USING COMPLEX-VALUED ICA TO EFFICIENTLY COMBINE RADAR POLARIMETRIC DATA FOR TARGET DETECTION

Mike Novey, Tülay Adalı

University of Maryland Baltimore County, Baltimore, MD 21250
{mnovey1, adali@umbc.edu}

ABSTRACT

Target detection in sea clutter is a challenging problem in radar detection, specifically, when the Doppler return of the target and clutter are collocated. Polarization diverse radars provide additional information that enhances target detection. In this paper, we use an effective independent component analysis (ICA) approach, adaptive complex maximization of non-Gaussianity (A-CMN) [1], to efficiently combine polarimetric radar data prior to detection. We show that A-CMN estimates the polarimetric scatter coefficients for the single target in clutter case, thereby providing matched-filter performance without the need for clutter or target models. The detection performance using ICA is evaluated with sea clutter collected with the McMaster IPix radar off the coast of Canada [2]. We also demonstrate that using the adaptable nonlinearity method in [1], allows the algorithm to adapt to various sea clutter conditions using simulation results.

Index Terms— Nonlinear estimation, radar detection, ICA

1. INTRODUCTION

For target detection in sea clutter, several methods have been described in the literature. Non-Gaussian models are used to formulate detection algorithms with polarimetric channels in [3] and Bayesian detector algorithms in [4]. In both cases, performance is tied to the validity of the target and clutter models. Data driven approaches, such as neural network implementations [5, 6] require a great number of training samples and are computationally intensive. The approach in [7] treats the spectrum as a probability density function (pdf) and forms a detection based on this approach. This exploits the line component nature of the target spectrum but does not utilize polarization diversity.

Independent component analysis (ICA) for separating complex-valued signals has found utility in many applications such as wireless communications [8], radar beamforming [9], data analysis in magnetic resonance imaging [10], and electroencephalograph [11]. It is also shown that adapting the nonlinearity to the source pdf improves performance [1]—we use this feature in A-CMN to adapt to the current sea state conditions. However to the best of our knowledge, application of ICA to radar target detection in sea clutter has not been demonstrated.

As pointed out in [12], the spectrum of sea clutter is relatively smooth, whereas the spectrum from a target return is a line component or in other words a peakier spectrum. This phenomena is exploited in ICA where the estimated independent components are non-Gaussian as possible, i.e., increasing non-Gaussianity results in a boosting of the line component. As we will show, this boosting results in an increase in signal to noise ratio (SNR) of the target providing increased probability of detection.

In this study, we show that using complex-valued ICA as a pre-processing step on radar polarimetric data improves detection performance. We compare the performance of using ICA as a preprocessing step versus a matched filter with the IPix radar data from McMaster University [2]. We also demonstrate that using the adaptable nonlinearity method in [1], allows the algorithm to adapt to various sea clutter conditions.

2. COMPLEX ICA

2.1. Complex preliminaries

A complex variable $z$ is defined in terms of two real variables $z^R$ and $z^I$, as $z = z^R + jz^I$. Statistics of a complex random vector $x = x^R + jx^I$ are defined by the joint pdf $p(x^R, x^I)$. The kurtosis of a zero mean complex random variable, often used as a quantitative measure of non-Gaussianity, is defined in [13] as $\text{kurt}(y) = E(|y|^4) - 2 (E(|y|^2))^2 - |E(y^2)|^2$ and reduces to

$$\text{kurt}(y) = E(|y|^4) - 2$$

where $y$ is circular with unit variance. Complex Gaussian random variables have kurtosis values of zero and kurtosis is nonzero for most non-Gaussian random variables [14]. Random variables that have a negative kurtosis are called sub-Gaussian and those with positive kurtosis are super-Gaussian. As stated earlier, peakier distributions such as the target spectrum, are super-Gaussian in nature.

2.2. ICA in the complex domain

In ICA, the observed data $x$ are typically expressed as a linear combination of latent variables such that

$$x = As$$

where $s = [s_1, \ldots, s_M]^T$ is the column vector of latent sources, $x = [x_1, \ldots, x_M]^T$ is the column vector of observed mixtures, and matrix $A$ is the $M \times M$ mixing matrix assumed invertible. We assume that the sources and mixing matrix are complex valued. ICA then identifies the statistically independent sources given the observed mixtures typically by estimating a demixing matrix $W$ so that the source estimates become $Wx$.

