BLIND CHANNEL ESTIMATION AND EQUALIZATION OF MULTIPLE-INPUT MULTIPLE-OUTPUT CHANNELS

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ABSTRACT
Channel estimation and blind equalization of MIMO (multiple-input multiple-output) communications channels is considered using primarily the second-order statistics of the data. We consider estimation of (partial) channel impulse response and design of finite-length MMSE (minimum mean-square error) blind equalizers. The basis of the approach is the design of a zero-forcing equalizer that whitens the noise-free data. We allow infinite impulse response (IIR) channels. Moreover, the multichannel transfer function need not be column-reduced. Our approaches also work when the “subchannel” transfer functions have common zeros so long as the common zeros are minimum-phase zeros. The channel length or model orders need not be known. The sources are recovered up to a unitary mixing matrix using higher-order statistics of the data. An illustrative simulation example is provided.

1. INTRODUCTION
Consider a discrete-time MIMO system with $N$ outputs and $M$ inputs:

$$y(k) = F(z)w(k) + n(k) = s(k) + n(k)$$  \hspace{1cm} (1-1)

where $y(k) = [y_1(k), y_2(k), \ldots, y_N(k)]^T$, similarly for $s(k)$, $w(k)$ and $n(k)$, and $z$ is the $Z$-transform variable as well as the backward-shift operator (i.e., $z^{-1}w(k) = w(k-1)$), etc.), $s(k)$ is the noise-free output, $n(k)$ is the additive measurement noise and the $N \times M$ matrix $F(z)$ is given by

$$F(z) := \sum_{i=0}^{\infty} F_i z^{-i} = A^{-1}(z)B(z),$$  \hspace{1cm} (1-2)

$$A(z) = I + \sum_{i=1}^{n_a} A_i z^{-i} \text{ and } B(z) = \sum_{i=0}^{n_b} B_i z^{-i}.$$  \hspace{1cm} (1-3)

We allow all of the above variables to be complex-valued. Such models arise in several useful baseband-equivalent digital communications and other applications [1]-[6],[8],[10]-[12],[14]. In these applications one of the objectives is to recover the inputs $w(k)$ given the noisy measurements but not given the knowledge of the system transfer function. A large number of papers (see [4],[5],[10],[11],[14]) have concentrated on a two-step procedure: first estimate the channel impulse response (IR) and then design an equalizer using the estimated channel. A fundamental restriction in these works is that the channel is FIR with no common zeros among the various subchannels. A few (see [1]and [12], e.g.) have proposed direct design of the equalizer bypassing channel estimation. Still they assume FIR (SIMO) channels with no common zeros.

In this paper we allow IIR channels. The MIMO channel does not have to be column-reduced. We will also allow common zeros so long as they are minimum-phase. Finally, in the presence of nonminimum-phase common zeros, our proposed approach equalizes the spectrally-equivalent minimum-phase counterpart of $F(z)$; it does not “fall apart” unlike quite a few existing approaches. Our proposed approach extends the SIMO results of [1] and [8] to MIMO channels.

2. PRELIMINARIES

2.1. FIR Inverses
Assume the following:

(H1) $N > M$.
(H2) $\text{Rank}\{B(z)\} = M \forall z$ including $z = \infty$ but excluding $z = 0$, i.e., $B(z)$ is irreducible [7, Sec. 6.3].
(H3) $\text{det}\{A(z)\} \neq 0 \text{ for } |z| \geq 1$.

It has been shown in [6] (using some results from [2]) that under (H1)-(H3) there exists a finite degree left-inverse (not necessarily unique) of $F(z)$:

$$G(z)F(z) = I_M$$  \hspace{1cm} (2-1)

where $G(z)$ is $M \times N$ given by

$$G(z) = \sum_{i=0}^{L_{se}} G_i z^{-i} \text{ for any } L_{se} \geq n_a + (2M - 1)n_b - 1.$$  \hspace{1cm} (2-2)

2.2. Linear Innovations Representation
Assume further the following:

(H4) Input sequence $\{w(k)\}$ is zero-mean, spatially independent and temporally i.i.d. with each of its components having non-zero fourth cumulants. Take $E\{w(k)w^H(k)\} = I_M$ where $I_M$ is the $M \times M$ identity matrix and the superscript $H$ is the Hermitian operation.

