ROBUST IMPULSE NOISE SUPPRESSION USING ADAPTIVE
WAVELET DE-NOISING

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ABSTRACT

It is widely acknowledged that the effect of
impulsive noise is a major source of performance
degradation within a wide range of communication
systems. This is due to the fact that non-Gaussian
interference is neglected within the system design
philosophy for reasons of complexity and tractability. In
this paper, we directly address this problem using a
novel ‘de-noising’ technique in which significant
performance gains are achieved with low-complexity.

1. INTRODUCTION

The performance evaluation of communication systems
has traditionally relied on the assumption of an
additive Gaussian noise channel. However, in
numerous circumstances this assumption is not always
justifiable and the communication medium can be more
accurately modelled by heavy-tailed, non-Gaussian
distributions. One process which is not adequately
described in terms of the Gaussian assumption is the
process that generates impulsive noise bursts. For
example, noise experienced on radio channels typically
comprises infrequent, high amplitude pulses associated
with either man-made or natural sources, superimposed on a more homogeneous (Gaussian)
background. Consequently, the presence of impulsive
noise is a major source of performance degradation
when discrete-time, linear detection schemes such as the
matched filter are used; this is largely due to the
existence of non-Gaussian interference being neglected
in the design philosophy.

A technique commonly used to suppress impulsive
interference involves passing the received data samples
through a memoryless nonlinearity. Typical nonlinear
functions are the hard-limiter and the Gaussian-tailed
nonlinearity. These suppress large excursions from the
wanted signal level by weighting the received data
samples prior to matched filter detection. Although this
approach to noise suppression is not based on any
optimal criteria, it is justified in that an increased
signal-to-noise ratio usually results when a suitable
threshold is chosen. In addition, limiting techniques
incur no excessive computational complexity.

Synthesis of the optimum receiver, based on the
Bayesian theory of signal detection, requires a priori
knowledge of the underlying noise process. Unfortunately, this information is generally not
available in any realistic application. Furthermore,
impulsive noise is highly dependent on the physical
environment and is also non-stationary. Consequently,
the process of obtaining an accurate statistical model
proves difficult and renders the optimum solution to the
signal detection problem impossible/unattainable.
Hence, the main objective of this paper is to present a
detector design which is robust in the presence of
impulsive noise, and can be implemented in a low-
complexity real-time architecture.

During recent years, the signal processing
community has developed a renewed interest in the
design of structured bases for the linear expansion of
signals. In particular, the subject of wavelets and
time-scale analysis has received increasing interest as a new
method of expanding functions onto a set of self-similar,
orthonormal basis functions [1]. This is largely due to
the fact that such techniques offer increased flexibility
over more traditional transform methods, combined
with the existence of efficient computational structures,
in the form of multirate filter banks, which allow rapid
calculation of the expansion coefficients.

Recent advances in the application of wavelets
include Donoho and Johnstone’s novel approach to
signal recovery in additive white Gaussian noise
(AWGN) [2]. Here, a simple thresholding technique
involving a ‘keep (shrink) or kill’ policy is applied to the
expansion coefficients. Both hard and soft-thresholding
procedures have been examined which kill the wavelet
coefficients corresponding to AWGN whilst retaining
the larger coefficients corresponding to signal features.
In this paper, we present a novel de-noising technique
which differs from that in [2] by focusing on the
suppression of non-Gaussian, impulsive noise.
Furthermore, when incorporated within a digital radio
receiver, considerable performance gains over a variety of
non-Gaussian radio channels are obtained.

The paper is organised as follows: in Section 2, the
noise model is discussed; in Section 3, the proposed
detector based on a de-noising technique specific to non-
Gaussian interference is explained in detail; in Section
4, simulation results are given; finally, in Section 5,
conclusions are drawn from the work described in this
paper.

