ROBUST NLOS MULTIPATH MITIGATION FOR TOA ESTIMATION

Chunpeng Yan*, and H. Howard Fan*
ECE Dept., University of Cincinnati
Cincinnati, OH, 45221

ABSTRACT
A technique that consists of correlation followed by thresholding is widely used to mitigate multipath effects, especially in the presence of non-line-of-sight (NLoS) effects, for TOA estimation. In practice, an accurate threshold that the technique relies on is difficult to obtain, since optimizing the threshold requires prior knowledge of channel statistics and signal/noise power, which are not always known a priori nor are easy to estimate. In this paper, we propose a new thresholding method that does not rely on such prior knowledge and is robust against an inaccurate threshold.

Index Terms—non-line-of-sight, threshold, TOA, multipath

1. INTRODUCTION
TOA or TDOA methods are important means for location estimation, but are often plagued by multipath propagation, especially in non-line-of-sight (NLoS) multipath channels where it is difficult to identify the obstructed LoS path for TOA estimation, since the LoS path is usually not the strongest path. In addition, noise causes spurious peaks that appear to be false paths that obscure a weak LoS path. Based on the fact that these false paths tend to be weaker than the LoS path, existing thresholding techniques remove the false paths using a threshold, which is optimized either on-line or off-line. The off-line method [1] relies on prior knowledge of the channel, such as the distribution of the LoS path gain. The on-line method [2] needs to estimate the signal-to-noise ratio (SNR), which is not a trivial task. Moreover, thresholding methods are not robust against an inaccurate threshold, and if the threshold is inaccurate, the possibility of missed and false detection of the LoS path can be large (cf. Fig. 5).

The central idea of our proposed method is that it is unlikely that two false paths, which are caused by a high level of noise, in two independent TOA estimates have the same time delay; while the time delays of the LoS path are likely to remain the same across different TOA estimates. In other words, the time delays of the false paths and those of the LoS path exhibit different kinds of statistics. Based on this property, we introduce an integer thresholding to optimize which the LoS path can be distinguished from false paths. Our method is not dependent on an accurate threshold and precise knowledge of the channel, and is therefore robust and can be applied to a wide range of signals.

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Richard K. Martin*
ECE Dept., Air Force Institute of Technology
WPAFB, OH, 45433

2. SIGNAL MODEL
We present a signal model for the reception of training sequences (TRS) in a multipath environment. The baseband transmitted signal $s_i(t)$ for the $i$ th TRS is formed by modulating symbols with the symbol waveform $g(t)$,

$$s_i(t) = \sum_{n=-N_i}^{N_i-1} b_{i,n-N_i} g(t-nT)$$

where the TRS $b_{i,n-N_i}$ of length $L$ is assumed to be a pseudo-random noise (PN) code, and $T$ is the symbol period. The $i$ th TRS starts from the $N_i$ th symbol in the data stream, where $N_i$ is unknown. The data stream is formatted into segments of lengths $(N_{i+1} - N_i)T$, with the same length $LT$ TRS at the same position in each segment (Fig. 1). We assume that the time duration $LT$ of one TRS is much smaller than the channel coherence time, and thus the time-varying fading channel gains remain unchanged within $LT$ but may vary over different TRSs. The channel impulse response (CIR) $h_i(t)$ corresponding to the $i$ th TRS is given by

$$h_i(t) = c_{0,i}\delta(t - \tau_{0,i}) + \sum_{k=1}^{K_p} c_{k,i}\delta(t - \tau_{k,i})$$

where the $k$ th path is parameterized by a path delay $\tau_{k,i}$ and a complex path gain $c_{k,i}$, and $K_p$ is the number of multipaths. The parameters $\tau_{0,i}$ and $c_{0,i}$ correspond to the propagation delay and the gain of the LoS path, respectively. The segment of the received baseband signal corresponding to the $i$ th TRS is given by [3]

$$r_i(t) = s_i(t) \otimes h_i(t) + z_i(t) = \sum_{k=0}^{K_p} c_{k,i}s_i(t - \tau_{k,i}) + z_i(t)$$

where the channel noise $z_i(t)$ is zero-mean additive white complex Gaussian with a constant power spectral density $N_0$. The carrier frequency is assumed to be estimated and compensated, so that there is no phase rotation term [3].

