ALTERNATIVES TO SPHERICAL MICROPHONE ARRAYS: HYBRID GEOMETRIES

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ABSTRACT

We present a novel theory and design for constructing microphone arrays to extract spherical harmonic components from soundfields. The proposed non-spherical array structure provides a flexible and alternative design to the traditional spherical microphone arrays with lesser restriction on sensor locations. We use the properties of the associated Legendre functions and the spherical Bessel functions to develop a systematic approach to place circular microphone arrays in three dimensions for hybrid array geometries. As an illustration, we design and simulate a fifth order spherical harmonic decomposition array using 70 microphones to operate over a frequency band of an octave.

Index Terms— Soundfield, microphone arrays, spherical microphone array, spherical harmonics, hybrid array

1. INTRODUCTION

Decomposition of three dimensional (3D) soundfields into spherical harmonics is a fundamental problem in acoustic signal processing. Whilst spherical microphone arrays [1–4] have been shown to be a natural choice for spherical harmonic decomposition, there are a number of limitations and constraints, which restrict their usefulness. Specifically, the sensor positions of spherical arrays need to meet a strict orthorollnormality condition resulting in a limited flexibility of array geometry. They also suffer from numerical ill conditioning at some frequencies. In this paper, we develop systematic theory to design alternative 3D structures consisting of circular arrays to decompose a given acoustic field into spherical harmonic components.

Meyer and Elko [5] proposed a method to use circular arrays of microphones on the x-y plane together with a centre microphone at the origin to extract spherical harmonic coefficients. Although, Meyer’s work gives some flexibility in controlling the vertical spatial response, fundamentally a 2D array on a x-y plane is not able to determine all of the spherical harmonic coefficients. We have extended [5] in [6, 7], where a number of circular arrays parallel to the x-y plane together with microphones on the z-axis are used to design a higher order (up to 5th) spherical harmonic decomposition array. In this paper, we show that by adding/ subtracting soundfield on two circles, which are placed equal distance above and below the x-y plane, we can eliminate odd / even spherical harmonics, respectively. We then exploit this property together with characteristics of the associated Legendre and the spherical Bessel functions to provide guidelines to design flexible harmonic extraction arrays.

1 Odd and even spherical harmonics are defined as when the sum of order and degree is odd and even, respectively.

2. SPHERICAL HARMONIC ANALYSIS

2.1. Harmonic Expansion

An arbitrary soundfield at a point \((r, \theta, \phi)\) within a source free region can be written as [6]

\[
S(r, \theta, \phi; k) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_{nm}(k)j_n(kr)P_{n|m}(\cos \theta)E_m(\phi)
\]

(1)

where \(E_m(\phi) \triangleq (1/\sqrt{2\pi})e^{im\phi}\), the normalized associated Legendre functions

\[
P_{n|m}(\cos \theta) \triangleq \sqrt{\frac{2n+1}{2} \frac{(n-|m|)!}{(n+|m|)!}} P_{n|m}(\cos \theta),
\]

(2)

and \(\alpha_{nm}(k)\) are the spherical harmonic coefficients of the soundfield. Using the orthonormality properties of the exponential functions and the normalized associated Legendre functions we can express

\[
\alpha_{nm}(k)j_n(kr) = \int_0^{2\pi} \int_0^\pi S(r, \theta, \phi; k) \times P_{n|m}(\cos \theta)E_{-m}(\phi) \sin \theta d\theta d\phi. \tag{3}
\]

Knowing the soundfield over angles on a radius \(r\), harmonic coefficients can be calculated using (3) provided \(j_n(kr) \neq 0\). The spherical microphone arrays are designed based on (3).

The representation (1) has an infinite number of terms. However, this series can be truncated \([8]\) to a finite number \(N = \lceil ekR/2 \rceil\) where \(R\) is the maximum dimension of the region.

