ sources and $L = 8$ relays. We observe that, compared to direct transmission, the RR-MUR scheme provides approximately 8-9 dB gain. Moreover, when $Q = 1$, RR-MUR outperforms $Q$-selection by 1 dB. When $Q = 2$, i.e. the case with no dimension reduction, the joint optimization of the precoders and the detector still leads to improved performances. It is interesting to note that in the $Q$-selection scheme, the case of $Q = 1$ is better than $Q = 2$ since, for $Q = 1$, power is allocated to only one symbol at each relay, which is likely the symbol corresponding to the best channels. Instead, for $Q = 2$, power must be divided evenly among the two users’ symbols, which may not be an efficient use of power.

In Fig. 4, the MSE performance of the three schemes are compared in a network with $K = 4$ sources and $L = 6$ relay nodes. It shows that RR-MUR outperforms $Q$-selection by 1.5-3 dB. In fact, the improvement of RR-MUR over $Q$-selection increases as the dimension $Q$ increases.

In Fig. 5, the MSE performance of the three schemes are compared in a network with $K = 4$ sources and $L = 4$ or $L = 6$ relays. Each relay retransmits $Q = 1$ symbol after precoding. When there are only 4 relays, the best $Q$-selection strategy is to have each source be served by a different relay. In this case, the diversity gains obtained due to cooperation are limited and thus the performance of RR-MUR and $Q$-selection are approximately the same. However, as the diversity gain increases, i.e. when $L = 6$, it is observed that the MSE improvement of RR-MUR exceeds that of $Q$-selection.

6. REFERENCES