![Fig. 1 Data symbol stream with TRSs](image)

3. ALIGNMENT OF CORRELATOR OUTPUTS
We cross-correlate the received signal with a locally generated TRS PN waveform via a channel correlator [3]. The locally generated PN waveform $\chi(t)$ is formed by multiplying the PN code with $g(t)$, i.e., $\chi(t) = \sum_{n=0}^{N-1} b_n g(t - nT)$. Assuming that the PN code has an ideal autocorrelation function, and that the
cross-correlation between the PN code and all other data symbols is zero, the cross-correlation can be written as a convolution:

\[ y_i(t) = r(t) \otimes x^*(-t) = A \sum_{k=0}^{N_i-1} p(t - N_i T + \tau_{0,i}) + n_i(t) \tag{4} \]

where \( A = L \cdot \mathbb{E}[h^2] \) is a constant, the correlation function is \( p(t) = g(t) \otimes g^*(-t) \) ( \( * \) denotes complex conjugate), and the Gaussian stationary noise \( n_i(t) \) is colored with the known autocorrelation \( r_0(\tau) = ANG \mathbb{P}(\tau) \).

We truncate and sample \( y_i(t) \) to obtain a finite length signal of duration \( 2L_{\text{max}} \) ( \( L_{\text{max}} \) is the maximum delay spread and is known a priori), which contains two \( L_{\text{max}} \) portions before and after the strongest path, since the weak LoS path may be located before the strongest path due to NLoS effect. In this way, the truncated correlator outputs \( \{y_i(t)\} \) are guaranteed to contain the LoS path, but they may also include false paths caused by channel noise. The finite length \( \{y_i(t)\} \) for different time segments \( i \) need to be aligned for the rest of the method to work properly. There are three cases to consider.

Case I: transmitter and receiver are stationary. It is seen from (2) and (4) that the TOA of the \( i \) th TRS is \((N_i T + \tau_{0,i})\). Although \( N_i T \) is unknown, in many cases the TRS repetition period \((N_{i+1} - N_i)T\) is known. And the LoS path propagation delay \( \tau_{0,i} \) is unchanged over different TRSs, because transmitter and receiver are stationary. Therefore the \((i+1)\) th correlator output can be aligned with that of the \( i \) th with a time shift of \((N_{i+1} - N_i)T\).

Case II: transmitter or receiver is moving. There is still a mismatch between consecutive correlator outputs after the time-shifting in Case I, since the LoS path propagation delay \( \tau_{0,i} \) changes as transmitter or receiver moves. The mismatch linearly increases or decreases if the motion is in constant speed. We can add a parameter \( \partial \tau_{0,i} / \partial t \) that characterizes the linearity, and jointly estimate \( \tau_{0,i} \) and \( \partial \tau_{0,i} / \partial t \), which can be tracked via a Kalman filter. In each of the iterations, the alignment is done via \( \partial \tau_{0,i} / \partial t \) that is estimated in the previous iteration. Due to space limit, we will not discuss this recursive approach further.

Case III: \((N_{i+1} - N_i)T\) is unknown. The methods in Case I and II can not be applied to this case. But if we assume there is a strong correlation between consecutive correlator outputs, i.e., two consecutive TRSs are within the channel coherence time, which means \((N_{i+1} - N_i)T\) is relatively small, a sequential alignment algorithm [4] can be applied.

In the remainder of the paper, we assume that the alignment of \( y_i(t) \) has been performed, which will be used in Section 5.

4. THEORETICAL OPTIMAL THRESHOLD

In NLoS channels, the TOA estimation approach in [1, 2] is to set a threshold \( \psi_b \) to remove all the weak paths below the threshold, and search over the finite duration \( 2L_{\text{max}} \) of the estimated CIR envelope \( \hat{h}(t - N_i T) \) estimated from \( y_i(t) \) for the first peak position, which is supposed to be the estimated TOA of the \( i \) th TRS.

However, if we set the threshold \( \psi_b \) too high, there is a possibility of the LoS path being removed. This probability of a missed detection of the LoS path can be written as [2]

\[ P_M = \Pr \left\{ y_i(N_i T + \tau_{0,i}) < \psi_b \right\} \tag{5} \]

where \( |y_i(N_i T + \tau_{0,i})| \) is the peak strength of the correlator output corresponding to the LoS path. On the other hand, if the threshold \( \psi_b \) is too low, we may fail to remove a false path positioned before the LoS path. This will result in a false detection. As shown in Fig. 4 (b) (c), a false path is a false peak caused by a high level of noise, and the probability of a false detection can be approximated by [1]

\[ P_F = \Pr \left\{ \sup_{t \in T} |n(t)| > \psi_b \right\} \tag{6} \]

where \( \Gamma \) (of length \( L_n \)) indicates the noise-only portion (or “noise portion” for simplicity, c.f., Fig. 4) of \( y_i(t) \). Eq. (6) is the probability of the envelop of the stationary process \( n_i(t) \) reaching above the level \( \psi_b \) in a time duration \( L_n \). With a probability of \( P_F \), \( P_M + P_F \), a false detection or a missed detection causes large errors (outliers) of TOA estimates. \( P_F + P_M \) can be viewed as a Bayes risk, by minimizing which we can determine the optimal threshold \( \psi_b \) to minimize the number of outliers.