\textbf{Lemma 1.} Under (H1)-(H4), $\{s(k)\}$ may be represented as

$$s(k) = - \sum_{i=1}^{K} D_i s(k-i) + I_s(k)$$  \hspace{1cm} (2-3)

where $K \leq n_a + M n_b$, $D_i$'s are some $N \times N$ matrices and $\{I_s(k)\}$ is a zero-mean white $N \times 1$ random sequence (linear innovations for $\{s(k)\}$) with

$$I_s(k) = F_0 w(k).$$  \hspace{1cm} (2-4)

\textbf{Proof:} See [16]. $\Box$

It follows from (1-1) and Lemma 1 that

$$D(z)s(k) = D(z)F(z)w(k) = F_0 w(k)$$  \hspace{1cm} (2-5)

where $D(z) = I_N + \sum_{i=1}^{K} D_i z^{-i}$. Since $w(k)$ is full-rank and white, it follows from (2-8) that

$$D(z)F(z) = F_0 \Rightarrow (F_0^H F_0)^{-1} F_0^H D(z) F(z) = I_M.$$  \hspace{1cm} (2-6)
Clearly the $M \times N$ polynomial matrix $G(z) := (F_0^0 F_0^T)^{-1} F_2^T D(z)$ is of degree $K \leq n_a + M n_b$ and it is a left inverse to $F(z)$. This result is summarized below.

**Lemma 2.** Under (H1)–(H4), there exists an integer $K \leq n_a + M n_b$ and a polynomial matrix $G(z) = \sum_{i=0}^K G_i z^{-i}$ of degree $K$ such that $G(z) F(z) = I_M$.

**Lemma 3.** Let $R_{zszL_e}$ denote a $[N(L_e + 1)] \times [N(L_e + 1)]$ matrix with its $i$-th block element as $R_{sz}(j-i) = E\{s(k+j-i)z^{-i}(k)\}$. Then under (H1)–(H4), $\rho(R_{zszL_e}) \leq NL_e + M$ for $L_e \geq n_a + M n_b$ where $\rho(A)$ denotes the rank of $A$.

**Sketch of proof:** It follows from Lemma 1 and (2-3) that

$$\begin{bmatrix} I_N & D_1 & \cdots & D_{n_a + M n_b} & 0 & \cdots & 0 \end{bmatrix} R_{zszL_e} \begin{bmatrix} F_0^0 & F_0^T & 0 & \cdots & 0 \end{bmatrix} = 0.$$  \hspace{1cm} (2-7)

Apply Sylvester’s inequality [7, p. 655] to (2-7) to deduce the desired result. \hfill \square

3. **EQUALIZATION: NO COMMON ZEROS**

Assume that (H1)–(H4) hold true. In addition assume the following regarding the measurement noise:

(H5) \{n(k)\} is zero-mean Gaussian with $E\{n(k + \tau) n^T(z^{-i}(k))\} = \sigma_n^2 \delta(\tau)$.

3.1. **Zero-Delay Zero-Forcing Blind Equalizer**

Using (1-2), (2-1) and (2-2), we have

$$\sum_{m=0}^{\infty} G_{m-1} F_i = \begin{cases} I_M, & m = 0, \\ 0, & m = 1, 2, \ldots. \end{cases}$$  \hspace{1cm} (3-1)

leading to

$$\begin{bmatrix} G_0 & G_1 & \cdots & G_{L_e} \end{bmatrix} S = \begin{bmatrix} 1 & 0 & \cdots & \cdots \end{bmatrix}$$  \hspace{1cm} (3-2)

where $S$ is the $(N(L_e + 1)) \times \infty$ matrix given by

$$S = \begin{bmatrix} F_0 & F_1 & F_2 & F_3 & \cdots & \cdots \\ 0 & F_0 & F_1 & F_2 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\ 0 & 0 & \cdots & \cdots & 0 & F_0 \end{bmatrix}. \hspace{1cm} (3-3)$$