3. SAMPLING SPACE BY CIRCLES

3.1. Circular harmonic decomposition

Let \(S(r_\|, \theta_\|, \phi_\|; k)\) be the the soundfield on a circle given by \((r_\|, \theta_\|)\). For the soundfield on this circle, we multiply (1) by \(E_{-m}(\phi)\) and integrate with respect to \(\phi\) over \([0, \pi]\) to obtain

\[
a_m(r_\|, \theta_\|; k) = \sum_{n=|m|}^{\infty} \alpha_{nm}(k)j_n(kr_\|)P_{n|m}(\cos \theta_\|) \tag{4}
\]

where

\[
a_m(r_\|, \theta_\|; k) \triangleq \frac{1}{2\pi} \int_0^{2\pi} S(r_\|, \theta_\|, \phi; k)E_{-m}(\phi) d\phi. \tag{5}
\]

We termed \(a_m(r_\|, \theta_\|; k)\) as the circular harmonics of a given field on a circle at \((r_\|, \theta_\|)\).
3.2. Sampling of circles

To evaluate the integral in (5) with a summation for practical purposes, we use the sampling theorem. For a radius $r_q$, the field is limited to $N_q = \lceil \text{ker}_q / 2 \rceil$ orders due to natural truncation. Hence, the maximum mode $m$ involved is $N_q$. Thus, $S(r_q, \theta_q, \phi; k)$ is mode limited to $N_q$, i.e., it contains terms with $e^{in\theta}$ with $m = 0, \ldots, N_q$.

According to Shannon’s sampling theorem, $S(r_q, \theta_q, \phi; k)$ can be reconstructed by its samples over $[0, 2\pi]$ with at least $(2N_q + 1)$ samples. Hence, we approximate (5) as

$$a_m(r_q, \theta_q, k) \approx \frac{2\pi}{V_q} \sum_{q=1}^{V_q} S(r_q, \theta_q, \phi; k) e^{-m(\phi)},$$  

where $V_q \geq (2N_q + 1)$ are the number of sampling points on the circle $(r_q, \theta_q)$.

3.3. Spherical harmonics decomposition: Least squares

Suppose our goal is to design a $N$th order microphone array to estimate $(N + 1)^2$ spherical harmonic coefficients. By placing $Q \geq (N + 1)$ circles of microphones on planes given by $(r_q, \theta_q)$, $q = 1, \ldots, Q$, for a specific $m$, we have

$$J_m a_m = a_m, \quad \text{for } m = -N, \ldots, N$$

where $a_m = [a_m(\alpha_1; \theta_1), a_m(\alpha_2; \theta_2), \ldots, a_m(\alpha_N; \theta_N)]^T$. The harmonic coefficients $a_m$ can be calculated by solving the linear equations (7) for each $m$. If $J_m$ has a valid Moore-Penrose inverse $J_m^+$, then $a_m$ can be calculated for each $m$ in the least squares sense as

$$a_m = J_m^+ a_m.$$

However, if we choose $(r_q, \theta_q)$ arbitrary, then there could be a number of singularities in (12). In our recent work [6, 7], we have given guidelines on how to avoid singularities. In this paper, we further advance our theory to give a systematic approach to design non-spherical arrays to estimate spherical harmonic coefficients.

4. CIRCULAR HARMONIC COMBINATION

Consider two circles placed at $(r_q, \theta_q)$ and $(r_q, \pi - \theta_q)$ where $0 \leq \theta_q \leq \pi / 2$. That is one circle above the x-y plane and the second circle below the x-y plane but equal distance $r_q$ from the origin. The circular harmonics of the soundfield on the circle on or above the x-y plane is given by (4) and the corresponding equation for the circle below the x-y plane is

$$a_m(r_q, \pi - \theta_q; k) = \sum_{n=-|m|}^{\infty} \alpha_{nm}(k) j_n(kr_q) P_n(\cos(\pi - \theta_q)).$$

Since $\cos(\pi - \theta) = -\cos \theta$ and $P_n(\cos \theta) = (-1)^n P_n(\cos \theta)$, we write

$$a_m(r_q, \pi - \theta_q; k) = \sum_{n=-|m|}^{\infty} (-1)^{n+|m|} \alpha_{nm}(k) j_n(kr_q) P_n(\cos \theta_q).$$

5. ARRAY OF CIRCULAR ARRAYS

There are number of different ways to construct an array of microphones consisting of pairs of circles to extract spherical harmonic coefficients from a 3D soundfield.