Due to the various differences of the background sea clutter distribution, we choose to use the A-CMN algorithm introduced in [1]. The cost function is

$$J(w) = E \left\{ |w^H x |^{2p} \right\} \quad \text{s.t.} \quad ||w|| = 1$$

where $p$ is the estimated shape parameter and $w \in \mathbb{C}^N$. The data model used in A-CMN assumes the signal’s modulus is described
by the generalized Gaussian distribution with shape parameter $p$. A shape parameter of one specifies a Gaussian distribution while a value less than one corresponds to super-Gaussian and a value greater than one is sub-Gaussian. The goal of A-CMN is thus to match the nonlinearity to the source distribution providing a more accurate estimate of non-Gaussianity. As in most ICA algorithms, A-CMN first prewhitens the data. The algorithm finds the demixing vector $w$ that maximizes the cost, or as shown in [1], the non-Gaussianity of the source estimates.

### 3. POLARIMETRIC RADAR MODEL

The polarimetric radar data used in this report was collected with the McMaster University IPIX radar in Dartmouth, Nova Scotia [2]. Details of the data are given in Table 1 where we use the designation $F_{19}$ and $F_{30}$ throughout this paper to specify the files we used.

We consider only two polarizations in this study, namely horizontally transmitted and received (HH) and horizontally transmitted and vertically received (HV). The complex-valued signal voltage at discrete time index $n = 1, \ldots, N$ becomes

$$y(n) = A s(n) + e(n) \quad (4)$$

where $y = [y_{hh}, y_{hv}]^T$ is the received polarization data corresponding to HH and HV polarizations, $A$ is the $2 \times 2$ complex-valued scattering matrix

$$A = \begin{bmatrix} a_{hh} & a_{hv} \\ a_{hv} & a_{hv} \end{bmatrix}$$

where the variables $a_{hh}$ and $a_{hv}$ represent the target and clutter backscatter amplitude and phase with HH polarization respectively, $a_{hh}$ and $a_{hv}$ are with HV polarization, $s = [s^h, s^v]^T$ is the $2 \times N$ source matrix consisting of the target and clutter baseband signal, and $e$ is thermal noise. Note that we are assuming that $s^h$ are the time samples of one target and $s^v$ represents the large collection of point scatters representing clutter [4] and the time index $n$ is ignored to simplify the notation.

### 4. ICA DETECTION ENHANCEMENT

As shown in [7, 12] the spectrum of sea clutter is relatively smooth as compared to the line component return from a target. Because the target spectrum is peaker, it has a distribution with a higher kurtosis value than clutter only and hence is more super-Gaussian. This is a natural application of ICA where the algorithm finds the demixing matrix $W$ to make the source estimates non-Gaussian, or in the case of a target, as super-Gaussian as possible. Therefore the spectrum of $y$ is used in the ICA algorithm by first transforming the data into the frequency domain using a discrete Fourier transform by

$$x = y F = A s F + e F$$

where $F$ is the $N \times N$ matrix of discrete Fourier transform coefficients, $N$ is the block size, and $x$ are the mixtures used in ICA. Note that the Fourier transform does not alter the scatter matrix $A$.

The result of ICA is then two source estimates, one with the target and one with clutter—we choose the source estimate with the highest super-Gaussianity measure to represent the target as seen in Figure 1. As we will show in the simulations, ICA increases the target’s SNR enhancing the detector’s performance, and surprisingly, closely matches a matched-filter response. This result, outlined in the appendix, shows that maximizing the cost given in (3) leads to a matched filter response assuming a single target in clutter. Note that if no target is present, the result may be a peakier clutter estimate which may increase the false alarm rate (FAR). This was taken into account in the simulations by adjusting the thresholds to keep the same false alarm rate among the simulations. Also if several targets are present performance will degrade, however we did not quantify this effect.

Figure 2 depicts how ICA increases the target SNR using a block size of 512. The data on the left is the amplitude versus Doppler filter of the received HH and HV data with a synthetic test target added to filter 245. The synthetic target was added to range gate four, a range gate with sea clutter only. The right side of the figure depicts the estimated sources using ICA. The top source shows the test target with an increased SNR while the second source estimate shows no target. The top source would be passed on to the detector since it has the highest super-Gaussianity measure.