Let $S^#$ denote the pseudoinverse of $S$. By [15, Prop. 1], $S^# = S^H(S S^H)^-$. Then the minimum norm solution to the FIR equalizer is given by [15, Sec. 6.11]

$$\begin{bmatrix} G_0 & G_1 & \cdots & G_{L_e} \end{bmatrix} = \begin{bmatrix} F_0^H & 0 & \cdots & 0 \end{bmatrix} (S S^H)^-.$$  \hspace{1cm} (3-4)

In a fashion similar to $R_{zszl_e}$ in Lemma 2, let $R_{zzl_e}$ denote a $[N(L_e + 1)] \times [N(L_e + 1)]$ matrix with its $i$-th block element as $R_{zz}(j-i) = E\{z(k+j-i) z^{-i}(k)\}$; define similarly $R_{nnl_e}$ pertaining to the additive noise. Carry out an eigendecomposition of $R_{zzl_e}$, the smallest $N - M$ eigenvalues of $R_{zzl_e}$ equal $\sigma_n^2$ because under (H1)–(H4), $\rho(R_{zzl_e}) \leq NL_e + M$ whereas $\rho(R_{nnl_e}) = NL_e + N = \rho(R_{zzl_e})$. Thus a consistent estimate $\hat{\sigma}_n^2$ of $\sigma_n^2$ is obtained by taking it as the average of the smallest $N - M$ eigenvalues of $\hat{R}_{zzl_e}$, the data-based consistent estimate of $R_{zzl_e}$.

Under (H4) and (H5),

$$\begin{bmatrix} \hat{S} \hat{S}^H \end{bmatrix} = R_{zzl_e} - R_{nnl_e} = R_{zzl_e} - \sigma_n^2 I.$$

Thus, $(\hat{S} \hat{S}^H)$ can be estimated from noisy data. However, we don’t know $F_0$. To this end, we seek an $N \times N$ FIR filter $G_a(z) := \sum_{i=0}^{L_a} G_a z^{-i}$ satisfying

$$G_{a_0} \quad G_{a_1} \quad \cdots \quad G_{a_{L_a}} = \begin{bmatrix} I_N & 0 & \cdots & 0 \end{bmatrix} R^#_{zszL_e}.$$  \hspace{1cm} (3-6)

Comparing (3-4) and (3-6) it follows that

$$\begin{bmatrix} G_0 & G_1 & \cdots & G_{L_a} \end{bmatrix} = F_0^H \begin{bmatrix} G_{a_0} & G_{a_1} & \cdots & G_{a_{L_a}} \end{bmatrix}$$  \hspace{1cm} (3-7)

leading to

$$\sum_{i=0}^{L_a} G_i z^{-i} = G(z) = F_0^H G_a(z).$$  \hspace{1cm} (3-8)

In practice, therefore, we apply $G_a(z)$ to the data leading to

$$v(k) := G_a(z) y(k) = v_{\ast}(k) + G_a(z) n(k)$$  \hspace{1cm} (3-9)

such that

$$F_0^H v_{\ast}(k) = w(k)$$  \hspace{1cm} (3-10)

where

$$v_{\ast}(k) := G_a(z) [y(k) - n(k)] = G_a(z) s(k).$$  \hspace{1cm} (3-11)