5.1. Calculating odd coefficients

In this section, we show how to extract $\alpha_{nm}(k)$ when $n + |m|$ is odd. Suppose, we have selected $Q$ pairs of $(r_q, \theta_q)$ such that

\[\text{odd, even, odd, even}, \ldots \]
$\mathcal{P}_{N|m}(|\cos \theta_q|) \neq 0$ when $n + |m|$ is odd for the required combinations of $n$ and $m$. Now we evaluate (10) for a given $m$ for $q = 1, \ldots, Q$ to write

$$J_m \alpha^*_m = b^*_m, \text{ for } m = -N, \ldots, N$$

where $\alpha^*_m = [\alpha_{(|m|+1)m}, \alpha_{(|m|+3)m}, \ldots, \alpha_{Nm}]^T$,

$$J^e_m = 2 \begin{bmatrix} d_1(|m|+1, m) & d_1(|m|+3, m) & \cdots & d_1(N, m) \\ \vdots & \vdots & \ddots & \vdots \\ d_Q(|m|+1, m) & d_Q(|m|+3, m) & \cdots & d_Q(N, m) \end{bmatrix}$$

with $d_q(n, m) = j_n(k_r)\mathcal{P}_{N|m}(|\cos \theta_q|)$, and

$$b^e_m = \begin{bmatrix} b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \\ b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \end{bmatrix}^T$$

if $m$ is even, and

$$b^e_m = \begin{bmatrix} b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \\ b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \end{bmatrix}^T$$

if $m$ is odd.

The odd harmonic coefficients $\alpha^*_m$ can be estimated by solving (11) using the least squares as $\alpha^*_m = J^e_m b^*_m$, where $J^e_m$ is the Moore-Penrose inverse of $J_m$. This solution exists only if $J^e_m$ is non-singular. We have the following guidelines to choose $(r_q, \theta_q)$ systematically such that $J^e_m$ is always non-singular:

1. Recently, we have shown that there are specific patterns of the normalized associated Legendre function when $n - |m| = 1, 3, 5, \ldots$ as depicted in Figures 1 and 2. There are many different radius combinations we can choose for $\theta_q$. Not that $\theta_q$ could be same for all $q$ or a group of values.

2. For a $N$th order system, there are $N(N+1)/2$ odd spherical harmonic coefficients from total of $(N+1)^2$ coefficients. We use $N$ (for $N$ odd) or $N - 1$ (for $N$ even) pairs of circular microphone arrays. We choose the radii of these circles as

$$r_q = 2 \frac{k_o}{k_o} \frac{N}{k_o} \frac{N}{k_o} \frac{N}{k_o} \ldots \frac{N}{k_o}$$

where $k_o$ is a carefully chosen frequency within the desired frequency band (octave), i.e., $k \in [k, 2k]$.

3. With this choice, the soundfield at frequency $k$ on a circle with $r_q$ is order limited to $N_q(k) = 2q \exp(1)k/k_o$ due to the properties of Bessel functions. This property limits the higher order components of the soundfield present at a particular radius $r_q$. Also, the lower order components are guaranteed to be present due to the choice of radii in (14) which avoids the Bessel zeros.

4. Thus, selecting $r_q$ according to (14) and $\theta_q$ from Figures 1 and 2, we can guarantee that $J^e_m$ is non-singular.

5.2. Calculating even coefficients

Suppose we have selected $Q$ pairs of $(r_q, \theta_q)$ such that $\mathcal{P}_{N|m}(|\cos \theta_q|) \neq 0$ when $n + |m|$ is even for the required combinations of $n$ and $m$. We evaluate (10) for a given $m$ for $q = 1, \ldots, Q$ to write

$$J_m \alpha^*_m = b^*_m, \text{ for } m = -N, \ldots, N$$

where $\alpha^*_m = [\alpha_{(|m|+1)m}, \alpha_{(|m|+3)m}, \ldots, \alpha_{Nm}]^T$,

$$J^e_m = 2 \begin{bmatrix} d_1(|m|+1, m) & d_1(|m|+3, m) & \cdots & d_1(N, m) \\ \vdots & \vdots & \ddots & \vdots \\ d_Q(|m|+1, m) & d_Q(|m|+3, m) & \cdots & d_Q(N, m) \end{bmatrix}$$

with $d_q(n, m) = j_n(k_r)\mathcal{P}_{N|m}(|\cos \theta_q|)$, and

$$b^e_m = \begin{bmatrix} b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \\ b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \end{bmatrix}^T$$

if $m$ is even, and

$$b^e_m = \begin{bmatrix} b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \\ b^0_n(r_1, \theta_1; k), \ldots, b^0_n(r_Q, \theta_Q; k) \end{bmatrix}^T$$

if $m$ is odd.