Calculation of \( P_M \) is not an easy task, since the distribution of \( |y_i(N_i T + \tau_{0,i})| \) relies on both signal/noise power and channel statistics (such as the distribution of the LoS path gain) [1, 2]. In contrast, it is relatively easy to compute \( P_F \). Note that since \( P_F \) is a function of a small time interval \( \Gamma \), it does not simply depend on the PDF of \( n_i(t) \). Rather, it depends heavily on the autocorrelation of \( n_i(t) \) which is known (the channel noise power can be estimated cf. the next paragraph). As shown in Fig. 2, we compute the \( P_F \) by modeling the noise process \( n_1(t) \) as a two-state Markov process, which falls in state 1 when \( n_1(t) > \psi_b \) for some \( \psi_b \), and otherwise is in state 0. This approach is analogous to the finite-state Markov channel model [5], where the continuous time-varying channel is quantized into a set of intervals (states). The difference is that the channel autocorrelation is usually characterized by the Jakes filter.

Fig. 2 Two-state Markov process for channel noise

Next we will determine the transition rates \( \lambda \) and \( \mu \) of the two-state Markov process. We define \( n_i(t) = n_1(t) + j n_2(t) \), where Gaussian processes \( n_1(t) \) and \( n_2(t) \) are uncorrelated with identical autocorrelation functions, i.e., \( r_0(\tau) = r_0(\tau) = (1/2)\sigma_0^2 \). The level-crossing rate of \( n_1(t) \) crossing the level \( \psi_b \) is given by [6]

\[ \text{LCR}(\psi_b) = \sqrt{\frac{\beta}{2\pi} \frac{\psi_b^2 \exp \left( \frac{-\psi_b^2}{2\sigma_0^2} \right)}} \tag{7} \]

where \( \sigma_2^2 = r_0(0) \) denotes the power of \( n_1(t) \) and \( n_2(t) \), and \( \beta = \exp(-r_0(0)) \left( r_0(\tau) = \int \sigma_0^2 \right) \). Note that \( \sigma_0^2 \) can be estimated by finding a level \( \rho \) that maximizes the number of level crossings of \( y_i(t) \) in the first half of \( 2L_{\text{max}} \) (consisting of mostly noise), since \( \arg \max_{\rho} \text{LCR}(\rho) = \sigma_0 \). The average length of the time intervals in which the envelop of \( n_1(t) \) is below a given level \( \psi_b \), i.e. the average period of state 0, \( 1/\lambda \), is given by [6]

\[ \frac{1}{\lambda} = \frac{\text{Pr} \left\{ n_1(t) \leq \psi_b \right\}}{\text{LCR}(\psi_b)} = \sqrt{\frac{2\pi}{\beta}} \frac{\psi_b \exp \left( \frac{-\psi_b^2}{2\sigma_0^2} \right)}{2\sigma_0^2} \tag{8} \]
Similarly, the average period of state 1 is given by 
\[ \frac{1}{\mu} \Pr \{ |r(t)| > \psi_{th} \} / \text{LCR} \psi_{th} \cdot \exp(-t) \]

The noise portion length \( L_n \) is assumed to be a fixed value since aligning \( y_i(t) \) makes \( L_n \) of different \( y_i(t) \) equal. Noticing (6) is the same as that of the two-state Markov process visiting state 1 at least once in the time duration \( L_n \), \( P_f \) that is needed in Fig. 5 is derived as (using the properties of the Markov process [5])

\[ P_f = 1 - \frac{\mu}{\lambda + \mu} e^{-X_n} \quad \text{(9)} \]

5. COARSE DETECTION OF LOS

As shown in Fig. 5, \( P_F + P_M \) vs. the threshold \( \psi_{th} \) is a U shaped curve, the bottom of which corresponds to the minimum \( P_F + P_M \). Decreasing \( L_n \), which is equivalent to making a coarse estimation of LOS TOA, not only reduces the minimum \( P_F + P_M \) but also makes a wider range for an acceptable threshold. Intuitively, directly summing the envelopes of aligned correlator outputs, i.e. \( \{ |y_i(t)| \} \), will increase SNR and therefore will have a similar effect. However, the peaks in the noise portion of the sum of \( \{ |y_i(t)| \} \) can be large due to increased variance by summation.