In (3-10) \{w(k)\} is a white $M$-vector sequence (by assumption (H4)), however, \{v_{\ast}(k)\} is not necessarily a white vector sequence. Given the second-order statistics of \{v_{\ast}(k)\}, how does one estimate $F_0$ so that \{w(k)\} satisfying (H4) is recovered? We need to have $R_{w(z)}(\tau) := E\{w(k + \tau) w^T(z^{-i}(k))\} = 0$ for $|\tau| \neq 0$ and $\rho(R_{w(z)}(0)) = M$. By (3-10), $R_{w(z)}(\tau) = F_0^H R_{v_{\ast}(z)}(\tau) F_0$ where $R_{v_{\ast}(z)}(\tau) := E\{v_{\ast}(k + \tau) v_{\ast}^T(z^{-i}(k))\}$. Define ($L > 0$ is some large integer)

$$\hat{R}_{v_{\ast}(z)} := [R_{v_{\ast}(z)}(1) \quad R_{v_{\ast}(z)}(2) \cdots \cdots R_{v_{\ast}(z)}(L) ; Q^T]^T$$  \hspace{1cm} (3-12)

where $Q = [q_{i-1}; \cdots; q_N]$, $r = \mu(R_{v_{\ast}(z)}(0))$ with $M \leq r \leq N_q$, $q_i$’s ($r + 1 \leq i \leq N$) are orthonormal eigenvectors of $R_{v_{\ast}(z)}(0)$ corresponding to zero eigenvalues, and the symbol $*$ denotes the complex conjugation operation. Note that if $r = N_q$, $Q$ is omitted from ([3-12]). Let $F_0$ and $F_{on}$ be the orthogonal projections of $F_0$ onto the $r$-dimensional ‘signal subspace’ (range space) and $(N - r)$-dimensional ‘noise subspace’ (null space), respectively, of $R_{v_{\ast}(z)}(0)$. By (3-10), $\rho(F_0) = M$. Then $F_0 = F_0 + F_{on}$ with $F_0^H F_{on} = 0$ and $R_{v_{\ast}(z)}(0)F_{on} = 0$. It then follows that $E\{F_0^H v_{\ast}(k) v_{\ast}^T(z^{-i}(k))F_{on}\} = F_0^H R_{v_{\ast}(z)}(0)F_{on} = 0$; hence, $F_0^H v_{\ast}(k) = 0$ with probability one (w.p.1) and (cf. (3-10))

$$F_0^H v_{\ast}(k) = w(k).$$  \hspace{1cm} (3-13)

**Lemma 4.** $R_{v_{\ast}(z)}$ is rank deficient for any $L \geq 1$ such that $R_{v_{\ast}(z)} F_0 = 0$ and $\rho(F_0) = M$.

**Proof:** By construction $Q^T F_{on} = 0$ as columns of $Q$ span the noise-subspace of $R_{v_{\ast}(z)}(0)$ and $F_0$ is the orthogonal projection of $F_0$ onto the $r$-dimensional signal subspace of $R_{v_{\ast}(z)}(0)$. Furthermore, we have

$$R_{v_{\ast}(z)}(\tau) = E\{w(k + \tau) w^T(z^{-i}(k))\} = 0 \quad \forall \tau \geq 1$$  \hspace{1cm} (3-14)

because $v_{\ast}(k)$ is obtained by causal filtering of $y(k)$, hence of $w(k)$. Using (3-13) in (3-14) it then follows that there exists a $N \times M$ matrix $F_{on} \neq 0$ such that

$$F_0^H R_{v_{\ast}(z)}(\tau) = 0 \quad \forall \tau \geq 1 \Rightarrow R_{v_{\ast}(z)} F_0 = 0 \quad \forall \tau \geq 1 \Rightarrow$$  \hspace{1cm} (3-15)
The desired result is then immediate. \(\blacksquare\)

Pick a \(N \times M\) column vector \(H_0\) to equal the rightmost \(M\) right singular vectors in a singular-value decomposition (SVD) \(R_m, \nu_m = U \Sigma N, \nu_m\), i.e., the right singular vectors corresponding to the \(M\) smallest singular values. Therefore, \(\rho(H_0) = M\). Then since ideally the \(M\) smallest singular values of \(R_m, \nu_m\) are zero, we have \(H^\nu_m R_m, \nu_m(\tau)H_0 = 0\) for \(\tau = 1, 2, \cdots, L\). This, in turn, implies that
\[
(H^\nu_m R_m, \nu_m(\tau)H_0)^\nu_m = 0 \quad \text{for} \quad \tau = 1, 2, \cdots, L. \quad (3 - 16)
\]