### 5. SIMULATIONS

Our goal in this section is to demonstrate the utility of using ICA as a preprocessing step for target detection, specifically, when the target occupies the same Doppler filter as the sea clutter. Our simulation uses the IPIX data outlined in Table 1 where we use a range gate that contains only sea clutter to inject the target. The synthetic target is placed in a Doppler filter centered in the sea clutter with an SNR based on the local mean noise level of the 16 surrounding filters. The scatter matrix is then randomly generated for the two polarizations.

The detector used in the simulations finds a threshold based on the squared magnitudes of the 16 Doppler filters above and below the target filter, more formally, the threshold $T = \max(L, R) k_{FAH}$, where $L$ is the mean of the 16 filters below the target, $R$ is the mean of the 16 filters above the target, and $k_{FAH}$ is the threshold multiplier for a given false alarm rate. Although the detector is not sophisticated, it has found use in numerous radar applications and it provides a means for quantifying our preprocessing stage. For each simulation run, we calculate the probability of detection (PD) while increasing the simulated target SNR. This is done with block sizes of 512 and 1024 using files $F_{19}$ and $F_{30}$. We run the detector on four channels: i) the Fourier transformed received data $x_{hh}$ and $x_{hv}$, ii) the output of ICA, $s^h$ and $s^v$, and iii) a matched filter designated as $x_m$.

A matched filter is used as a benchmark since it maximizes the SNR [15] at the output of a linear filter and is optimal with Gaussian noise. The $2 \times 1$ matched filter coefficients are $m = C^{-1} r$, where $C = E\{e e^H\}$ is the covariance of the zero-mean noise between polarization channels and $r = [a_{hh}, a_{hv}]^T$ is the steering vector. The covariance is estimated on each block of data while the steering vec-

<table>
<thead>
<tr>
<th>File number</th>
<th>Wave height</th>
<th>Range resolution</th>
<th>Frequency</th>
<th>PRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{19}$</td>
<td>2 m</td>
<td>30 m</td>
<td>9.39 GHz</td>
<td>1 kHz</td>
</tr>
<tr>
<td>$F_{30}$</td>
<td>0.9 m</td>
<td>30 m</td>
<td>9.39 GHz</td>
<td>1 kHz</td>
</tr>
</tbody>
</table>

Table 1. IPIX radar parameters
tor is known a priori for the synthetic target. The resulting matched filter channel is thus \( x_m = m^Hx \). Note that the steering vector is unknown for real targets. Prior to each simulation run, the threshold multiplier, \( k_{\text{FAR}} \), is determined to keep the false alarm rate at \( 10^{-2} \), where a false alarm is defined as at least one detection per block.

Figures 3 and 4 depict the PD results with a block size of 512 for files \( F_{19} \) and \( F_{30} \) for the four channels: ICA, matched filter, HH, and HV data. What we glean from the figures is that ICA performs about 1 dB better than no preprocessing, i.e., using \( x_{19} \) and \( x_{30} \), and only 0.25 dB less than the matched filter. These values were calculated at PD = 0.5. File \( F_{30} \) shows ICA performs approximately 0.5 dB less than the matched filter but still 0.5 dB better than no preprocessing. Figures 5 and 6 show similar results when the block size is 1024, however with file \( F_{30} \), ICA performs almost as good as the matched filter. This result is surprising in that ICA was able learn the steering vector to be able to perform as well as the matched filter, we show how ICA estimates the matched filter in the appendix.

Figure 7 illustrates the importance of using the adaptive non-linearity intrinsic to the A-CMN algorithm. The figure shows the shape parameter estimate, \( p \), for the two sources with and without a target using \( F_{19} \). Figure 7(a) clearly indicates the nonstationarity of sea clutter with the shape parameter oscillating between sub and super-Gaussian indications. However with a target injected, the shape parameter becomes \( \sim 0.3 \) indicating a super-Gaussian source as expected. Figure 8 depicts the shape parameter using \( F_{30} \). Here we see that the sea clutter is more super-Gaussian with \( p < 1 \). Again with the target present the shape parameter becomes \( \sim 0.3 \). This evidence along with the PD curve results suggests that file \( F_{30} \) has a higher state than file \( F_{19} \). One can conclude that adaptability is important when used in an application such as this due to the variable nature of sea environment.