The even harmonic coefficients $\alpha^*_m$ can be estimated by solving (15) using the least squares as $\alpha^*_m = J^e_m b^*_m$, where $J^e_m$ is the Moore-Penrose inverse of $J^e_m$. As for the case of odd harmonics, the solution exists only if $J^e_m$ is non-singular. We have the following guidelines to choose $(r_q, \theta_q)$ systematically such that $J^e_m$ is always non-singular:

1. As in the case of odd coefficients, we can choose range of values for $\theta_q$ from Fig. 3, which plots $\mathcal{P}_{N|m}(|\cos \theta|)$ for $n + |m|$ even.

2. Note that on the $x$-$y$ plane ($\theta = \pi/2$), all even associate Legendre functions are non zero. Thus, placing circles on the $x$-$y$ plane seems to be an obvious choice to estimate even coefficients, where we do not need pairs of circles.

3. However, we may still choose circles on other planes.

4. Depending on our choice, we can design different array configurations, which will be capable of estimating spherical harmonic coefficients.

5. For a $N$th order system, we place $N/2 (N$ even) or $(N+1)/2 (N$ odd) circles on the $x$-$y$ plane. We choose the radii of these circles as in the case off odd coefficients (see (14)).

5.3. Broadband performance

The spherical harmonic decomposition method proposed in this paper is reliant on constructing matrices $J^e_m$ and $J^e_m$ by appropriately placing circular arrays. We have chosen $\theta_q$ and $r_q$ such that these matrices are non-singular. However, they are dependent on the operating frequency $k$ and the design parameter $k_o$. It can be shown that (our simulation support this claim) by choosing $k_o = k_o \exp(1)/2$ where $k_o$ is the lower end of the design band, the array can work over an octave of $[k_o, 2k_o]$. 

![Fig. 2: Magnitude of the normalized associate Legendre functions $\mathcal{P}_{N|m}(|\cos \theta|)$ in dB when $n + |m| = 3$.](image-url)
6. SIMULATIONS

To illustrate the new design guidelines, we simulate a 5th order system. The guideline provides a number of different array configurations. We only show one such configuration here. According to Section 5.1, we first place four circular arrays (two pairs) with 11, 11, 7 and 7 microphones at \((4/k_o, \pi/3)\), \((4/k_o, \pi - \pi/3)\), \((5/k_o, \pi/6)\), and \((4/k_o, \pi - \pi/6)\). Then we place a pair of microphones at \((5/k_o, 0)\) and \((5/k_o, \pi)\). This sub array consists of 38 microphones are designed to calculate all odd spherical harmonics up to the 5th order (total of 15 coefficients). Most of these microphones positions could be reused for even coefficients estimation. However, in this design we place three circular arrays on the x-y plane together with a single microphone at the origin to complete the design. We have 7, 11, and 13 microphones in three arrays on x-y plane at radial distances \(2/k_o\) and \(4/k_o\) respectively. We use \(k_o = k_l\exp(1)/2\) to enable the array to operate over an octave of \((k_o, 2k_l)\). We use a total of 70 sensors for the fifth order spherical harmonic extraction array. Note that we could reduce the number of microphones used by reusing some of the circles used for odd coefficients calculations for even coefficients. Also the operating bandwidth could be extended by using the concepts of nested arrays [9].

For simulation, our chosen octave is 3000Hz to 6000Hz \((k_l = 55.44)\) and the speed of sound \(c = 340m/s\). We apply 40dB signal to noise ratio (SNR) at each sensor, where the noise is additive white Gaussian (AWGN). We test our design by estimating all odd spherical harmonics \(\alpha_{n,m}\) for a plane wave sweeping over the entire 3D space and for all frequencies within the desired octave. We plot the real and imaginary parts of \(\alpha_{n,m}\) against the azimuth and elevation of the sweeping plane wave for lower, mid, and upper end of the frequency band. From 36 coefficients, we only show \(\alpha_{5,4}\) in Fig. 4 in this paper. It is evident from Fig. 4 that the array can operate over an octave with measurement noise level of 40dB.

7. REFERENCES