Moreover \(Q^\nu_m H_0 = 0\). An eigendecomposition yields
\[
R_m, \nu_m(0) = \sum_{i=1}^r \sigma_i q_i^\nu_m Q i \Sigma D Q^T \quad (3 - 17)
\]

where \(\tilde{Q} = [q_1, \cdots, q_r]\), \(q_i\)'s \((1 \leq i \leq r)\) are orthonormal eigenvectors of \(R_m, \nu_m(0)\) corresponding to non-zero eigenvalues \(\sigma_i\)'s and \(\Sigma = \text{diag} [\sigma_1, \cdots, \sigma_r]\). Thus columns of \(\tilde{Q}\) span the signal subspace of \(R_m, \nu_m(0)\) and the columns of \(Q\) span the noise subspace of \(R_m, \nu_m(0)\). Since \(Q^\nu_m H_0 = 0\), we have \(H_0 = \tilde{Q} C\) where \(C = \tau \times M\) and \(\rho(C) = M\). Then \(H^\nu_m R_m, \nu_m(0) H_0 = C \tilde{Q} C\) where \(C = \Sigma^{-1/2} C\) and \(\rho(C) = M\). Finally, by result R11 on p. 261 of [9], \(\rho(C \tilde{Q}) = \rho(\tilde{Q})\), and therefore, by Sylvester's inequality [7, p. 655], \(\rho(H^\nu_m R_m, \nu_m(0) H_0) = M\).

The overall system with \(W\) as input and \(H^\nu_m v_m(\tau)\) as output has the transfer function
\[
H^\nu_m G_\nu_m(z)A^{-1}(z)B(z) \quad (3 - 18)
\]

and therefore, is an autoregressive moving average (ARMA) model with autoregressive (AR) order no more than \(N a\) and moving average (MA) order no more than \(N a + L a + (N - 1) a\), denoted by ARMA\((N a, N a + L a + (N - 1) a)\). Therefore, it follows from (3-16) that \(H^\nu_m v_m(0)\) is zero-mean white if \(L \geq N a + L a + (N - 1) a\). Moreover, since \(v_m(\tau)\) is obtained by causal filtering of \(w(k)\), it follows that \(H^\nu_m v_m(\tau) = \sum_{\tau = 0}^\infty P_m w(k-\tau)\) where \(M \times M\) \(P_m = \sum_{\tau = 0}^\infty P_m z^{-\tau}\) is stable satisfying \(P_m(1) = I_M\) \(\forall M, 0 < a\). Therefore, we have \(E(H^\nu_m w(k-\tau)w^H(k-\tau)) = P_m\). Using (3-13), we have \(E(H^\nu_m v_m(\tau)w^H(k-\tau)) = H^\nu_m R_m, \nu_m(\tau)F_{m,s} = 0\) (by construction of \(H_0\) for \(\tau \geq 1\)). Thus \(P_m\) verifies the above. Therefore, we have
\[
H^\nu_m v_m(\tau) = P_m w(k) \quad \text{such that} \quad \rho(P_m) = M. \quad (3 - 19)
\]

By (3-11) and (3-19), we have
\[
H^\nu_m G_\nu_m(z)y(k) = P_m w(k) + H^\nu_m G_\nu_m(z)u(k) \ni \rho(P_m) = M. \quad (3 - 20)
\]