Notation: A convenient way of analysing multirate
filter banks is in terms of a time-domain operator.
Here, for example, we have adopted the following
notation; A column vector \(\mathbf{x}\) of length \(n\) is denoted by
\(\mathbf{x}^{(n)}\), while filtering with \(h_i(n), i \in \{0,1\}\) followed by
two-fold subsampling is denoted by
\(\mathbf{H}_i^{(m,n)}, i \in \{0,1\}\),
where \(\mathbf{H}_i^{(m,n)}, i \in \{0,1\}\) is a matrix of dimension \(m \times n\)
whose row entries are even shifted versions of the filter
impulse response \(h_i(n), i \in \{0,1\}\). We consider the case

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when the filter coefficients are purely real, and when
the filter bank computes an orthonormal discrete
wavelet transform. Now for a signal containing \( N = 2^n \)
samples, the filter bank structure comprises
\( J = \log_2(N) - 1 \) stages, and the wavelet expansion
coefficients at the \( j \)th level are given by
\[
\alpha_j^{(N/2^j)} = H_j^{(N/2^j, N/2^{j-1})} \prod_{i=j-1}^{1} H_i^{(N/2^i, N/2^{i-1})} \mathbf{x}^{(N)}, \quad j = 1, ..., J,
\]
while the approximation coefficients are computed as
\[
\beta_j^{(N/2^j)} = \prod_{i=j}^{1} H_i^{(N/2^i, N/2^{i-1})} \mathbf{x}^{(N)}, \quad j = 1, ..., J.
\]

2. THE NOISE MODEL

The model describing impulsive noise is based on the
expanded noise model in [3]. Here, the background
noise component is described statistically by a zero-
mean, Gaussian random process having variance \( \sigma_n^2 \),
while the non-Gaussian, impulsive component is
described by a train of randomly arriving delta
functions whose amplitude is governed by a heavy-
tailed density. Hence, the \( n \)th sample at the receiver
front-end, \( y(n) \), can be expressed as
\[
y(n) = x(n) + w(n) + i(n),
\]
where \( x(n) \) is the \( n \)th transmitted sample, \( w(n) \) is
AWGN and \( i(n) \) is the impulsive noise. Here, all
components of \( y(n) \) are assumed to be mutually
uncorrelated. The impulsive noise component \( i(n) \) is
modelled as
\[
i(n) = s(n) h(n),
\]
where \( h(n) \) is an ever present impulse whose amplitude
is governed by a Laplacian density, and \( s(n) \) is a
switching process of ones and zeros. If \( s(n) \) is a one
(zero) then an impulse is (is not) present. The switching
mechanism is chosen to be a Poisson random process,
where the mean time of impulse arrival is governed by
the variance \( \lambda_p \).

Unless otherwise stated, we will adopt a Laplacian
density having zero-mean and variance 100, while the
mean time of impulse arrival is 30 ms.

3. THE PROPOSED DETECTOR

Donoho and Johnstone have recently developed a
powerful noise reduction technique [2] in which they
adopt a thresholding strategy in conjunction with the
discrete wavelet transform. The technique has been
applied successfully to both 1 and 2-dimensional data,
and is proven to be near-optimal for a wide class of
signals corrupted by AWGN. Summarising their
results, they consider a discretised signal \( x \) of length
\( N \), which is corrupted by zero-mean AWGN having
variance \( \sigma^2 \). Hence, using the matrix notation defined
previously, we can formulate the problem more explicitly as
\[
y^{(N)} = x^{(N)} + \sigma \mathbf{w}^{(N)},
\]
where the main objective is to recover the signal vector
\( x \) from the noisy observations \( y \). On computing the
wavelet expansion coefficients of \( y \) using
\[
\alpha_j^{(N/2^j)} = H_j^{(N/2^j, N/2^{j-1})} \prod_{i=j-1}^{1} H_i^{(N/2^i, N/2^{i-1})} \mathbf{y}^{(N)},
\]
\[
j = 1, ..., \log_2(N) - 1,
\]
a 'keep (shrink) or kill' policy is then applied to the
individual wavelet coefficients prior to computing the
inverse transform.

Donoho and Johnstone consider two thresholding
strategies; hard-thresholding in which \( \alpha_j \) is kept if it is
above some threshold \( T \), else it is set to zero, i.e.
\[
\hat{\alpha}_j^{(k)} = T_H(\alpha_j^{(k)}, T) = \begin{cases} \alpha_j^{(k)}, & |\alpha_j^{(k)}| \geq T \\ 0, & |\alpha_j^{(k)}| < T \end{cases},
\]
\[
j = 1, ..., J, \quad k = 1, ..., N/2^j ;
\]
and soft-thresholding which additionally 'shrinks' those
values of \( \alpha_j \) by \( T \) which are not set to zero, i.e.
\[
\hat{\alpha}_j^{(k)} = T_S(\alpha_j^{(k)}, T) = \begin{cases} \text{sgn}(\alpha_j^{(k)}) \left| \alpha_j^{(k)} \right| - T, & |\alpha_j^{(k)}| \geq T \\ 0, & |\alpha_j^{(k)}| < T \end{cases},
\]
\[
j = 1, ..., J, \quad k = 1, ..., N/2^j .
\]
The threshold \( T \) is chosen as \( T = \sigma \sqrt{2 \ln(N)/N} \), where
an estimate of the noise standard deviation is derived
from the wavelet expansion coefficients. Furthermore,
soft-thresholding is usually chosen in favour of its
hard-thresholding counterpart, in order to avoid the
generation of spurious oscillations when computing the
inverse transform.