In our method we will reduce this variance by quantizing \( \{ |y_i(t)| \} \) to the two values of the Markov process and perform a coarse detection of the LOS path, under the condition that only the noise power is estimated and known. Because of this condition, \( P_F + P_M \) is unknown and thus the optimal threshold cannot be computed. However, the transition rates \( \lambda \) and \( \mu \) of the two-state Markov process are controllable by \( \psi_{th} \). We choose a threshold \( \psi_{th} \) that satisfies 
\[ \psi_{th} = \sqrt{2 \log(2) \sigma_i^2} = 1.18 \sigma_i \]

which makes \( \lambda / \mu < 1 \) thereby \( P_F \) small (cf. the factors of \( \lambda / \mu \) in (12)). Threshold \( \psi_{th} \) also needs to be small enough, so that the LoS path is unlikely to miss, i.e., \( P_M \) is small. In our method some rough knowledge, such as in the worst case scenario \( P_M \) is 10% when \( \psi_{th} \) is less than \( 4 \sigma_i \) (computed from (5)), i.e., \( P_M < 10 \% \) for \( \psi_{th} < 4 \sigma_i \), is sufficient for the rest of the method to work. This kind of rough knowledge of a tolerable \( P_M \) is usually available in practice. The choice of this threshold is rather subjective and a wide range of the threshold can be accepted.

Using this rough threshold \( \psi_{th} \), each correlator output \( y_i(t) \), which contains both the noise portion and the signal portion (the region containing signals, the leading edge of which corresponds to the LoS path. cf. Fig. 4(d)), is quantized into 2 states to generate a two-state on-off process \( v_i(t) \), i.e. \( v_i(t) = 0 \) when the envelop of \( y_i(t) \) is below the threshold, i.e. \( |y_i(t)| \leq \psi_{th} \) for some \( t \), and otherwise \( v_i(t) = 1 \) for some other \( t \). The quantization of \( y_i(t) \) into \( v_i(t) \) effectively reduces the variance of the noise portion, therefore \( v_i(t) \) is more robust than \( y_i(t) \) even for a very rough estimate of \( P_M \). Thus, rather than \( y_i(t) \), we sum \( v_i(t) \) to obtain a discrete valued process \( v(t) \):

\[ v(t) = \sum_{i=1}^{M} v_i(t) \quad \text{(10)} \]

where \( \{ v_i(t) \} \) are aligned and thus the noise portion of \( v(t) \) contains only noise. In the viewpoint of packet multiplexing [5], the noise portion of \( v(t) \) is the sum of \( M \) identical two-state on-off mini-sources, and thus it can be modeled by an \( (M+1) \)-state Markov chain with dependent state transition rates (cf. Fig. 3).

The noise portion and the signal portion of \( v(t) \) show different statistical characteristics (cf. Fig. 4(d)). In the noise portion, \( v(t) \) tends to stay in states of small numbers, since the false paths (corresponding to state 1 of \( v(t) \)) over different \( v(t) \) are unlikely to locate at the same position and therefore their sum is small. By contrast, in the signal portion, with a high probability, \( v(t) \) stays in states of large numbers, e.g. state \( M \). Therefore, we define an integer threshold \( m_{th} \) (0 \( \leq m_{th} \leq M-1 \)) for \( v(t) \), by minimizing a new Bayes risk defined below, and the first transition time from state \( m_{th} \) to \( (m_{th} + 1) \) is a coarse detection of the LOS path arrival time.

Similar to the analysis in Sec. 4, we define new probabilities of false and missed detection for the discrete \( v(t) \). The probability of detecting the LOS in each \( v_i(t) \), corresponding to state 1, is \( (1 - P_F) \), and fails with probability \( P_M \). A missed detection of \( v(t) \) occurs when the number of events of detecting the LoS path for each \( v_i(t) \) is less than \( (m_{th} + 1) \). Therefore the number of such events (the sum of state 1 of \( v_i(t) \)) is binomial distributed, and the probability of missed detection by \( v(t) \) is given by

\[ P_M' = \sum_{k=0}^{M} \left( \begin{array}{c} M \cr k \end{array} \right) (1 - P_F)^k P_F^{M-k} \quad \text{(11)} \]

A false detection happens, when the first transition from state \( m_{th} \) to \( (m_{th} + 1) \) occurs in the noise portion. This is a first-passage-time problem for finite-state Markov chains. The closed-form expression for this kind of problem is not always available and further discussion is beyond the scope of this paper. Instead, for the case that \( M \) is moderately large, we approximate the false detection probability as (this derivation is omitted here due to space limitation.)

\[ P_F' \approx \sum_{k=m_{th}+1}^{M} \left( \begin{array}{c} M \cr k \end{array} \right) \left( \frac{\lambda}{\mu} \right)^k \left( 1 + \frac{\lambda}{\mu} \right)^{-M} \quad \text{(12)} \]

Similar to the analysis in Sec. 4, we can minimize \( P_F' + P_M' \) to obtain an optimal integer threshold \( m_{th} \). As shown in Fig. 6, the minimal \( P_F' + P_M' \) is much smaller than \( P_F + P_M \) and the threshold \( m_{th} \) can be easily chosen from a small set of integers.