Since \(P_m, U^H P_m^H = P_m, P_m^H\) for any unitary matrix \(U\), one can not uniquely determine \(P_m\) from (3-20) given second-order statistics of data \(y(k)\) and knowledge (estimates) of \(H_0, filter G_\nu_m(z)\) and noise variance \(\sigma_n^2\). One has to exploit higher-order statistics (HOS) of data. The model (3-20) is an instantaneous mixture model [13], therefore, any existing method may be applied to estimate \(P_m\) given (3-20). In this paper we have used the joint diagonalization procedure of [13]. The required estimate of \(P_m\) is obtained as
\[
P_m = W^{-1} U \quad (3 - 21)
\]

where \(W\) "diagonalizes" \(H^\nu_m R_m, \nu_m(0)H_0\) into an identity matrix and \(U\) is a unitary matrix obtained via the joint diagonalization procedure of [13] using fourth-order cumulants of \(W H^\nu_m G_\nu_m(z)y(k)\). Let \(L_i (i = 1, 2, \cdots, M)\) denote the orthonormal eigenvectors of \(H^\nu_m R_m, \nu_m(0)H_0\) with the corresponding eigenvalues \(\gamma_i\)'s. Set \(L = [l_1, \cdots, l_M] \text{ and } \Gamma = \text{diag}(\gamma_1, \cdots, \gamma_M)\). Then
\[
W = \Gamma^{-1/2} L^H \quad (3 - 22)
\]
diagonalizes \(H^\nu_m R_m, \nu_m(0)H_0\) to an identity matrix: \(W (H^\nu_m R_m, \nu_m(0)H_0) W^H = I_M\). Finally, we have
\[
H^\nu_m v_m(\tau) = w(k) \quad (3 - 23)
\]

**Remark 1.** Using (3-11) and (3-23) it follows that \(H^\nu_m \sum_{i=0}^{L_m} G\nu_m s(k - i) = w(k)\). The filter \(H_0, G_\nu_m(z)\) whitens the noise-free received signal. Moreover, the derivation of this filter was based upon whitening of \(H^\nu_m G_\nu_m(z)\).

These considerations motivate the name a whitening approach for the proposed technique. Our approach is far more structured and different than that of [12]. \(\blacksquare\)

### 3.2. MMSE Equalizer with Delay \(d\)

Using the orthogonality principle, the MMSE equalizer of length \(L_a + 1\) to estimate \(w(k - d) (d \geq 0)\) based upon \(y(n), n = k, k - 1, \cdots, k - L_a\), satisfies
\[
[ \mathbf{G}_{d,0} \mathbf{G}_{d,1} \cdots \mathbf{G}_{d,L_a} ] = \begin{bmatrix} F_{d}^T & F_{d-1}^T & \cdots & F_{0}^T \end{bmatrix} \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix} \mathbf{R}^{-1}_{yy} \mathbf{L}_a. \quad (3 - 24)
\]

Clearly one can obtain a consistent estimate of \(R_{yy} L_a\) from the given data. It remains to estimate \(F_d\)'s to complete the design. Here the discussion of Sec. 3.1 becomes relevant. From (3-11) and (3-23) we have
\[
H_0^\nu_m v_m(n) = \sum_{i=0}^{L_m} H_0^\nu_m G\nu_m s(n - i). \quad (3 - 25)
\]

Using (3-25) and taking expectations we have
\[
E\{s(n) v_m(n - \tau)\}H_0 = \sum_{i=0}^{L_m} L_{s,\tau}(\tau + i) G\nu_m s(n - i) \quad (3 - 26)
\]

Using (1-1), (1-2) and (3-23) we have
\[
E\{s(n) v_m(n - \tau)\}H_0 = F_{\nu_m} \quad (3 - 27)
\]

Hence, we have from (3-26) and (3-27)
\[
F_{\nu_m} = H_0^\nu_m \sum_{i=0}^{L_m} G\nu_m s(n - i) \quad (3 - 28)
\]

Substitute the results of (3-28) for \(\tau \geq 0, 1, \cdots, d\) in (3-24) to complete the design. The MMSE estimate \(\hat{w}(t - d)\) of \(w(t - d)\) is then given by \(\hat{w}(t - d) = \sum_{i=0}^{L_a} \mathbf{G}_{d,i} y(t - i)\).