6. CONCLUSIONS

In this paper we present a method for using complex-valued ICA as a preprocessing step in target detection in sea clutter. Our method uses ICA to combine polarimetric data to increase the targets’ SNR prior to detection. We show, using a synthetic target embedded in real sea clutter, that the gain using this method approaches that of a matched filter. We did not compare our results with the Bayesian receiver in [4] due to the disparate block sizes. The Bayesian approach uses block sizes of 64 to 128, whereas ICA requires greater then 256 to operate effectively. However, with efficient Bayesian approaches, the ICA step can be used to improve the detection performance of the Bayesian receiver, as it can other detection techniques.

We also show that the adaptable ICA algorithm, A-CMN presented in [1], is able to modify the nonlinearity to match the distribution of the changing sea clutter conditions.

A. APPENDIX

We show that maximization of the ICA cost function (3) with the constraint \( w^Hw = 1 \), results in the matched filter coefficients with the simplifying assumption that a target resides in one Doppler filter and shape parameter \( p = 2 \). Our data vector \( x \) is \( M \times N \), where \( M \) is the number of polarization channels and \( N \) is the number of Doppler filter observations. Doppler filters, \( x(1), \ldots, x(N) \), contain whitened noise, due to ICA preprocessing. We assume a target is added to one filter with steering vector \( r = Qa \), where \( a \) is the target’s true steering vector and \( Q = C^{-1/2} \) is the whitening transform. We replace the expectation in (3) with the mean ergodic theorem obtaining

\[
J(w) = \sum_{n=1}^{N} |w^Hx(n)|^4 + |w^Hr|^4 + \lambda(w^Hw - 1)
\]

where \( \lambda \) is the Lagrange multiplier. Although \( J(w) \) is real-valued and hence can be written as a function of \( w \) and \( w^* \) allowing us to use Wirtingen calculus [16] to find the derivative as

\[
\frac{\partial J}{\partial w^*} = \sum_{n=1}^{N} 2|w^Hx(n)|^2 \left[ w^*x(n)^* \right] x(n) + 2|w^Hr|^2(w^*r^*)r + \lambda w.
\]
We find the \( w \) that maximizes the cost by setting the derivative to zero and use the expectation of the noise term yielding

\[
\mathbf{w} = \frac{2NE\{\mathbf{w}^H \mathbf{x} (\mathbf{w}^T \mathbf{x}^*) \mathbf{x} \} + 2|\mathbf{w}^H \mathbf{r}|^2 (\mathbf{w}^T \mathbf{r}^*) \mathbf{r}}{ \lambda - 2N}. \tag{5}
\]

The term in the expectation can be rewritten as \( E\{\mathbf{w}^H \mathbf{xx}^H \mathbf{w} (\mathbf{xx}^H \mathbf{w}) \} \). Due to the initial whitening of \( \mathbf{x} \), we use the approximation from [14] allowing us to split the expectation into \( E\{\mathbf{w}^H \mathbf{xx}^H \mathbf{w} (\mathbf{xx}^H \mathbf{w}) \} \approx E\{\mathbf{w}^H \mathbf{xx}^H \mathbf{w}\} E\{\mathbf{xx}^H \mathbf{w}\} \). Noting that \( \mathbf{x} \) is white with zero mean, we can further simplify to \( E\{\mathbf{w}^H \mathbf{xx}^H \mathbf{w}\} E\{\mathbf{xx}^H \mathbf{w}\} = \mathbf{w} \).

Using the constraint \( \mathbf{w}^H \mathbf{w} = 1 \), we pre-multiply both sides of (5) by \( \mathbf{w}^H \) and solve for \( \lambda \) obtaining

\[
-\lambda = 2N\mathbf{w}^H \mathbf{w} + 2|\mathbf{w}^H \mathbf{r}|^2 (\mathbf{w}^T \mathbf{r}^*) \mathbf{w}^H \mathbf{r}. \tag{6}
\]

Substituting (6) into (5) and solving for \( \mathbf{w} \) results in

\[
\mathbf{w} = \frac{2N\mathbf{w} + 2|\mathbf{w}^H \mathbf{r}|^2 (\mathbf{w}^T \mathbf{r}^*) \mathbf{r}}{2N + 2|\mathbf{w}^H \mathbf{r}|^2 (\mathbf{w}^T \mathbf{r}^*) \mathbf{w}^H \mathbf{r}}.
\]

After some algebraic manipulations we obtain our final result \( \mathbf{w} = \frac{\mathbf{r}}{\mathbf{w}^H \mathbf{r}} \), where \( \mathbf{r} \) is the steering vector after whitening and the denominator scales \( \mathbf{w} \) to unit norm.

### B. REFERENCES