Fig. 3 (M+1)-state Markov process for noise portion of \( v(t) \)

Our coarse detection method is theoretically more tractable than directly summing \( \{ |y_i(t)| \} \) which will involve calculating a continuous threshold that is not trivial without knowledge of signal power and channel. In contrast, determining an integer threshold is much easier as shown above. The coarse detection is more robust and can detect the LoS TOA to within a few symbols in simulation.

6. REFINEMENT OF LOS TOA

If more precise TOA of the LoS path is required, we can refine the above result by using again the correlator output \( y_i(t) \), but only in the vicinity of the coarsely detected LoS time. Since we are now so close to the actual LoS TOA, there is no longer any issue of false or missed detection – the only issue now is accuracy. So thresholding is no longer needed and therefore \( y_i(t) \) can now be used. But \( y_i(t) \) is also subject to inter-multopath interferences caused by the tails of the correlation function \( p(t) \) (cf.(4)). If the pulse \( p(t) \) is relatively narrow compared to path delay differences,
e.g., the waveform $g(t)$ is wideband and the paths are not very dense, the multipath interferences between paths are small. In such a case, the estimated CIR $\hat{h}(t-NT)$ can be approximated by the peaks of $y(t) / (A[p(0)])$, and the closest peak of $\hat{h}(t-NT)$ to the coarsely detected LoS TOA corresponds to the refined TOA. Otherwise, we need to further process $y(t)$ to obtain accurate CIR via e.g. the time-domain approach [1] or the frequency domain approach [7], and obtain a refined LoS TOA by the closest peak to the coarsely estimated LoS TOA.

7. SIMULATION RESULTS

The simulation channel is modeled as a 12-path time-varying Rayleigh fading channel with NLoS propagations, and the channel varies over different TRSs. Due to the NLoS effect, the LoS path is 4 dB below the strongest path, as shown in Fig. 4(a). Each time-varying path gain is generated according to the Jakes model with the Doppler frequency 100 Hz. The signal is the ATSC digital TV signal, in which a training sequence (511 symbol PN code) varies over different TRSs. Due to the NLoS effect, the LoS path is weakened. Rayleigh fading channel with NLoS propagations, and the channel statistics (which are impractical). It is seen that our TOA results, calculated based on the known SNR and the known channel statistics, are close to those of [1] although the thresholding method [1] with an optimal threshold that is pre-computed, to obtain accurate CIR within the signal bandwidth (6MHz). With the known channel statistics, $P_F + P_M$ is numerically evaluated, as shown in Fig. 5.

Fig. 4  (a) Delays and relative powers of a 12-path channel. (b)(c) Aligned correlator outputs (d) Discrete valued process $x(t)$ for $M=15 ; \psi \equiv 3\sigma_0$ and SNR = 0 dB. The integer threshold $m_0 = 5 , \psi \equiv 3\sigma_0$ is denoted by a horizontal strait line. (x-axes for (a) (b) (c) (d) are in sampling periods, i.e., $T/2$.)

Fig. 5 $P_F + P_M$ vs. threshold $\psi_0$ normalized by $\sigma_0$ for $L_n = \{2, 32, 62, 92, 122, 152\} \times T$ and SNR $= \{-5, 0\}$ dB

Fig. 6 $P_F + P_M$ vs. integer threshold $m_0$ for $M = \{5, 6, 7, 8, 9, 10\}% and $\psi_0 = 3\sigma_0$

Tab. 1 shows the TOA RMSE (root mean square error) results of our method (both coarse and refined TOA) and those of the thresholding method [1] with an optimal threshold that is pre-computed based on the known SNR and the known channel statistics (which are impractical). It is seen that our TOA results, with much less information, are close to those of [1] although the threshold $\psi_0 = 3\sigma_0$ is not optimal. It is also seen at low SNR ( -10dB), the refined TOA is worse than the coarse TOA. This is because there are many false peaks caused by the high level of noise in the vicinity of the LoS, the refinement (Sec. 6) that picks the closest peak to the coarse LoS TOA tends to choose false peaks instead of the peak corresponding to the actual LoS path.

8. REFERENCES