### 4. COMMON MINIMUM-PHASE ZEROS

Here the MIMO transfer function is
\[
\mathcal{X}(s) = A^{-1}(s)B(z)B_s(z), \quad B_s(z) = \sum_{i=0}^{N_{sc}} B_{sc} z^{-i} \quad (4 - 1)
\]

where \(B(z)\) satisfies (H2) and \(B_s(z)\) is a finite-degree \(M \times M\) polynomial that collects all the common zeros/factors of the subchannels. Assume that
Given model (4-1), \( \det(B_c(z)) \neq 0 \) for \(|z| \geq 1 \).

Then while \( A^{-1}(z)B(z) \) has a finite left-inverse, \( B_c^{-1}(z) \) is IIR though causal under (H6). Then (3-2) holds true approximately for “large” \( L_e \), the approximation getting better with increasing \( L_e \). Similarly Lemmas 1 and 2 hold true approximately for “large” \( K \) and Lemma 3 also holds true approximately for \( L_e \geq K \). Note also that \( \rho(F_0) = M \) where \( F_0 = B_cB_{c0} \) since, by (H6), \( \rho(B_c) = M \) (evaluate \( \det(B_c(z)) \) at \( z = \infty \)). It is then readily seen that the developments of Sec. 3 apply to the current case also.

\[
\begin{align*}
\text{Prob. of Symbol Error} \\
\text{SNR (dB)}
\end{align*}
\]

\[
\begin{align*}
\text{Equalization MSE} \\
\text{SNR (dB)}
\end{align*}
\]

Fig. 1. Normalized MSE and probability of symbol detection error \( P_e \) for the two users after MMSE equalization with \( d = 3 \). Record length \( T = 500 \) symbols for equalizer design. Averages over 100 Monte Carlo runs. The designed equalizer was applied to record lengths of 3000 symbols for performance evaluation.

5. SIMULATION EXAMPLE

We consider a wireless communications scenario with two \((M = 2)\) 4-QAM user signals arriving at a uniform linear array (half-wavelength spacing) of \( N \) = 4 sensors via a frequency selective multipath channel. The signaling pulse shape for both the users was a raised-cosine pulse with a roll-off factor of 0.5, the pulse being truncated to a length of \( 4T_s \), where \( T_s \) = symbol duration. The array measurements are assumed to be sampled at baud rate with sampling interval \( T \), seconds and the two sources have the same baud rate. The relative time delay \( \tau \) (relative to the first arrival), the angle of arrival \( \theta \) (in degrees w.r.t. the array broadside) and the relative attenuation factor (amplitude) \( \alpha \), \( (\tau, \theta, \alpha) \), for the two sources were selected as:

\[
\begin{align*}
w_1 : & \quad (0T_s, 40^\circ, 1.0), (0.3T_s, 20^\circ, 1.0), (0.6T_s, -20^\circ, 1.0) \\
w_2 : & \quad (0T_s, 10^\circ, 1.0), (1.1T_s, -15^\circ, 1.0), (1.6T_s, -1^\circ, 1.0).
\end{align*}
\]

Sampling of received signal at the array leads to a discrete-time MIMO FIR model \( B(z) \) with \( N = 4 \), \( M = 2 \) and \( n_0 = 4 \) such that \( B_{c1} \neq 0 \) and \( B_{c2} \neq 0 \) (see Sec. 1). The effective MIMO channel was taken to be

\[
F(z) = B(z)B_c(z) \quad \text{where} \quad B_c(z) = (1 - 0.5z^{-1})T_s.
\]

The part \( B_c(z) \) in (5-2) may represent some filtering at the transmitter or receiver, and it leads to a system with common zeros in the two subchannels.

An MMSE equalizer of length \( L_e = 7 \) (8 taps per sub-channel, totaling 32 taps) was designed with a delay \( d = 3 \) for each of the two sources. Fig. 1 shows the results of simulations for a record length of \( T = 500 \) symbols. It is seen that the proposed approach works quite well. Also presence of a common (minimum-phase) zero has not caused any problems.

6. REFERENCES