The de-noising technique presented so far applies
specifically to the problem of signal recovery in AWGN.
However, we are concerned with the design of a low-
complexity receiver, which is robust in the presence of
impulsive noise typically encountered on radio
channels. Hence, the work presented here differs from
that in [2] by considering a thresholding strategy which
operates on large wavelet expansion coefficients,
generated by impulsive phenomena. The advantages
of this method include improved receiver performance in
non-Gaussian conditions, along with robustness to a
wide variety of contaminating densities giving rise to
outliers, and varying degrees of impulsivity.
We first assume that the detector has a reliable estimate of the background noise variance $\sigma^2$, which is derived from a real-time channel evaluation (RTCE) procedure [4]. Following this, a running estimate is computed by separating the received data into Gaussian and impulsive components using a threshold test, based on the current noise variance estimate. It is important to remember that this adaptive estimation process is based on the assumption that the noise is independent, and that the receiver has a priori knowledge of the information bearing waveform’s variance.

The proposed detector, in the first instance, reads in $N = 2^{k-1}$ samples, and performs the following test

$$\left| y^{(N)} \right| \geq T,$$

where $T$ is a threshold derived from the estimate of the background noise variance. If the test fails, no further action is taken. However, if the test holds, then a contaminating density giving rise to outliers is present. The block of received data samples is then transformed using an orthonormal wavelet basis. Here, the tiling of the time-frequency plane differs from that induced by the traditional discrete wavelet transform, by iterating off the highpass component of the filter bank structure, i.e. the wavelet expansion coefficients are computed as

$$a^{(N/2^j)}_j = \mathcal{H}_0^{(N/2^j)} \left[ \prod_{k=1}^{t} \mathcal{H}_1^{(N/2^j,N/2^j-1)} \right] y^{(N)},$$

where $\gamma \in \mathbb{R}^+$. Hence, corrupted coefficients are simply replaced by a scaled version of Donoho and Johnstone’s universal threshold. Selection of the threshold $T$ is chosen on the grounds that $3\sigma = \sigma$ encompasses all but 0.3% of the Gaussian density’s support. Hence, if any coefficients exceed $3\sigma$, then it is assumed that a contaminating density giving rise to outliers is present.

Finally, the inverse transform is computed. On removing the impulsive component, the preprocessed signal is then assumed to be contaminated by AWGN only. Based on this assumption, the problem of detection in the impulse environment is reduced to detection in a Gaussian environment. Consequently, the preprocessed signal is demodulated using matched filter detection.

### 4. SIMULATION RESULTS

We now test the proposed detector by simply observing the noise statistics at the input and output of the de-noising algorithm. Here, we have adopted the Haar basis as a result of its time-localisation properties; furthermore, minimal computational complexity is incurred. Figure 2 contains the probability density function (pdf) of simulated impulsive noise, where the background noise level results in $\frac{\sigma}{\sigma_n} = 10$ dB, while Figure 3 contains the resulting noise pdf after de-noising. Overlaid on each of these is the pdf of Gaussian noise. From Figures 2 and 3, it can be seen that while the pdf of impulsive noise is heavier in the tails compared with Gaussian noise, after de-noising the resulting pdf shows a good fit.
matched filter performance is possible when the noise process of the communicating medium is non-Gaussian.

REFERENCES


ACKNOWLEDGEMENT

The authors wish to express their thanks to the Defence Research Agency at Portsdown UK for supporting the work detailed in this paper.

5. CONCLUSIONS

A novel de-noising algorithm for suppressing impulsive noise has been presented. The resulting algorithm serves as a preprocessing unit to enhance matched filter performance and is modulation independent. The algorithm has been incorporated within a digital radio receiver, and simulation results show that extended